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RESEARCH ARTICLE

The Ancients and the Moderns: Chasles on Euclid's lost *Porisms* and the pursuit of geometry

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Abstract
Of Euclid's lost manuscripts, few have elicited as much scholarly attention as the *Porisms*, of which a couple of brief summaries by late-Antiquity commentators are extant. Despite the lack of textual sources, attempts at restoring the content of this absent volume became numerous in early-modern Europe, following the diffusion of ancient mathematical manuscripts preserved in the Arabic world. Later, one similar attempt was that of French geometer Michel Chasles (1793–1880). This paper investigates the historiographical tenets and practices involved in Chasles' restoration of the porisms, as well as the philosophical and mathematical claims tentatively buttressed therewith. Echoes of the Quarrel of the Ancients and the Moderns, and of a long-standing debate on the authority and usefulness of the past, are shown to have decisively shaped Chasles' enterprise—and, with it, his integration of mathematical and historical research.

Keywords: Chasles; Comte; Euclid; Pappus; Simson; Halley; Poncelet; geometry; ancients; moderns; historiography; porisms

> *I owe to the conjunction of a mirror and an encyclopedia the discovery of Uqbar.*
> J. L. Borges, *Tlon, Uqbar, Orbis Tertius.*

1. Introduction

In 1870, whilst writing a report requested by the French Ministry of Education, an aging Michel Chasles (1793–1880) reflected upon the great strides made throughout the nineteenth century towards the perfection of geometry, a science to which he himself had devoted the better part of the past forty years. Throughout his long and distinguished career, Chasles had constantly interwoven mathematical research with studies in the history of this science, to an extent where both endeavors were often hard to separate. One specific claim he made in this *Rapport* clearly demonstrates this:

> Nowadays, in the various branches of Mathematics, statements of *incomplete* theorems are admitted; these theorems are called Porisms. . . . Were it not for Pappus' defense of its utility, [the doctrine of Porisms] would have disappeared in the sciences. Instead of its disappearing, however, science incorporated this doctrine, modifying unbeknownst to us the ancient form of theorems and replacing it with that of the Porisms. (Chasles 1870, 238–239)[1]

[1]All translations, unless otherwise noted, are ours. For biographical information on Chasles, see Boudin 1869; Koppelman 1971.

The *Porisms* in question were a lost treatise of Ancient Greek geometry, presumably authored by Euclid. All that was known about this treatise at the time were two short descriptions given by late-Antiquity commentators Pappus of Alexandria (c. 290–350 C.E.) and Proclus Lycaeus (412–485 C.E.). In the seventh book of his *Mathematical Collections*, Pappus provided a list of rather cryptic mathematical statements supposed to encapsulate Euclid's porisms, whose status *qua* mathematical propositions (*viz.* the question of deciding whether they were theorems, construction problems, or something else entirely) had elicited much discussion, at least since Federico Commandino's famous Latin translation in 1588 (and, in fact, still do).[2] Chasles' interest in this lost book goes back at least to the 1830s, and is in fact not at all unique: many geometers of his generation, and of his milieu, shared the opinion that a restitution of this mysterious text's meaning would be of great importance both for the history of geometry, and for its pursuit by means other than algebra.

Chasles' continued engagement with this lost treatise spans a large part of his scientific career. In his first book, an extensive historical survey of geometrical methods published in 1837 under the title *Aperçu historique sur l'origine et le développement des méthodes en Géométrie*, he included a putative characterization of the philosophical nature of porisms as the Ancient Greeks' equivalent to algebraic (*viz.* Cartesian) equations. In 1860, he published his definitive interpretation of the porisms by way of a standalone book on the subject. In the second half of this book, having fleshed out and altered his interpretation of the nature of the porisms, he proposed nothing less than a "divination" of Euclid's lost treatise (Chasles 1860a, 1).

On the basis of Pappus' few cryptic statements, Chasles had restored the totality of Euclid's 171 porisms—and even provided proofs thereof. These divined porisms, in turn, closely resembled the propositions and equations which Chasles himself had constructed and taught in his courses on "higher geometry" (*Géométrie supérieure*) at the Faculté de Paris since 1846. Chasles was keenly aware that the generality of his own methods went far beyond what could be expected from ancient sources, and consequently his proofs of Euclid's supposed porisms were adapted to the level of generality he attributed to Greek mathematics (for instance, by leaving out any mention of imaginary points or points at infinity). However, the content of Chasles' teaching served both as a tool to regroup and categorize Euclid's propositions, and, more crucially, as the interpretative key to the question of their meaning, a question which had puzzled historians and mathematicians for several centuries. Indeed, a key feature of Chasles' historical work was his thesis that the meaning of Euclid's porisms would only emerge once the methods of which they were the germ had fully developed. This is why, in his 1870 *Rapport*, Chasles insisted on the prevalence, in modern mathematics (and beyond geometry), of propositions of a similar nature to those of Euclid. A few paragraphs below the quote above, Chasles commented on the difficulty which many of Pappus' statements had presented to earlier mathematically-competent readers, writing, "thus, the study and development of the elementary theories of modern Geometry were necessary to successfully restore the three books of Euclid's Porisms" (Chasles 1870, 240). The reason Pappus' fragments had been undecipherable for centuries, he argued, was simply that one had to wait for geometry to develop and, unbeknownst to its practitioners, draw out the latent mathematical methods nested in Euclid's books.

It is nowadays all too common to deride such historiographical practices as Whiggish, presentist, or teleological—a series of anathemas which, as Michael Gordin pointedly noted, often serve to "define the history of science against other fields," and to constitute this field "as a discipline" (Gordin 2014, 417, 420). This is particularly the case for the history of Ancient Greek mathematics, which, in the 1970s, was taken by storm when several papers and books collectively argued that it was time to do away with most of the existing research, on the grounds that it had

[2]On Commandino's translation of Pappus' *Mathematical Collections*, and its transformative role in early-modern mathematics, see Bos 2001. For a more recent translation of Pappus' book, see Jones 1986, especially pp. 547–572 for the section on the porisms. Further discussions of the history of the interpretations of this text can be found in Hogendijk 1987.

