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WEYL'S THEOREM FOR CLASS $A(k)$ OPERATORS

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Abstract. In this paper we shall show that Weyl's theorem holds for class $A(k)$ operators T where $k \geq 1$, via its hyponormal transform $\hat{T}$. Next we shall prove some applications of Weyl's theorem on class $A(k)$ operators.

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1. Preliminaries. Let H be a complex Hilbert space and $B(H)$ the algebra of all bounded linear operators on H. An operator $T \in B(H)$ has a unique polar decomposition $T = U|T|$, where $|T| = (T^*T)^{\frac{1}{2}}$ and U is the suitable partial isometry satisfying $\mathrm{Ker}\,U = \mathrm{Ker}(T) = \mathrm{Ker}(|T|)$ and $\mathrm{Ker}(U^*) = \mathrm{Ker}(T^*)$.

An operator $T \in B(H)$ is said to be *hyponormal* if $T^*T \geq TT^*$, where T^* is the adjoint of T. As a generalisation of hyponormal operators, p-hyponormal and log hyponormal operators have been introduced in [**2**] and [**13**], respectively. An operator T is said to be *p-hyponormal* if $(T^*T)^p \geq (TT^*)^p$ for a positive number p and log-hyponormal if T is invertible and $\log(T^*T) \geq \log(TT^*)$. Furuta et al. [**13**] defined a new class of operators; namely class $A(k)$, where $k > 0$. T belongs to class $A(k)$ if $(T^*|T|^{2k}T)^{\frac{1}{k+1}} \geq |T|^2$, where $k > 0$. A class $A(1)$ operator T is known as a class A operator and satisfies an operator inequality $|T^2| \geq |T|^2$. As a generalisation of class $A(k)$ operators, Fujii et al. [**12**] introduced class $A(s, t)$ operators. For positive numbers s and t, T belongs to class $A(s, t)$ if $(|T^*|^t|T|^{2s}|T^*|^t)^{\frac{t}{s+t}} \geq |T^*|^{2t}$. It has been shown that a class $A(k, 1)$ operator is a class $A(k)$ operator [**22**]. Since many properties of hyponormal operators are known, by giving a hyponormal transform from a class $A(k)$ operator T to a hyponormal operator $\hat{T}$, we can study the properties of T via $\hat{T}$ [**18**].

The following inclusion relation holds among these operators.

$$\begin{aligned}\{hyponormal\} &\subset \{p-hyponormal, 0 < p < 1\}\ [\mathbf{17}]\\ &\subset \{classA(s, t), s, t \in (0, 1]\}\ [\mathbf{12}]\\ &\subset \{classA\}\ [\mathbf{17}]\\ &\subset \{classA(k), k \geq 1\}\ [\mathbf{13}]\end{aligned}$$

Now $T \in B(H)$ is called a *Fredholm operator* if TH is closed and both $\mathrm{Ker}T$ and $\mathrm{Ker}T^*$ are finite dimensional. For any Fredholm operator T, there corresponds an integer called the *index of* T denoted by $\mathrm{ind}(T) = \mathrm{dimKer}T - \mathrm{dimKer}T^*$. Let F_0 denote the class of all Fredholm operators in $B(H)$ with index 0. Then

$w(T) = \{\lambda \in C : T - \lambda \notin F_0\}$ is called the *Weyl spectrum* of T. We denote the spectrum, the point spectrum, the normal point spectrum, the approximate point spectrum, the normal approximate point spectrum and the set of all isolated eigenvalues of finite multiplicity by $\sigma(T)$, $\sigma_p(T)$, $\sigma_{np}(T)$, $\sigma_a(T)$,$\sigma_{na}(T)$, and $\pi_{00}(T)$, respectively.

Weyl's theorem. According to Coburn [7], Weyl's theorem holds for T if $\sigma(T)\backslash w(T) = \pi_{00}(T)$.

In general, Weyl's theorem does not hold for all operators. Some examples are given below.

