A NON-ROTATING ORIGIN ON THE INSTANTANEOUS EQUATOR

N. Capitaine
Observatoire de Paris (UA 1125 du CNRS)
61 Avenue de l'Observatoire, 75014-Paris, FRANCE

B. Guinot
Bureau International des Poids et Mesures
Pavillon de Breteuil, 92310-Sèvres, FRANCE

ABSTRACT. In order to give an exact and clear description of the angle of rotation of the Earth, we propose to use, as the reference point in space, a "non-rotating origin" (Guinot 1979) such that its hour angle, reckoned from the origin of the longitudes (or "non-rotating origin" in the Earth), represents strictly the specific rotation of the Earth. The position of this origin on the instantaneous equator depends only on the trajectory of the pole of rotation. We show that the estimation of the deduced angle of rotation is not critically affected by the precision with which this trajectory is known. We give therefore the formulae to obtain the non-rotating origin, at any date t, from a chosen fixed reference, and we propose a definition of the Universal Time which will remain valid, even if the adopted model for the precession and the nutation is revised. We show that the use of the non-rotating origin also simplifies the transformation of coordinates between the terrestrial and celestial reference systems.

INTRODUCTION

The classical definition of the angle of rotation of the Earth through the Apparent Greenwich Sidereal Time (GST) is not satisfactory for two main reasons. First, this angle is referred to the true equinox of date which is an inadequate and unnecessary intermediate reference point because present observations of the Earth orientation in space (by VLBI or SLR) are practically insensitive to the orientation of the ecliptic and consequently to the position of the equinox. Secondly, the expression of the GST neglects some cross terms between precession and nutation which are of the order of 0.001" and should have now to be considered as the present observations reach such a precision.

This definition has therefore to be improved in order that the chosen reference point on the moving equator be more adapted to the new methods of observations and that the reached accuracy be of the order of 10^{-4}".

This paper recalls the necessary concepts and gives the main conclusions which have been obtained in a more detailed study (Capitaine et al. 1986).

A. K. Babcock and G. A. Wilkins (eds.), The Earth's Rotation and Reference Frames for Geodesy and Geodynamics, 33–38.

1. CONCEPTUAL DEFINITIONS

1.1. The non-rotating origin

In order to represent the specific rotation of the Earth around its axis of rotation, which has itself a displacement with respect to space and to the Earth, it is necessary to clearly separate these two kinds of motions. In this purpose Guinot (1979) used explicitly the concept of the "non-rotating" origin (fig.1) as the reference point in space on the instantaneous equator. This concept was also implicitly used (Xu et al. 1984) in the definition of UT1 as recommended by IAU (Aoki et al.1982).

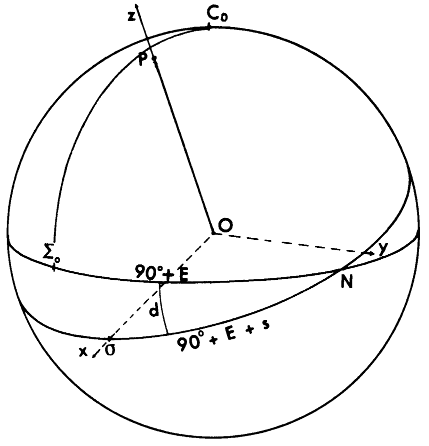

Fig. 1 : The non-rotating origin

The Celestial Reference System (CRS) is represented by a celestial sphere of center O with a fundamental great circle of pole C_0 and an origin Σ_0 on this circle. If Oxyz is a trirectangular coordinate system such that its z-axis is along the axis of instantaneous rotation and its x-axis is directed towards the non-rotating origin on the moving equator, the kinematical definition of this point, denoted σ , is such that Oxyz has no motion of rotation around Oz with respect to the CRS when P moves in the CRS. A similar definition can be given for the non-rotating origin, denoted ϖ , with respect to the Terrestrial Reference System (TRS), this point being in fact the instantaneous origin of longitudes.