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not followed proper methods. "History of mathematics is history, not mathematics" was the famous and influential sentence enunciated by the Israeli historian Sabetai Unguru (Unguru 1979, 563).[3] Chasles' work on the porisms exemplifies all that is now deemed wrong in the outdated historiography of mathematics: it relies on anachronistic notations and locates them on a "royal road" leading to Chasles himself.[4]

However, while exhaustive critiques of so-called Whiggish historiographies have been produced, there remains much to gain by paying attention to the variety of scholarly practices these historiographies actually entailed.[5] As we shall see, Chasles' engagement with the porisms was a dynamic one: it nourished his mathematical research as much as it was nourished by it. The intellectual work accomplished by Chasles was not a passive translation of old mathematics into newer notations, but rather the back-and-forth adaptation of a notational technology, and an interpretation of ancient sources. We shall, moreover, argue that the analytical categories and the theoretical justifications at the heart of Chasles' historiographical practice only become meaningful when cast against the backdrop of early nineteenth-century cultural and intellectual trends. Much of what Chasles set out to achieve by reinterpreting the porisms reverberated with the echoes of an old quarrel whose embers had not yet gone entirely cold, namely that of the Ancients and the Moderns. It was also informed by the discussions of philosophical theses on the modalities of scientific progress brought forth by the positivist philosopher Auguste Comte (1798–1857).

Furthermore, a philosophical reflection on the writing of generality and the type of propositions which ought to be chosen as the foundation for proper geometrical methods, which Chasles had conducted in the course of his mathematical work, would largely motivate the goals with which he approached this historiographical reconstruction. By thus situating Chasles' historiography, our goal in the following pages is not to rehabilitate his historical methods or his conclusions. Rather, this essay reconstructs the epistemic framework, that is, the conditions under which the intellectual labor and mathematical transformations he performed on ancient sources were meaningful.[6] By not downplaying the depth and internal coherence of Chasles' project (however problematic it may be by modern standards), we then show that a certain use of the past was central to his scientific practice as a whole, and to the very intelligibility of his "divination" of Euclid's lost books.

2. Ancients and Moderns revisited

"The spirit of modern Geometry is to always elevate truths, whether old or new, to the greatest possible generality" (Chasles 1837, 269).[7] Chasles significantly chose this quotation from Bernard

[3]This text came in conclusion of a polemic exchange (Unguru 1975; van der Waerden 1976; Freudenthal 1977; Weil 1978), which pitted career-mathematicians turned historians of their discipline against an emerging group of self-identifying professional historians. On the social context of this polemic, see Schneider 2016. The Unguru affair was an epiphenomenon of a larger transformation, which bespeaks the contemporary publication of Knorr 1975. On this transformation of the historiography of Greek mathematics at the turn of the 1970s, see Saito 1998, and the introduction of Netz 1999. These discussions are still active today, see for instance Blåsjö 2014, Fried 2014, and Blåsjö and Hogendijk 2018.

[4]The question of notations is one that largely polarized the aforementioned debates on the historiography of Greek mathematics. The expression "the Royal Road to us" was mobilized by Ivor Grattan-Guinness in his essay (Grattan-Guinness 2004, 165), to characterize the approach that views mathematics of the past as heritage, as opposed to an object of historical analysis proper.

[5]The way scientists write the history of their own discipline and use such historical practices in their own research has elicited growing attention in recent years; see, for instance, Wilson 2017, and ten Hagen 2021, 265. The fruits of such studies, in the case of the history of historiography of mathematics, have been illustrated in recent years in various publications such as Goldstein 2010; Chemla 2012; Remmert, Schneider and Sørensen 2016; Wang 2022. On Chasles' historiography of mathematics more specifically, see Chemla 2016; Smadja 2016; Michel 2020; Michel and Smadja 2021a; Michel and Smadja 2021b.

[6]This has also been called the "infrastructure" of historiography; see Goldstein 1976. This concept was used to analyze early nineteenth-century interpretations of ancient Sanskrit mathematical sources in Smadja 2016.

[7]See Fontenelle [1704] 1707, 60 for the original excerpt from Fontenelle, which Chasles is quoting, with a slight distortion, in the passage cited here. Fontenelle used the French term "universalité," which Chasles replaced by "généralité."

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Le Bovier de Fontenelle as a finale for his own 1837 prize-winning essay, the *Aperçu historique*. The sentence is excerpted from the tract "Sur les spirales à l'infini," presented at the Académie Royale des sciences in 1704. One of the main protagonists of the French quarrel between the Ancients and the Moderns, Fontenelle took advantage of his friend Pierre Varignon's memoir on spirals, published that same year, to draw an eloquent parallel between ancient and modern geometry and compare their respective merits. Varignon had indeed generalized Fermat's parabolic spirals into spirals of all kinds, obtained from any generating curve.[8] From the so-called "general equation of infinitely many spirals"[9] (Varignon 1707, 72), he could then retrieve the equation in polar coordinates of any specific spiral, by merely substituting for the ordinate a certain algebraic expression yielded by the equation of the generating curve,[10] and thus, in each case, compute the subtangents and spiral areas algebraically. As Fontenelle pointed out,

> Therein lies the great advantage of modern Geometers over ancient ones. Infinitely many more truths cost us infinitely less, not because we have any superior genius, but because we have excellent methods. The glory of the Ancients is to have been able to achieve without the help of our Art whatever little they achieved, & the glory of the Moderns is to have found such a wonderful Art. The Ancients resemble the natives of *Mexico* & *Peru* who, having no cranes nor any such instruments, & ignoring how to put up scaffolding, still raised up buildings by hand, & the Moderns are the *Europeans* who build incomparably better, but with machines. (Fontenelle [1704] 1707, 65)

Chasles took up similar metaphors, but diverted them from their original meaning by making them serve another purpose: instead of promoting analytic Geometry as Fontenelle intended, he offered pure Geometry, enhanced by the new methods he had created. Duality and homography would henceforth endow modern Geometry with new modes of geometric transformations which, although comparable in generality to the formula-type transformations of algebra, would nevertheless grant it with a character of easiness and universality setting it apart from ancient Geometry. Chasles presented these modes of transformation as "true instruments," allowing anyone to produce infinitely many geometrical truths from previously known ones, thus ultimately making genius dispensable in mathematics (Chasles 1837, 269).[11]

[8]Archimedes' spiral had been obtained by moving a point along a half-straight line rotating around its end point, so that the radii of the spiral be in proportion to the corresponding arcs of revolution. Fermat later made the generation of this curve more general by assuming that the radii be set in proportion not to the arcs themselves, but to any power of these, whether integral or fractional. Varignon then made a further step and conceived of a way to form an inexhaustible wealth of new spirals by observing that Fermat's idea to take the radii of the spiral in proportion to any arbitrary power of the corresponding arcs, would amount to taking the arcs of revolution themselves in the same ratio as the ordinates of a parabola, whose abscissæ would be set equal to the radii of the spiral, since "the nature of the parabola in general consists in its abscissæ being as some power of its ordinates" (Varignon [1704] 1707, 60). But why should the parabola be granted such a privilege? Why not take instead any other curve as a generating curve? As Fontenelle aptly noted, with Varignon, "the generation of the spiral was no more enclosed in any boundaries" (Fontenelle [1704] 1707, 61), although this open-ended extension to infinitely many spirals had its source in the very nature of Archimedes' spiral, upon which the new spirals were modelled.