THEOREM R_1 [7]. *If T is hyponormal, then* $w(T)$ *consists of all points in* $\sigma(T)$ *except the isolated eigenvalues of finite multiplicity.*

THEOREM R_2 [**4**]. *Let T be a p-hyponormal operator on H, where* $0 < p < 1$. *Then Weyl's theorem holds for T.*

THEOREM R_3 [**20**]. *If T belongs to class A and* $KerT|_{[TH]} = \{0\}$, *then Weyl's theorem holds for T.*

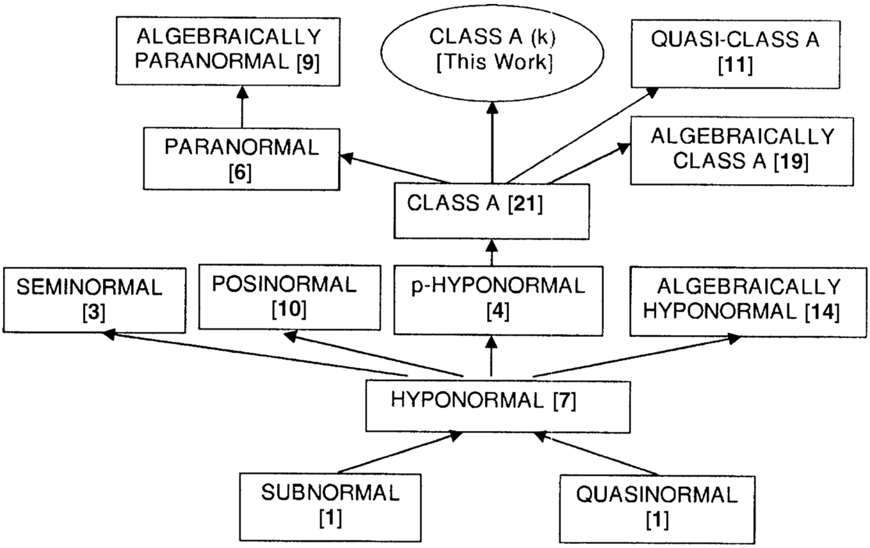

Figure 1. Operators satisfying Weyl's theorem.
(References are shown within parentheses.)

Figure 1 shows the inclusion relation between operators that satisfy Weyl's theorem.

In [**18**] [Theorems 6 and 7, Corollaries 3 and 5], we have proved that if a class $A(k)$ operator T with $k > 1$ satisfies the Limit Condition, then (i)$\sigma_a(T) = \sigma_{na}(T) = \sigma_{na}(\hat{T})$, (ii) $\sigma_p(T) = \sigma_{np}(T) = \sigma_p(\hat{T})$ and (iii) $\sigma(T) = \sigma(\hat{T})$ hold. However, these results hold for class $A(1)$ operators without any such condition [**5**, Theorem 2 and Corollary 5]. Since we need these results to prove Weyl's theorem, we first prove Weyl's theorem for class $A(1)$ operators (class A operators) without Limit Condition, as a particular case and then for class $A(k)$ operators $k > 1$ with Limit Condition, as a general one.

2. Weyl's theorem for class A **operators.** The main result of the paper follows.

THEOREM 1. *Weyl's theorem holds for class A operators.*

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We say that $T \in B(H)$ is *isoloid* if every isolated point of $\sigma(T)$ is in the point spectrum of T [**3**]. Also if every restriction $T|_M$ to its reducing subspace M is *isoloid*, then we say that T *satisfies the condition* (α''') [**3**]. We say that T is *reduction isoloid* if it satisfies the condition (α''').

We need the following propositions to prove Theorem 1.

PROPOSITION 1 [Berberian [**3**]]. *If $T \in B(H)$ satisfies the condition (α''') and if every finite dimensional eigenspace of T reduces T, then Weyl's theorem holds for T.*

PROPOSITION 2 [Hansen's inequality [**16**]]. *If $A \geq B \geq 0$, then $(B^*AB)^\delta \geq B^*A^\delta B$, for all $\delta \in (0, 1]$.*

PROPOSITION 3 [**21**]. *If T is a class A operator and M is an invariant subspace of T, then $T|_M$ is also a class A operator.*

PROPOSITION 4 [**18**]. *If $T = U|T|$ is the polar decomposition of a class $A(k)$ operator, where $k \geq 1$, then $\hat{T} = WU\left||T|^kT\right|^{\frac{1}{k+1}}$ is hyponormal, and $|T||T^*| = W||T||T^*||$ is the polar decomposition.*

PROPOSITION 5 [**5**, Theorem 2]. *If T is a class A operator, then $\sigma(T) = \sigma(\hat{T})$.*

PROPOSITION 6 [**5**, Corollary 5]. *If T is a class A operator, then $\sigma_p(T) = \sigma_p(\hat{T})$.*