1.2. The Stellar Angle

Let θ be the hour angle $\widehat{\varpi O \sigma}$ of the non-rotating origin σ from the prime meridian $P\varpi$. This angle, reckoned positive westwards, is designated (Guinot 1979) as the stellar angle. It is proposed to use this angle as the representation of the angle of rotation of the Earth in place of the Apparent Greenwich Sidereal Time, GST.

https://doi.org/10.1017/S0074180900119229 Published online by Cambridge University Press

1.3. UT1

UT1 should be defined (Guinot 1979) as an angle proportional to θ, such that:

$$\theta = k\,(UT1 - UT1_0) \quad , \qquad (1)$$

the two constants $UT1_0$ and k being chosen such that UT1 corresponds to its present definition.

With such a definition, UT1 strictly increases linearly with time if the rotation of the Earth is uniform.

1.4. The quantity s

The non-rotating origin is not observable, but its position (at any date t) on the moving equator can be computed through the quantity $s = \sigma N - \Sigma_0 N$ (fig.1), when taking as zero its constant value at the chosen epoch of reference t_0.

If E and d are the spherical coordinates ($d = C_0P$, $E = \widehat{\Sigma_0 C_0 P}$) of the pole of instantaneous rotation P in the CRS, this quantity can be expressed as:

$$s = \int_{t_0}^{t} (\cos d - 1)\,\dot{E}\,dt \quad . \qquad (2)$$

A similar expression can be given for the quantity s′ relative to the non-rotating origin ϖ in the CRS.

If the celestial motion (E,d) of the pole of instantaneous rotation is known between the epoch t_0 and the date t, the quantity s can be computed and the position of σ in the moving equator can thus be given. Similarly, the position of ϖ on the moving equator can be obtained as soon as the coordinates of the pole of rotation are known in the TRS.

2. REALIZATION OF THE DEFINITIONS

2.1. Expected errors

As a conceptual definition has no value if it cannot be implemented with sufficient accuracy, the expected errors on the realization of these definitions have to be evaluated.

Provided that the CRS has no residual rotation in space, the non-rotating origin σ does not depend on the selection of the axes of reference of the CRS. The conclusion is similar for the non-rotating origin ϖ and the TRS.

The computation of the quantity s is dependent on the representation of the pole trajectory and a changeover of the realized pole from P_a to P_b at date t, introduces a finite rotation ρ in the CRS and ρ' in the TRS.

The difference $\theta_b - \theta_a$ between the two realized stellar angles θ_b and θ_a, corresponding respectively to P_b and P_a, can be evaluated through $\rho - \rho'$. Such an evaluation shows that θ is not sensitive (up to 10^{-4}") to the reasonable errors of the pole trajectory . One consequence is

https://doi.org/10.1017/S0074180900119229 Published online by Cambridge University Press

that the Celestial Ephemeris Pole (CEP) can be used in the computations of the quantity s or s´ in place of the instantaneous axis of rotation.

2.2. Development of the quantities s and s´ as functions of time

The development of the quantity s is obtained by using the expression (2) with the assumption that the polar origin and the fixed origin Σ_0 on the equator of reference are coincident respectively with the mean pole and mean equinox of epoch J2000.0. Its numerical coefficients are based on the numerical precession quantities (Lieske et al.1977) and nutation coefficients (Seidelmann 1982). The resulting development up to 10^{-3}" in a century contains one term due to precession, one term due to nutation and cross terms between precession and nutation; it is given by:

$$s = 0.036'' t^3 + 0.004'' t - 0.003'' (\sin\Omega - \sin\Omega_0) - 0.045'' t \cos\Omega - 0.003'' t \cos 2\Theta, \quad (3)$$

Ω being the mean tropic longitude of the Moon´s node, Ω_0 its value at epoch J2000.0 and Θ the mean tropic longitude of the Sun.

The development of s, up to 10^{-5}" in a century shows that only a few terms (twelve of them) have an amplitude larger than 10^{-4}".

A numerical development of the quantity s´ can also be obtained for a given polar motion. For instance, for a Chandlerian wobble of amplitude 0.50" and an annual wobble of amplitude 0.10", it can be written up to 10^{-5}" in a century:

$$s' = - 0.00034'' t , \quad (4)$$

which cannot be neglected when a precision of 10^{-4}" is required.