[9]In Varignon's construction, the abscissæ of the generating curve (y) are set equal to the radii of the spiral, while the ordinates of that curve (z) are in proportion to the corresponding circular arcs of the spiral (x). Assuming that c is the circumference of the circle of revolution, and b the ordinate of the generating curve corresponding to a complete revolution on the circle, one then obtains the proportion $c : x :: b : z$, hence the general equation $cz = bx$.

[10]For instance, if the generating curve is a parabola of equation $za^{m-1} = y^m$, by substituting $z = \frac{y^m}{a^{m-1}}$ in the general equation $cz = bx$, one obtains the equation of Fermat's parabolic spirals $cy^m = a^{m-1}bx$.

[11]The significance of Chasles' dismissal of genius in the new geometry should be assessed with reference to the bifurcated pattern of the cultural and social history underlying the shaping of that notion. Understood as an outstanding capacity for creative and original thought, the modern concept of *genius,* which was given prominence and codified in the Age of the Enlightenment, can be traced back to the classical Latin *ingenium* meaning a natural quality or disposition. Still, as Edgar Zilsel pointed out in his famous 1926 monograph, *ingenium* as an inborn quality that cannot be learned was unknown in the Middle Ages and only appeared in the Renaissance when, Zilsel observed after Jacob Burckhardt, the discovery of individuality

The comparison between ancient and modern learning in the sciences and the arts had been at the center of the Quarrel between the Ancients and the Moderns—the late seventeenth- and early eighteenth-century controversy that polarized the European *République des Lettres*—in which Fontenelle played an important part. More than a century after the dust had settled, Chasles' grand opus, the *Aperçu historique*, resounded with echoes of those earlier concerns.[12] His historiography of mathematics placed the primary emphasis on the comparison of methods, ancient and modern. As we shall see, his long-lasting interest in restoring the lost meaning of Euclid's *Porisms*, and his claim that the *Porisms* contained the "germ" from which pure geometry should grow in its modern form, owed much to his close reading of the British seventeenth- and eighteenth-century mathematicians, whose reflections on ancient geometry were articulated in the midst of this scholarly whirlwind.

The Quarrel began in 1687 in France, ignited, under the gilding of the Louvre, by the public reading of a poem by Charles Perrault glorifying the century of Louis le Grand. The following year, Fontenelle published his influential *Digression sur les Anciens et les Modernes,* which stated:

> Mathematics and physics are sciences whose yoke weighs ever more heavily upon the *savants*. One should eventually renounce them, but the methods multiply at the same time. The same spirit that improves our grasp of things by adding new insights to them, also improves the way to learn them by abridging it, and provides new means to embrace the new extent it gives to the sciences. A *savant* of this century contains ten times a *savant* of the age of Augustus, but he also had ten times more conveniences [*commodités*] to become *savant.*" (Fontenelle [1688] 2001, 309)

In his series of *Parallèles des Anciens et des Modernes* (1688–1692), Perrault claimed the superiority of the Moderns over the Ancients in all quarters of scholarship, from mathematics and philosophy to poetry and eloquence. The greater perfection of Huygens's clock over the clepsydra was largely conceded, the former outmatching the latter owing to the mathematics of the cycloid being inbuilt. However, Cicero and Quintilien still reigned supreme in rhetoric, while architecture would provide a disputed middle ground.[13]

crystallized (see Zilsel [1926] 1972, 110). However, etymology and cultural history suggest that another significant thread unfolds from the Latin *ingenium* to the French word *engin* meaning machine, instrument, apparatus (especially in the context of warfare), and from there to the modern French term *ingénieur*, *engineer*, as is averred in the early eighteenth-century dictionary by Furetière or Diderot and d'Alembert's *Encyclopédie* (see Vérin 1984, 22). From Vauban and Belidor to Gaspard Monge, eighteenth-century French military engineering enhanced the *ingénieur*'s capacity for inventing new material devices and machines with the help of mathematics. In contrast to the modern concept of genius, emphasis would increasingly be laid on talent as something that could be acquired and learned in military schools (see Langins 2004, 166, 179–180; Belhoste 2003, 131–140). Chasles belonged to that tradition. On the "social epistemology of Enlightenment engineering," and the role of mathematics in the shaping of the social identity of engineers from ancien régime artillery schools to the Revolutionary educational institutions, and on the social significance of Monge's descriptive geometry in this context, see also Alder 2010.

[12]Chasles' continuing curiosity for the twists and turns of this episode of cultural history seems to be also attested by the fact that he possessed a copy of the first authoritative scholarly *Histoire de la Querelle des Anciens et des Modernes* (1856) by Hippolyte Rigault (see Claudin 1881, 41, Ref. 425).

[13]Charles Perrault relied on the expertise of his brother Claude, the famous architect known for the controverted colonnade of the Louvre's eastern façade, as well as for his commented translation of Vitruvius' ten books on architecture, in which he disparaged the "excessive respect for the Antique." The Ancients, Charles Perrault noted about the so-called *reticulatum* or *maillée* (one of their techniques of construction), "had a very bad way of building which consisted in laying down the stones upon one another diamond-shaped, for each stone put in this way was like a wedge tending to set apart the two stones upon which it was laid" (Perrault 1684, 42–49). By contrast, "there is perhaps nothing more ingenious in all the Arts, he added, nor anything to which mathematics had more contributed than the drawing and the cutting of stones. . . . There shows the ingenuity of an Architect, who knows how to use the gravity of the stone against itself, & how to make it support up in the air the same weight that makes it fall. This is what the Ancients never knew, who far from knowing how to hold stones suspended in this way, never invented any good machine to elevate them" (Perrault 1688, 169–172). Claude Perrault's books on ancient and modern architecture were read and disputed in Restoration England, where they attracted the attention of Christopher Wren (see Levine 1999). While tracing back Gaspard Monge's