LEMMA 1. *Let T be a class A operator. Then $\lambda \in \sigma(T)$ is an isolated point $\Longleftrightarrow$ λ is an isolated point of $\sigma(\hat{T})$.*

Proof. $\lambda \in \sigma(T)$ is an isolated point
$\Longleftrightarrow \exists$ a neighbourhood V of λ such that $(V \cap \sigma(T)) - \{\lambda\} = \phi$
$\Longleftrightarrow (V \cap \sigma(\hat{T})) - \{\lambda\} = \phi$ by Proposition 5
$\Longleftrightarrow \lambda$ is an isolated point of $\sigma(\hat{T})$. □

LEMMA 2. *If T is a class A operator and λ is a complex number, then $(T - \lambda)x = 0$ implies that $(T - \lambda)^*x = 0$, where $x \in H$.*

Proof. We have $(T - \lambda)x = 0$. By Proposition 6, $(\hat{T} - \lambda)x = 0$. Since $\hat{T}$ is hyponormal, $(\hat{T} - \lambda)^*x = 0$ and hence $(|\hat{T}|^2 - |\lambda|^2)x = 0$. By Proposition 4, $\hat{T} = WU|T^2|^{\frac{1}{2}}$ and $(\hat{T})^* = |T^2|^{\frac{1}{2}}(WU)^*$. We obtain $|\hat{T}|^2 = |T^2|$ and $(|T^2| - |\lambda|^2)x = 0$. That is $(T^*)^2T^2x = |\lambda|^4x$. Since, by hypothesis, $T^2x = \lambda^2x$, we have $(T^*)^2x = (\overline{\lambda})^2x$. It follows that $(T - \lambda)^*x = 0$. □

LEMMA 3. *If T is a class A operator, then T is isoloid and satisfies the condition (α''').*

Proof. Let λ be an isolated point of $\sigma(T)$. Then the range of the Riesz projection $E = \frac{1}{2\pi i}\int_{\partial D}(zI - T)^{-1}dz$ is a closed invariant subspace for T and $\sigma(T|_{EH}) = \{\lambda\}$. Here D is a closed ball with center λ such that $\sigma(T)\bigcap D = \{\lambda\}$ and ∂D is the boundary of D described once counterclockwise. By Lemma 1, λ is an isolated point of $\sigma(\hat{T})$. Since $\hat{T}$ is hyponormal, and hence isoloid, λ is in the point spectrum of $\hat{T}$. By Proposition 6, $\sigma_p(T) = \sigma_p(\hat{T})$ and this implies that $\lambda \in \sigma_p(T)$. Therefore T is isoloid. By Proposition 3, $T|_{EH}$ is a class A operator and hence isoloid. Therefore T satisfies the (α''') condition. □

Proof of Theorem 1. If T is a class A operator, then by Lemma 2 every finite dimensional eigenspace of T is a reducing subspace of T. Also T satisfies the (α''')

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condition by Lemma 3 and hence, according to Berberian's result (Proposition 1), Weyl's theorem holds for class A operators. Hence the proof is complete. □

3. Weyl's theorem for class $A(k)$ operators, where $k > 1$. The main result of this section is as follows.

THEOREM 2. *Let T be a class $A(k)$ operator and $\hat{T}$ its hyponormal operator transform such that for each $\lambda \in \sigma_a(T)$ and a corresponding sequence $\{y_n\}$ of unit vectors, $\hat{T}$ satisfies the condition $\lim_{n\to\infty} \||\hat{T}|^2 y_n\| = |\lambda|^2$. Then Weyl's theorem holds for T.*

Limit Condition [**18**]. For each $\lambda \in \sigma_a(T)$ and a corresponding sequence $\{y_n\}$ of unit vectors, $\hat{T}$ satisfies the condition $\lim_{n\to\infty} \||\hat{T}|^2 y_n\| = |\lambda|^2$ where T is a class $A(k)$ operator, $k > 1$ and $\hat{T}$ is its hyponormal operator transform. □

The following Propositions will be used to prove Theorem 2.