3. PRACTICAL USES OF THE NON-ROTATING ORIGIN

3.1. Numerical relation between θ and UT1

The conceptual relation (1) can lead to a definitive expression of θ in the case where no residual rotation is introduced in the realization of the CRS. In order that this expression lead as an implementation of the conventional relation between the Greenwich Mean Sidereal Time and UT1 in the case of the IAU 1976 System of Astronomical Constants and the FK5 system, it is given as:

$$\theta = 0.779\ 057\ 273\ 264 + 1.002\ 737\ 811\ 911\ 354\ (\text{UT1} - 2000 \text{ Jan. } 1, 12 \text{ h UT1}) \quad (5)$$

UT1 being in days and θ in revolutions, or a similar expression of θ at 0h UT1 in seconds.

https://doi.org/10.1017/S0074180900119229 Published online by Cambridge University Press

3.2. Transformation from the CRS to the TRS

The transformation from the CRS to the TRS, at any date t, can be expressed as a product of rotation matrixes P(t) for precession, N(t) for nutation and S(t) for Earth rotation (including polar motion).

In the classical representation (Mueller 1981), the two matrixes:

$$P(t) = R_3(-z_A)R_2(\theta_A)R_3(-\zeta_A), \quad N(t) = R_1(-\epsilon - \Delta\epsilon)R_3(-\Delta\psi)R_1(\epsilon),$$

include various ecliptic and equatorial parameters for precession and nutation.

In the proposed representation, using the non-rotating origin, the product of rotation matrixes for precession and nutation is replaced by:

$$NP(t) = R_3(-s)R_3(-E)R_2(d)R_3(E)$$

where the only necessary developments to be used are the conventional series of the celestial pole in the CRS.

The matrix for Earth rotation, which is classically given by:

$$S(t) = R_2(-x_p)R_1(-y_p)R_3(GST)$$

x_p and y_p being the "polar coordinates" in the CRS, is thus replaced by:

$$S'(t) = R_2(-x_p)R_1(-y_p)R_3(\theta)R_3(s').$$

The resulting transformation, which is conceptually more simple, only requires the knowledge (for the date t) of the pole coordinates in the CRS and in the TRS and of the stellar angle.

CONCLUSION

The concept of the non-rotating origin should be employed when dealing with the Earth's rotation and in particular to give a clear definition of UT1; it could replace advantageously the equinox especially for the observations which are not sensitive to the orientation of the ecliptic. Its full advantages will be taken only when the CRS will be realized by positions of quasars.

REFERENCES

Aoki, S., Guinot, B., Kaplan, G.H., Kinoshita, H., Mc Carthy, D.D., Seidelmann, P.K., 1982, 'The new definition of Universal Time', Astron. Astrophys. 105 ,359.

Aoki, S. and Kinoshita, H., 1983, 'Note on the relation between the equinox and Guinot's non-rotating origin', Celestial Mechanics 29 , 335-360.

https://doi.org/10.1017/S0074180900119229 Published online by Cambridge University Press

Capitaine,N., Guinot,B., Souchay,J.,1986, ´A non-rotating origin on the instantaneous equator: definition, properties and use´, accepted for publication in Celestial Mechanics 39 , 283-307.

Guinot, B., 1979, ´Basic problems in the kinematics of the rotation of the Earth´ in Time and the Earth´s rotation, 7-18, D.D. Mc Carthy and J.D. Pilkington (eds), IAU Copyright.

Lieske, J., Lederle, T., Fricke, W., Morando, B., 1977, ´Expressions for precession quantities based upon the IAU (1976) system of Astronomical constants´, Astron.Astrophys. 58 , 1-16.

Mueller, I.I., 1981, ´Reference coordinate systems for Earth Dynamics: a preview ´ in Reference coordinate systems for Earth Dynamics , ed. by E. M. Gaposchkin and B. Kolaczek, D. Reidel Publishing company, p 1-22.

Seidelmann, P.K., 1982,´1980 Theory of nutation: the final report of IAU working group on nutation´, Celestial Mechanics 27 , 79-106.

Xu B.X., Ahu, S.Y., Zhang H., 1984, ´Discussion of meaning and definition of UT´ in Astrometric Techniques, H.K. Eichhorn and R.J. Leacok (eds), D.Reidel Publishing Company.