Although these comparisons had been a common trope of European scholarship since the fifteenth century, in setting out to systematically inventory the strengths and weaknesses of ancient and modern learning, the late seventeenth-century scholars lay bare some of the tensions inherent in the legacy of the Renaissance. As the Quarrel progressively died out on the French arena in the 1690s with the public reconciliation of Boileau and Perrault, the two leading figures of the rival factions, it passed to England, where it was kindled anew by an essay written by Sir William Temple, partly against Fontenelle's *Digression*. This soon prompted a reply from William Wotton, a friend of Robert Boyle, Robert Hooke and Isaac Newton, who found support in Perrault to complement an array of arguments derived from the new science. Thus started what Jonathan Swift would call the Battle of the Books.[14]

Although the Quarrel between the Ancients and the Moderns was fought with different agendas in different contexts across borders, certain shared features, which are significant for our current concern, may be thrown into relief. The main dividing line opposed two kinds of human endeavors, those that were conceived as depending on imitation, such as poetry and the arts, and those that would proceed by accumulation, such as Baconian philosophy and the sciences. The former implied the shaping of taste and sense by dealing with examples, while the latter relied on methods, appliances, devices, and machinery of all ilks, both material and intellectual, supposedly abridging and facilitating learning. Though this tension between imitation and accumulation did not break into open conflict until the late seventeenth century, its sources lay in the achievements of Renaissance humanism. The efforts to reconstruct and elucidate the works of the Ancients had equipped classical scholarship with an impressive arsenal of new techniques of grammatical and textual criticism, which in return would increasingly conflict with classical emulation. By enhancing our understanding of ancient texts, philology progressively instated a critical distance with respect to them that would eventually hinder imitation.[15]

On the whole, the debate hinged on history, authority, and criticism.

> The quarrel was always and everywhere about history, about the meaning and use of the past and about the method of apprehending it . . . . History was the nub of the contest, because wherever one started, whether it was with literature or philosophy, the arts or the sciences, the dispute was always about the purposes of the past, about its usefulness and authority in the present. (Levine 1981, 84–85)

While revisiting a Baconian theme,[16] Swift vividly, albeit not impartially, captured the gist of the dispute with his well-known prosopopoeia of the spider and the bee, which, jocularly put in the

descriptive geometry to its source, Chasles also pointed out stone cutting, an art based on the theory of projections, which Desargues and Frezier treated in a general and abstract geometrical way, and which Monge later showed to be dependent upon "a few principles and a few rules, even more elementary . . . more or less like the four rules of arithmetic are the common tools for all operations of computation" (cf. Chasles 1837, 355).

[14]See Levine 1991 for a detailed account of the British Quarrel and the role therein played by Jonathan Swift, natural son of Sir John Temple and secretary to his half-brother Sir William Temple.

[15]In the British Battle of the Books, the debate significantly focused on the authenticity of the so-called *Epistles of Phalaris*, now known to be a forgery by a late Greek sophist, but which William Temple had considered genuine and pinnacled in his praise of the Ancients. In response, Richard Bentley, the most brilliant representative of this new trend of textual criticism aiming at reconstructing texts, displayed his dazzling philological erudition by offering a formidable array of arguments ranging from grammar to numismatics to prove that these letters could not have been written in the sixth century BCE in Syracuse. On Richard Bentley's so-called digamma theory and his other philological *tours de force*, see Turner 2014, 67–71.

[16]*Novum Organum* (1610, I:95): "Empiricists, like ants, simply accumulate and use; Rationalists, like spiders, spin webs from themselves; the way of the bee is in between; it takes material from the flowers of the garden and the field; but it has the ability to convert and digest them." Joseph M. Levine emphasizes that the British Quarrel was not only about the rise of the new Baconian science and the Royal Society, but that it was at least as much about the authority and the value of ancient texts, hence "cannot be understood without its continental origins" (Levine 1981, 74). See also Levine 1991.

mouth of a fictional Aesop, was to become emblematic of the Moderns opposing the Ancients (Fumaroli 2001). In contrast to the wandering bee, visiting flowers in all fields and turning the past into honey, the spider, for all "his great skill in architecture and improvement in the mathematics," boasts of being obliged to no predecessor, only "drawing and spinning out all from [himself]" (Swift [1704] 1824, 237). Mathematics, however, had a history of its own, as ancient texts bear witness, and as such it appealed to both antagonistic dispositions—accumulative as well as imitative. In the context of Restoration Britain, most mathematicians hardly fitted among either the Moderns or the Ancients. On the one hand, they were engaged in the grand advancement of the experimental sciences, but on the other hand, they were also committed to reading and appropriating ancient Greek texts.

Edmond Halley (1656–1742) is representative of such a form of dual allegiance. He was both a modern empirical natural philosopher—establishing the comet's periodicity, charting the stars or mapping the terrestrial magnetic variation—*and* a learned classical scholar, who launched into editing, translating or reconstructing Apollonius' canonical texts.[17] In 1704, Halley came to the Savilian Chair of Geometry at Oxford, where he succeeded John Wallis, who had just started an edition of Apollonius when he died. Halley agreed to take up this project together with David Gregory. His first foray in the realm of Apollonius was with *On the Cutting off of a Ratio*, a text only extant in an Arabic manuscript that had been copied and partially translated by his Oxford colleague Edward Bernard. Possibly relying on his knowledge of Hebrew, Halley compared Bernard's Latin version with the Arabic and completed it with the help of Pappus' lemmas.[18] The whole text was published in 1706, together with a translation of Book VII of Pappus' *Mathematical Collection* and his own reconstruction of another of Apollonius' problem books, *On the Cutting off of an Area* (Apollonius 1706). As for the *Conics*, the original plan was that Gregory would edit the first four books for which there is a Greek text, while Halley would deal with the last four, books V to VII being extant only in Arabic, and book VIII being lost and only known from Pappus' lemmas and an Arabic epitome. Gregory died in 1708, and Halley alone published all eight books of Apollonius's *Conics* in 1710, the first four in both the original Greek and Latin translation, the next three in Latin translation alone, and the last one as a conjectural reconstruction.[19]

Halley's scholarship partly grew out of the legacy of Renaissance humanism. His work on Apollonius was based on Federico Commandino's Latin translations of Apollonius's first four books of the *Conics* (1566), and of Pappus' *Mathematical Collection* (1588), but these were read and augmented in a context in which textual criticism and textual reconstruction had gathered unprecedented momentum. Another trait was the pondered balance Halley maintained between the respective merits of the Moderns and the Ancients. In his preface to *On the Cutting off of a Ratio*, Halley praised the *Algebra speciosa*, the Arithmetic of the Infinites, or the doctrine of fluxions, while also insisting that "nothing, however, is thereby removed from the glory of the Ancients, who brought Geometry to such perfection that it would perhaps have been easier for posterity to admire it, than to reach it in its investigations without the writings of the Ancients" (Apollonius 1706, i).[20] In the same vein, Halley dismissed Descartes' claim that Apollonius had failed to solve the four-line locus problem,

[17]For a detailed analysis of Halley's work on Apollonius, see Cook 1998, 334–340, and Fried 2011.