PROPOSITION 7 [**18**, Theorem 6, Corollaries 3 and 5]. *Let T be a class $A(k)$ operator. Suppose that $\{y_n\}$ is a sequence of unit vectors in H such that $(T-\lambda)y_n \to 0$ and $\||\hat{T}|^2 y_n\| - |\lambda|^2 \to 0$ as $n \to \infty$. Then $\lim_{n\to\infty}(T-\lambda)^* y_n = 0$ and $\sigma_{na}(T) = \sigma_{na}(\hat{T})$.*

PROPOSITION 8 [**18**, Theorem 7]. *Let T be a class $A(k)$ operator. Suppose that $\lambda \in \sigma_a(T)$ and $\{y_n\}$ is a corresponding sequence of unit vectors such that $\||\hat{T}|^2 y_n\| - |\lambda|^2 \longrightarrow 0$ as $n \longrightarrow \infty$. Then $\sigma(T) = \sigma(\hat{T})$.*

LEMMA 4. *Let T be a class $A(k)$ operator such that the Limit Condition is satisfied. Then $\lambda \in \sigma(T)$ is an isolated point $\Longleftrightarrow$ λ is an isolated point of $\sigma(\hat{T})$.*

Proof. $\lambda \in \sigma(T)$ is an isolated point
$\Longleftrightarrow$ $\exists$ a neighbourhood V of λ such that $(V \cap \sigma(T)) - \{\lambda\} = \emptyset$
$\Longleftrightarrow$ $(V \cap \sigma(\hat{T})) - \{\lambda\} = \emptyset$ by Proposition 8
$\Longleftrightarrow$ λ is an isolated point of $\sigma(\hat{T})$. □

LEMMA 5. *If T is a class $A(k)$ operator, where $k > 1$ and M is an invariant subspace of T, then $T|_M$ is also a class $A(k)$ operator.*

Proof. Let $\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$ on $H = M \oplus M^{\perp}$ and P the projection onto M. Then we have $P\{(T^*|T|^{2k}T)^{\frac{1}{k+1}} - (T^*T)\}P \geq 0$. By Hansen's inequality, we see that

$$A^*A = P(T^*T)P \leq P(T^*|T|^{2k}T)^{\frac{1}{k+1}}P \leq (PT^*|T|^{2k}TP)^{\frac{1}{k+1}} = (A^*|A|^{2k}A)^{\frac{1}{k+1}}.$$

It follows that A is a class $A(k)$ operator. That is, $T|_M$ is a class $A(k)$ operator. □

LEMMA 6. *Let T be a class $A(k)$ operator and $\hat{T}$ its hyponormal transform such that the Limit Condition is satisfied. Then the eigenspace of T reduces T.*

Proof. We have $\sigma_p(T) = \sigma_p(\hat{T})$, by Proposition 7. That is $(T-\lambda)x = 0$ implies that $(\hat{T}-\lambda)x = 0$. Since $\hat{T}$ is hyponormal $(\hat{T}-\lambda)^*x = 0$. We shall show that $(T-\lambda)^*x = 0$. When $\lambda = 0$, we have $\|Tx\| = 0$. Since T is a class $A(k)$ operator, we have $\|T^*x\| \leq \|Tx\|$ and so $\|T^*x\| = 0$.

On the other hand, when $\lambda \neq 0$ we have $(\hat{T}-\lambda)x = 0$ and $(\hat{T}-\lambda)^*x = 0$, so that

$$(|\hat{T}|^2 - |\lambda|^2)x = 0 \quad \text{and} \quad (|(\hat{T})^*|^2 - |\lambda|^2)x = 0. \tag{1}$$

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Since

$$|\hat{T}|^2 = \left||T|^k T\right|^{\frac{2}{k+1}} = (T^*|T|^{2k}T)^{\frac{1}{k+1}}$$

and

$$|(\hat{T})^*|^2 = \left|T^*|T|^k\right|^{\frac{2}{k+1}} = (|T|^k|T^*|^2|T|^k)^{\frac{1}{k+1}},$$

we obtain from (1) that

$$((T^*|T|^{2k}T)^{\frac{1}{k+1}} - |\lambda|^2)x = 0 \text{ and } \left((|T|^k|T^*|^2|T|^k)^{\frac{1}{k+1}} - |\lambda|^2\right)x = 0. \quad (2)$$

Since T belongs to class $A(k)$,

$$(T^*|T|^{2k}T)^{\frac{1}{k+1}} \geq |T|^2 \geq (|T|^k|T^*|^2|T|^k)^{\frac{1}{k+1}}$$

and hence, by (2), we have

$$((|T|^2 - |\lambda|^2)x, x) = 0. \quad (3)$$

Also,

$$\left\|\left[(T^*|T|^{2k}T)^{\frac{1}{k+1}} - |T|^2\right]^{\frac{1}{2}}x\right\|^2 = \left(\left[(T^*|T|^{2k}T)^{\frac{1}{k+1}} - |\lambda|^2\right]x, x\right) - ([|T|^2 - |\lambda|^2]x, x.).$$

It follows from (2) and (3) that $\|[(T^*|T|^{2k}T)^{\frac{1}{k+1}} - |T|^2]^{\frac{1}{2}}x\|^2 = 0$.

Consequently we obtain

$$(|T|^2 - |\lambda|^2)x = \left[|T|^2 - (T^*|T|^{2k}T)^{1/k+1}\right]x + \left[(T^*|T|^{2k}T)^{1/k+1} - |\lambda|^2\right]x = 0.$$

That is $(T^*T - \overline{\lambda}\lambda)x = 0$. Since $(T - \lambda)x = 0$, we have $(T - \lambda)^*\lambda x = 0$ and $\lambda \neq 0$ implies $(T - \lambda)^*x = 0$. This shows that every finite dimensional eigenspace of T is invariant under T and T^* and hence the proof is complete. □

LEMMA 7. *If T is a class $A(k)$ operator satisfying the Limit Condition, then T is isoloid and satisfies the condition* (α''').

Proof. Let λ be an isolated point of $\sigma(T)$. Then the range of the Riesz projection $E = \frac{1}{2\pi i}\int_{\partial D}(zI - T)^{-1}dz$ is a closed invariant subspace for T and $\sigma(T|_{EH}) = \{\lambda\}$. Here D is a closed ball with center λ that satisfies $\sigma(T)\bigcap D = \{\lambda\}$ and ∂D is the boundary of D described once counterclockwise.

By Lemma 4, λ is an isolated point of $\sigma(\hat{T})$. Since $\hat{T}$ is hyponormal and hence isoloid, λ is in the point spectrum of $\hat{T}$. By Proposition 7, $\sigma_p(T) = \sigma_p(\hat{T})$ and this implies that $\lambda \in \sigma_p(T)$. Therefore T is isoloid. By Lemma 5, $T|_{EH}$ is a class $A(k)$ operator and hence isoloid. Therefore T satisfies the (α''') condition. □

Proof of Theorem 2. If T is a class $A(k)$ operator, then by Lemma 6 every finite dimensional eigenspace of T is a reducing subspace of T. Also T satisfies the (α''') condition by Lemma 7 and hence, according to Berberian's result (Proposition 1), Weyl's theorem holds for a class $A(k)$ operator T. □

4. Applications of Weyl's theorem on class $A(k)$ operators.

DEFINITION 4.1. An operator $T \in B(H)$ is said to be *normaloid* if $r(T) = \|T\|$ and *transaloid* if $(T - \lambda)$ is normaloid for any λ in C, where $r(T) = \sup\{|\lambda| : \lambda \in \sigma(T)\}$ is the spectral radius of T.

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THEOREM 3. *Let T be a class $A(k)$ operator and $\hat{T}$ its hyponormal operator transform such that the Limit Condition is satisfied. Then the following properties hold.*

(i) $w(T) = w(\hat{T})$.
(ii) *If iso* $\sigma(T) = \phi$, *then* $\sigma(T) = w(T) = \sigma(\hat{T}) = w(\hat{T})$, *where iso* $\sigma(T)$ *is the set of all isolated points in* $\sigma(T)$.
(iii) *If* $w(T) = \{0\}$, *then T is compact and normal.*
(iv) *If* $\pi_{00}(T) = \emptyset$, *then T is extremally noncompact.*
(v) $r((T-\lambda)^{-1}) = r((\hat{T}-\lambda)^{-1}) = \|(\hat{T}-\lambda)^{-1}\|$.
(vi) $p(w(T)) = w(p(T))$, *for every polynomial p.*
(vii) *Weyl's theorem holds for* $f(T)$, *for every* $f \in H(\sigma(T))$, *where* $H(\sigma(T))$ *is the space of functions analytic in an open neighbourhood of* $\sigma(T)$.

Proof. (i) By Proposition 8, $\sigma(T) = \sigma(\hat{T})$ and, by Lemma 4, $\pi_{00}(T) = \pi_{00}(\hat{T})$. Since Weyl's theorem holds for T, $w(T) = \sigma(T) - \pi_{00}(T) = \sigma(\hat{T}) - \pi_{00}(\hat{T}) = w(\hat{T})$.