[18]Although Halley says nothing about Hebrew in this connection, one may surmise that his learning it at school (Cook 1998, 26) later helped him cope with the Arabic texts of Apollonius. See also Fried 2011, 5: "When he entered Queens College, Oxford, [Halley] came, as Aubrey says, 'well versed in Latin, Greek, and Hebrew,' … and that was more than a mere ornament of a classical education: he valued and used his knowledge."

[19]On the meaning of Halley's reconstruction of Apollonius' lost book of the *Conics*, see Fried 2011.

[20]"Nihil tamen inde Veterum gloriæ detrahitur, qui Geometriam ad eam provexere perfectionem, quam facilius forsan fuisset posteris mirari, quam absque Antiquorum scriptis investigando assequi."

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and instead pointed out Newton's alternative geometrical construction in Part V, Book I of the *Principia*.[21]

In the next generation, the Scottish mathematician Robert Simson (1687–1768), a professor at the university of Glasgow, made a further step in the path Halley had opened. In 1711, Simson spent a year in London. There he met Halley, who encouraged him to study the ancient geometers and gave him a copy of his 1706 opus, including a translation of Pappus enriched with his own personal handwritten notes. For all his knowledge of ancient mathematics, in this book Halley had explicitly acknowledged that he could not make sense of Pappus' comments on Euclid's lost *Porisms* in Book VII of the *Mathematical Collection*. In a footnote which Simson (and Chasles after him) would recall, Halley had indeed avowed that "the description of the Porisms given up to this point was neither understood by me nor will it be of use to the reader" (Apollonius 1706, XXXVII).[22] Halley's remark soon caught the attention of Simson, who embraced the task set forth for him "with the sanguine expectation of a young man" (Robison 1817, 371).[23] Directing all his efforts to the restoration of Euclid's lost treatise, he made a first breakthrough in 1723 with a partial interpretation of Pappus' lemmas to the *Porisms*, later to be completed by a full treatise published posthumously in 1776 (Simson 1723, 1776).

In the 1830s, Chasles thoroughly studied Halley's and Simson's previous attempts at restoring ancient geometrical texts, but he approached the problem posed by the comparison between ancient and modern geometry from a new perspective, which was connected to the context of early-nineteenth century mathematics. Among the many topics covered in the *Aperçu historique*, Chasles only dealt with Euclid's lost work in a provisional or indirect way, in connection with Pappus and ancient geometry, as well as with the tentative seventeenth-century reconstructions thereof from Fermat to Stewart, or from the vantage point of the new geometry that Monge had initiated.[24] However, the reference to the ancient *Porisms* played a pivotal role in his reflections on geometry, and underlay his conception of mathematics and history of mathematics as being tightly intertwined. In a well-known page of the *Aperçu historique*, Chasles distinguishes three branches of geometry: firstly, "ancient Geometry;" secondly, what he called "mixed Geometry," or "analytic Geometry," or "Descartes' Geometry;" and eventually "a third kind of Geometry," *viz*. the Geometry "whose first seeds were in Euclid's *Porisms* and were preserved by Pappus in his Mathematical collections" (Chasles 1837, 116–117). About the latter, Chasles articulates his point in the following terms:

> This third branch of Geometry which today constitutes what we call the new Geometry is exempt from algebraic computations, although it makes good use of the metric relations of figures as well as of their descriptive relations [*relations de situation*]; but it only takes into account ratios of rectilinear distances of a certain kind, which require neither the symbols nor the operations of algebra.
>
> This Geometry is the continuation of the geometrical analysis of the Ancients, from which it does not differ as to the goal and the nature of its speculations; but over which it presents immense advantages, owing to the generality, uniformity and abstraction of its conceptions and methods, which have replaced the particular propositions, incomplete and without connection, which formed all the science and the unique resource of the Ancients. (Chasles 1837, 117)

[21]Michael N. Fried describes Halley's relationship with the Greek mathematical past in the form of a distant dialogue with the Ancients. In contrast to a mere "completion," a "restoration," as Halley meant it, would imply an awareness of the difference between the past and the present (Fried 2011, 28).

[22]"Hactenus Porismatum descriptio nec mihi intellecta nec lectori profutura." For Simson's and Chasles' reference to Halley's point, see Simson 1776, 318; Tweddle 2000, 14; Chasles 1837, 12.

[23]The entry titled "Simson, Dr. Robert," since the third edition of the *Encyclopedia Britannica*, attributed to a Dr. Robison, is the main source of biographical information about Robert Simson (see Trail 1812, 6).

[24]In addition to Note III, "Sur les Porismes d'Euclide" (Chasles 1837, 274–284). Chasles touched upon the topic of porisms in the *Aperçu historique* in connection with Pappus (§8, 11–15), Simson (§30–37, 33–43), Fermat (§14, 67–68), and Matthew Stewart (§31–36, 180–183).