(ii) Assume that *iso* $\sigma(T) = \emptyset$. By Proposition 8, $\sigma(T) = \sigma(\hat{T})$ and hence we have *iso* $\sigma(T) =$ *iso* $\sigma(\hat{T})$. Since T is reduced by each of its finite dimensional eigenspaces we have $\sigma(T) = w(T)$ [**15**, Corollary 1.3]. Since $\hat{T}$ is hyponormal, $\sigma(\hat{T}) = w(\hat{T})$. It follows that $\sigma(T) = w(T) = \sigma(\hat{T}) = w(\hat{T})$.

(iii) Since Weyl's theorem holds for T, by Theorem 2, and $w(T) = \{0\}$, by assumption and by Proposition 7, every non-zero point of $\sigma(T)$ is an isolated normal eigenvalue with finite dimensional eigenspace which reduces T. Hence $\sigma(T)\backslash w(T)$ is a finite set or a countably infinite set whose only accumulation point is 0.

Let $\sigma(T)\backslash w(T) = \{\lambda_n\}$ with $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3| \geq \ldots \geq 0$ and let E_n be the orthogonal projection onto $\text{Ker}(T-\lambda_n)$. Then $TE_n = E_nT = \lambda_n E_n$ and $E_nE_m = 0$ if $n \neq m$. Put $E = \oplus_n E_n$. Then $T = \oplus_n \lambda_n E_n \ominus T|_{(1-E)H}$ and $\sigma(T|_{(1-E)H}) = \{0\}$. Since EH is a reducing subspace of T, $T|_{(1-E)H}$ also belongs to class $A(k)$. It is known that every class $A(k)$ operator is normaloid. Since $\sigma(T|_{(1-E)H}) = \{0\}$ we have $T|_{(1-E)H}) = 0$. Hence $T = \oplus \lambda_n E_n$ is normal. The compactness of T follows from the finiteness or the countability of $\{\lambda_n\}_n$ satisfying $|\lambda_n| \downarrow 0$ and each E_n is a finite rank projection.

(iv) [**15**, Corollary 1.7] says that if $T \in B(H)$ is normaloid and $\pi_{00}(T) = \emptyset$ then T is extremally noncompact. Since a class $A(k)$ operator is normaloid and by assumption $\pi_{00}(T) = \emptyset$, T is extremally noncompact.

(v)

$$
\begin{aligned}
r((T-\lambda)^{-1}) &= \sup|\sigma((T-\lambda)^{-1})| \text{for any } \lambda \notin \sigma(T) \\
&= \sup \frac{1}{|\sigma(T-\lambda)|} \qquad (\lambda \notin \sigma(T)) \\
&= \sup \frac{1}{|\sigma(T)-\lambda|} \qquad (\lambda \notin \sigma(T)) \\
&= \sup \frac{1}{|\sigma(\hat{T})-\lambda|} \qquad (\lambda \notin \sigma(T)) \\
&= \sup \frac{1}{|\sigma(\hat{T}-\lambda)|} \qquad (\lambda \notin \sigma(T)) \\
&= \sup|\sigma((\hat{T}-\lambda)^{-1})| \qquad (\lambda \notin \sigma(T)) \\
&= r((\hat{T}-\lambda)^{-1}) \qquad (\lambda \notin \sigma(T)) \\
&= \|(\hat{T}-\lambda)^{-1}\|.
\end{aligned}
$$

(vi) [**15**, Corollary 1.5] says that if $T \in B(H)$ is reduced by each of its finite-dimensional eigenspaces, then $(p(w(T)) = w(p(T))$ for every polynomial p. Hence, by Lemma 6, the result follows.

(vii) According to [**8**, Theorem 2.5], suppose that $T \in B(H)$ has SVEP and is transaloid, then Weyl's theorem holds for $f(T)$, for every $f \in H(\sigma(T))$. We shall show that a class $A(k)$ operator is transaloid. It is well known that a hyponormal operator is transaloid and hence $\hat{T}$ is transaloid. That is, $\hat{T} - \lambda$ is normaloid and hence

$$\begin{aligned} r(T-\lambda) &= \sup\{|\lambda| : \lambda \in \sigma(T-\lambda)\} \\ &= \sup\{|\lambda| : \lambda \in \sigma(T) - \lambda\} \\ &= \sup\{|\lambda| : \lambda \in \sigma(\hat{T}) - \lambda\} \\ &= \sup\{|\lambda| : \lambda \in \sigma(\hat{T} - \lambda)\} \\ &= r(\hat{T} - \lambda) = \|\hat{T} - \lambda\| \\ &= \|T - \lambda\| \quad \text{since } \|\hat{T}\| = \|T\| \text{ [18, Corollary 8].} \end{aligned}$$

This shows that $T - \lambda$ is normaloid and hence T is transaloid. Also T has the SVEP [**18**, Theorem 11]. Therefore Weyl's theorem holds for $f(T)$, for every $f \in H(\sigma(T))$.

REFERENCES

1. P. Aiena and C. Carpintero, Weyl's theorem, α- Weyl's theorem and single valued extension property, *Extracta Math.* **20** (2005), 25–41.

2. A. Aluthge, Some generalised theorems on p-hyponormal operators, *Int. Eq. 0p. Th.* **24** (1996), 497–501.

3. S. K. Berberian, An extension of Weyl's theorem to a class of not necessarily normal operators, *Michigan Math. J.* **16** (1969), 273–279.

4. M. Chō, M. Itoh and S. Oshiro, Weyl's theorem holds for p-hyponormal operators, *Glasgow Math. J.* **39** (1997), 217–220.

5. M. Chō and T. Yamazaki, An operator transform from class A to the class of hyponormal operators and its application, *Int. Eq. 0p. Th.* **53** (2005), 497–508.

6. N. N. Chourasia and P. B. Ramanujan, Paranormal operators on Banach space, *Bull. Austral. Math. Soc.* **21** (1980), 161–168.

7. L. A. Coburn, Weyl's theorem for nonnormal operators, *Michigan Math. J.* **13** (1966), 285–288.

8. R. E. Curto and Y. M. Han, Weyl's theorem, a-Weyl's theorem and local spectral theory, *J. London Math. Soc.* (2) **67** (2003), 499–509.

9. R. E. Curto and Y. M. Han, Weyl's theorem for algebraically paranormal operators, *Int. Eq. 0p. Th.* **47** (2003), 307–314.

10. B. P. Duggal and C. Kubrusly, Weyl's theorems for posinormal operators, *J. Korean. Math. Soc.* **42** (2005), 529–541.

11. B. P. Duggal, I. H. Jeon and I. H. Kim, On Weyl's theorem for quasi-class A operators, *J. Korean. Math. Soc.* **43** (2006), 899–909.

12. M. Fujii, D. Jung, S. H. Lee, M. Y. Lee and Nakamoto, Some class of operators related to paranormal and log-hyponormal operators, *Math. Japan.* **51** (2000), 395–402.

13. T. Furuta, M. Ito and T. Yamazaki, A subclass of paranormal operators including class of log-hyponormal and several related classes, *Sci. Math.* **1** (1998), 389–403.

14. Y. M. Han and W. Y. Lee, Weyl's theorem holds for algebraically hyponormal operators *Proc. Amer. Math. Soc.* **128** (2000), 2291–2296.

15. Y. M. Han and W. Y. Lee, Weyl spectra and Weyl's theorem, *Studia Math.* **148** (2001), 193–206.

https://doi.org/10.1017/S0017089507003904 Published online by Cambridge University Press

16. F. Hansen, An inequality, *Math. Ann.* **246** (1980), 249–250.

17. M. Ito and T. Yamazaki, Relations between two inequalities $(B^{\frac{r}{2}}A^pB^{\frac{r}{2}})^{\frac{r}{p+r}} \geq B^r$ *and* $A \geq (A^{\frac{p}{2}}B^rA^{\frac{p}{2}})^{\frac{p}{p+r}}$ and their applications, *Int. Eq. Op. Th.* **44** (2002), 442–450.

18. J. S. I. Mary and S. Panayappan, Some properties of class $A(k)$ operators and their hyponormal transforms, *Glasgow Math. J.* **49** (2007), 133–143.

19. S. Mechrei, Weyl's theorem for algebraically class A operators, *Bull. Belg. Math. Soc. Simon Stevin* **14** (2007), 239–246.

20. J. Stampfli, Hyponormal operators, *Pacific J. Math.* **12** (1962), 1453–1458.

21. A. Uchiyama, Weyl's theorem for class A operators, *Math.Inequal. Appl.* **4** (2001), 143–150.

22. T. Yamazaki, On powers of class $A(k)$ operators including p-hyponormal and log hyponormal operators, *Math. Inequal. Appl.* **3** (2000), 97–104.