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How this new Geometry, to which Chasles himself contributed in no small measure, was to be conceived as the "continuation" of ancient geometrical analysis will hopefully appear clearly, once the original way in which the French geometer connected to the work of his seventeenth-century British predecessors is made explicit. With regards to Euclid's lost *Porisms*, Chasles observed that the interpretive problem on the sole basis of ancient sources (*viz*. Pappus and to a far lesser extent Proclus) was so perplexing that "the famous Halley, so deeply versed in ancient Geometry, confessed that he understood none of it" (Chasles 1837, 12). Hence Chasles gave full credit to Simson for "discover[ing] the meaning of several of these enigmas, as well as the form of these statements which was characteristic of this kind of propositions" (Chasles 1837, 13). However, he judged Simson's work largely incomplete for, despite all the ingenuity to which it bore witness, it nevertheless failed to address such crucial issues as the relationship between ancient porisms and the would-be substitutive modern methods into which they presumably transformed (Chasles 1837, 14). Raising this question on Chasles' part was a barely hidden performative step, since these modern methods were themselves shaped to a great extent by his own instrumented reading of Pappus, owing to the interpretive tools he himself created, a notational technology which proved efficient on both scores, for "restoring" the ancient porisms as well as for advancing the new Geometry. In this sense, carrying on in the direction indicated by Simson was only made possible owing to Chasles' suitably transposing the ancient practice into another key, in conformity with his resolute stance in nineteenth-century debates over a controverted issue.

3. A nineteenth-century dilemma

In the *Aperçu historique*, when presenting the work of Desargues, Chasles inserted a significant note, in which he claimed that there were no considerations based on the principles of perspective in ancient mathematics. This statement, however, would open a breach in contemporary consensus, for the question of whether the Ancients ever knew the uses of perspective was generally settled in a different way. As a rule, Chasles first observed, one would be tempted, *prima facie,* to tip the scale in favor of an affirmative answer, for the method of perspective, once acquired, could not but appear as a "natural" one—a seeming fact apparently supported by the way the Ancients conceived of the generation of conics in a cone with a circular base. But this view, he argued, was not only prevalent among geometers,

> it was [also] strengthened in recent times by the particular sentiment voiced by M. Poncelet on Euclid's porisms, which would presumably have been obtained by this method. . . . But, with all due respect for the opinion of this famous geometer, we must avow that no trace of it is to be found in the writings of the Ancients, no clue would allow us to share it in this circumstance. We believe, on the contrary, that the method of perspective, as we practise it today in rational Geometry, was not being used among the Greek school. Hence, until a more detailed and broader survey, we will attribute this method to the Moderns, and we will say that Desargues and Pascal have the merit to have been the first to apply it to the theory of conics. (Chasles 1837, 74)

Desargues was thus credited with two important innovations in the study of conics: First, (rightly or wrongly), with considering conics in the cone in all possible positions of the intersecting plane, without using the axial triangle, as the Ancients did,[25] and second, with the conception of the

[25]Jan P. Hogendijk observes that Chasles was partially wrong in his appreciation of what was new in Desargues in comparison with Apollonius: "Chasles believed that Apollonius had only intersected the (oblique) cone with planes in special positions. Therefore, Chasles believed that Desargues' main contribution was the fact that he had intersected the cone with arbitrary planes. Later authors have shown that this had in fact been done by Apollonius" (Hogendijk 1991, 3). Chasles indeed thought that the Ancients only intersected the cone with planes perpendicular to the so-called axial triangle (Chasles 1837, 79),

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properties of the circle at the base of the cone as being "appropriated" by the conic sections themselves. "This [latter] idea," Chasles insisted, "which seems to us so simple and so natural today because we are accustomed to the procedures of perspective, and various other modes of transformation of figures, did not come to the minds of the geometers of Alexandria, for no trace of it can be found in their works" (Chasles 1837, 75). While on the one hand Chasles explicitly opposed Poncelet's view that Euclid's lost *Porisms* would attest to the fact that the Ancients had relied on considerations pertaining to perspective in their geometrical analysis, on the other hand he nevertheless embraced a pattern of discourse about the various kinds of geometry that he derived from Poncelet and, to a lesser extent, Gergonne.

In the introduction to the *Traité des propriétés projectives des figures* (1822), Poncelet produced a string of subtle arguments to delineate the characteristics of what he took to be distinct ways to practice geometry, namely, in his own terms, analytic Geometry, Monge's "general Geometry," and "ordinary or ancient Geometry." Poncelet had already engaged in a public debate with Joseph-Diez Gergonne over the strengths and weaknesses of both pure and analytic Geometry a few years earlier, in the pages of the *Annales de mathématiques pures et appliquées*.[26] In an open letter to Gergonne, Poncelet replied to some thoughts by the former on the comparison of the competing geometries, included in two papers published in the seventh issue of the *Annales* (Gergonne 1816–1817a, 1816–1817b). Gergonne had indeed offered solutions to some well-known geometrical problems, supposedly demonstrating that when "suitably handled," analytic geometry surpasses pure geometry in elegance and simplicity (Gergonne 1816–1817a, 289). Poncelet responded that although he admired the elegant way in which Gergonne was able to bend algebraic analysis to the most difficult questions of geometry, he could not accept the demotion of purely geometric methods. In his view, distinctions had to be made. If pure geometry were to be understood only as that of the Ancients,[27] Poncelet conceded that he would have sided with Gergonne. But if, by pure geometry, one meant "that geometry in which one simply refrains from using the method of coordinates, or even any kind of computation whatsoever which would allow to momentarily lose sight of the figure one deals with," or in other words, "that geometry cultivated by the Moderns," then Poncelet felt compelled to starkly disagree, substantiating his view with a purely geometric proof of a theorem, generalizing that which Gergonne had adduced in support of his own (Poncelet 1817–1818, 142–143).[28]

Gergonne wittily made amends by hinting at what he referred to as an apology of exaggeration, a frame of mind for which hitting hard is to be preferred to hitting fair, for a curved stick can only be straightened when forcefully bent in the other direction. Basically, however, Gergonne expressed his full agreement with Poncelet about the equal merits of analytic and pure geometry in their own respective ways, provided the latter refers to Monge's geometry, rather than to the geometry of the Ancients and their modern followers. Gergonne added that each of these two branches of geometry would in fact benefit from being cultivated in isolation, without the help of the other. For this reason, he expressed the wish that, in pure geometry, one could even "evade all use of proportions, because at bottom these are nothing more than disguised computations"

which was not the case, as Zeuthen later pointed out (Zeuthen 1886, 65). Hence Chasles' point was partly right and partly wrong, for it was "based on a misinterpretation of the *Conics* and not of the *Brouillon Project*. [Besides] Pascal said correctly [and so did Chasles after him] that Apollonius used the axial triangle in his proofs and that Desargues did not" (Hogendijk 1991, 42). Chasles' wrong belief that Apollonius would only consider the intersecting plane perpendicular to the axial triangle, does not seem to interfere whatsoever with his main contention here that the Greeks did not resort to considerations of perspective in their geometry as Poncelet claimed.

[26]For a comprehensive discussion of the Gergonne-Poncelet debate, see Lorenat 2015.

[27]Poncelet assumes that the Geometry of the Ancients may be taken to range from Euclid and Apollonius to Vieta, Fermat, Viviani, and Halley.

[28]As an example of the way analytic geometry may also deal with a problem depending only on the geometry of the ruler, Gergonne had provided the construction of a problem taken from Pappus, albeit in an extended form, *viz.* "in a line of the second order, to inscribe a triangle whose prolonged sides go through three given points" (Gergonne 1816–1817b, 325).

(Gergonne 1817–1818, 160). He exhorted Poncelet to make his case even more compelling by publishing his results.

Reminiscences of this exchange are clearly perceptible in the 1822 *Traité des propriétés projectives des figures*, in which Poncelet pursued even further the comparison of the various ways to practice geometry. Not only did Poncelet hold that "ancient or ordinary geometry" lacks generality, but also that it can be reproached for "a too frequent and too extended use of the mechanism of proportions, which at bottom is nothing but a disguised computation, as a knowledgeable geometer, M. Gergonne, observed" (Poncelet 1822, XIX). In reference to the well-known schemata of inference characterizing the language of ratios and proportions of Euclidean geometry (*viz*. *invertendo*, *permutando*, *ex aequali*, etc.), Poncelet coined the phrase, "mechanism of proportions." He always used this phrase as a badge of the inherent limitation of ancient geometry, and turned Gergonne's point about proportions being "disguised computations" into one of the cornerstones of his criticism of ancient geometry (Poncelet 1822, XXVII, 17). On the other hand, he pointed out that Monge's geometry did not develop as pure geometry proper, cut out from analysis, but still owed part of its generality to "the mixing, the inner fusion so to speak, of the two ways to deal with the magnitude in figure" (Poncelet 1822, XI). It begged the question of how exactly to characterize that in which consisted the power of analysis thus transfused into geometry, why "ordinary or ancient geometry" was constitutively deprived thereof and, eventually, how this defect could be remedied so as to endow it after all with an equivalent power.

The core of Poncelet's elaborate answer to these questions lies in his working out the notion of implicit reasoning, that is the kind of reasoning that occurs when one reasons with indeterminate magnitudes, as one does in algebra when one uses abstract signs to represent magnitudes which are left entirely undetermined. The essential weakness of "ordinary Geometry, which is often called *synthesis*," consists in the fact that,

> the figure is described, [that] one never loses sight of it, [that] one always reasons with magnitudes, real and existing forms, [hence that] one never draws consequences that cannot be depicted to the imagination or the sight by perceptible objects … in one word, in this restricted Geometry, one is forced to recommence the whole series of the initial reasonings, as soon as one line, or one point is shifted from the left to the right of another. (Poncelet 1822, XIII)

This being the main reason why such geometry ranked lower than both Monge's or analytic geometries, Poncelet's research program aimed at finding ways to implement implicit reasoning in pure geometry by gaining new leeway with respect to the figure.

However, Poncelet noted, this was not wholly unprecedented in the history of geometry, and here Euclid's porisms began to come into new focus.

> [*Implicit reasoning*] is what occurs in Geometry when the figure gets complicated, or when the relations between its parts increase, because it is no more possible to make out, by merely looking at the figure, the order of magnitude or the respective position of these parts. It is again what occurs when certain of these parts are the object of an inquiry on the basis of the figure, or when they are assumed to be unknown in both magnitude and position; and this is why the way which the Ancients called analytical and to which they attached so much importance, was not entirely devoid of this generality, or force that belongs to algebra. (Poncelet 1822, XII)

Poncelet's view of ancient Geometry evinced a tension between the essential weakness of traditional synthesis being riveted to the actual figure, and the suggestion that ancient geometrical analysis, to a certain extent, adumbrated modern projective geometry. Indeed, in this connection, Poncelet explicitly assumed that "Euclid had composed a Treatise in three books, on *Porisms*, whose considerations were probably very close to those of the theory of transversals and the geometry of the ruler" (Poncelet 1822, XXIV). Referring to Commandino's edition of Pappus, he observed that the meaning of Euclid's *Porisms* had been obscured and lost over the centuries, and, while recalling the many attempts at recovering it, he did not acknowledge Simson's work as particularly outstanding. However, without having any pretensions to explaining Pappus' text, he claimed that,

> one will nevertheless remain convinced, or at least one will strongly tend to believe, that Euclid's Treatise on Porisms hardly dealt with any other topic than those general and abstract properties of figures whose character would be defined with difficulty in the language of ancient Geometry; in one word, that the Porisms were genuine projective properties, which Euclid derived from considerations pertaining to perspective which had become familiar to him, judging by a Treatise that he published on this very topic. (Poncelet 1822, XXV)

Such emphasis on Euclid's lost *Porisms* being an integral part of Poncelet's whole pattern of discourse (*viz.* the correlation between implicit reasoning, freedom with regard to figures, and the role these propositions supposedly played), it comes as no surprise that, since Chasles embraced this very pattern, he was also led to pay heed to this ancient text. However, he approached it in a more cautious and reflective way than Poncelet. Unlike Poncelet, Chasles did not ignore the historical distance so to speak, in compliance with some Comtean philosophical insights.

His elder by five years, Chasles had always been in friendly terms with Auguste Comte (Pickering 2009, 415), to whom he referred in print as his "old comrade from the École Polytechnique"[29] (Chasles 1837, 415). In spite of their offbeat start, they shared the same scientific training and belonged to the same milieu. However, Comte advanced faster than Chasles who, for a few years, spent the leisurely life of a young man attending Parisian theaters while working as a stockbroker, before actively engaging in mathematics with his first publications around 1827. At that time, Comte had already made a name for himself with his *Système de politique positive* (published in 1824), his series of open lecture courses, which began on April 2, 1826, in the presence of such major figures of the scientific elite as Arago, Fourier or Humboldt, and the first volume of his *Cours de Philosophie positive* (eventually completed in 1830). With regards to innovative views on the problem posed by the comparison between ancient and modern geometries, Comte was also, at first, a good length ahead of Chasles. As early as 1818, young Comte had already translated John Leslie's *Geometrical Analysis* (1809) into French.[30] This book by the Scottish mathematician was essentially based on the reconstructive work of Halley, Simson, and Playfair, and drew from ancient sources to present a comprehensive view of Greek analysis in the form of a modern textbook. In a three-page footnote at the end of the volume, Leslie listed his sources for the eight ancient treatises, which, according to Pappus, belonged to the domain of analysis, that is mainly Halley's 1706 and 1710 editions of Apollonius and Simson's work on Euclid's lost *Porisms* (Hachette 1818, 439–442). However, all these historical references disappeared in the French translation, which, as Comte confided in his correspondence, would eventually result in a "bad book" (Valat 1870, 36). In later years, he would take another approach to this old topic.

In the tenth *Leçon* of his *Cours de Philosophie Positive*, Comte defined geometry as the science whose object is the measure of extension (*la mesure de l'étendue*), but he considered that there

[29]Although, due to the break of the War of the Sixth Coalition and the ensuing suspension, Chasles had already left when Comte was first admitted in November 1814.

[30]Jean Nicolas Hachette, Monge's successor as professor of descriptive geometry and Comte's teacher at the École Polytechnique, wanted the translation of Leslie's book to be published as an appendix to his second supplement to descriptive geometry (Hachette 1818). He proposed this menial task to young Comte, a fellow Republican, to help him earn money and make himself known in being useful to the sciences.

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were essentially two different ways of dealing with geometrical questions, two different methods, hence two geometries whose philosophical character he set out to accurately define. Ordinarily, Comte observed, one referred to these two geometries by using the expressions *synthetic* and *analytic* geometry, but he added that he "would prefer by far the purely historical denominations *Geometry of the Ancients* and *Geometry of the Moderns*, which, at least, have the advantage to prevent one from disregarding their true characteristic" (Comte 1830, Leçon 10, 383–384). However, he proposed to call them *special* and *general* geometry respectively, as these alternative labels were more in step with a dogmatic presentation. As is well known, in Comte's view, any science is susceptible to two distinct modes of exposition. According to the historical mode, one presents knowledge in succession, in the same order in which the human mind effectively acquired it, and following—as far as possible—the same series of steps. According to the dogmatic mode, by contrast, one presents the system of ideas as it could be conceived today by one single mind, so that the whole science is, so to speak, "remade" from this privileged standpoint.

In striving to organize an ever-increasing amount of knowledge into a coherent whole, the human mind displays a constant tendency to substitute the dogmatic order for the historical one. However, Comte warns, it would not be possible to eliminate the historical mode completely. Indeed, the dogmatic mode alone would not permit a perfectly rigorous exposition. Since, as a rule, it requires a new elaboration of knowledge already acquired, it cannot be suitably applied, in each and every epoch of science, to the most recent parts, which can only be presented according to the historical mode. Thus, "any effective mode of exposition inevitably proves to be a certain combination of the dogmatic order with the historical one" (Comte 1830, Leçon 2, 80). In each science, an adequate balance must therefore to be found, although, Comte insisted, any rational classification of the sciences "will always necessarily contain something, if not arbitrary, at least artificial" (Comte 1830, Leçon 2, 76). The need to find the right balance between both modes, as well as the recourse to well-chosen artificial devices to perform such balance, will also prove essential in the relationship between ancient and modern geometry. Indeed, it will be seen below that, on this score, Chasles put Comte's ideas to good use. As for Comte's own agenda, the previous general considerations would apply to geometry as to any other science, so as to make a perfectly rational exposition of geometry possible. However, for our present concern, only the following salient features of Comte's doctrine will be of interest.

The fundamental difference between the "Geometry of the Ancients" and the "Geometry of the Moderns" does not consist in the kind of deductive tools that are employed, but in the nature of the questions that are dealt with. Contrary to a common view, Comte claimed, it is not the use of calculus that distinguishes modern geometry from ancient geometry. For one thing, he noted, the Greeks did not totally ignore all kinds of calculus in geometry, for they heavily relied on the theory of proportions which "to them was, as a means of deduction, some sort of actual equivalent of our current algebra, albeit a very imperfect and above all an extremely limited one" (Comte 1830, Leçon 10, 384). In this, Comte concurred with Gergonne, without, however, ever mentioning the debate that had unfolded in the *Annales* more than a decade earlier. Besides, Comte remarked, certain methods exist, such as Roberval's method for finding tangents, which he deemed essentially modern in character, although they owe nothing to calculus whatsoever. Considered as a whole and in a state of perfection, he explained, geometry should embrace all imaginable figures on the one hand, and on the other hand, it should be able to discover all properties of each figure. Thus, geometry can be organized according to two different plans: either by gathering all questions, however diverse, about one and the same figure, and by separating those that bear on different figures; or by uniting all similar enquiries under one and the same point of view, no matter what figures they apply to, and setting apart those pertaining to different properties, even if they hold for one and the same figure.

In short, geometry can go by the figures, or by the properties. Ancient geometry would adopt the first plan, whereas modern geometry, after Descartes, distinctly favored the second one. The Greeks, for instance, would exhaustively study the properties of one curve before passing to the

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next, and would then start all over again, despite considering the same properties as before. By contrast, modern geometers tend to develop uniform methods to deal with any single problem, whether of tangents, rectification, curvature, or anything else, whatever the curve considered, whether it be a conic section, a cissoid, or a transcendental curve. This is the original meaning Comte attached to the terms *special* and *general,* which he used to refer to these different geometries. What form, then, would the necessary balance between them take, and beyond, why would such a balance be needed in this particular instance?

> Owing to the very nature of the subject matter, it is necessarily impossible to do without the ancient method absolutely, for, no matter what we do, it will always dogmatically serve as a preliminary basis to the science, as it was the case historically. The reason for this is easy to understand. General geometry being essentially founded . . . upon the use of the calculus, that is upon the transformation of geometrical considerations into analytical ones, such a way to proceed could not take hold of the topic immediately from the outset. (Comte 1830, Leçon 10, 393)

In his third *Leçon*, Comte had indeed elaborated a specific notion of "calculus," which he used as a synonym for "abstract mathematics," a central notion for his whole dogmatic exposition of the mathematical sciences. However, in spite of its centrality, Comtean "calculus" as such could not unfold its operations out of nothing; it had to start from some input received from outside its own sphere.

> Its starting point is, constantly and necessarily, the knowledge of precise relations, that is *equations*, between various magnitudes considered simultaneously, which by contrast is the ending point of concrete mathematics. However complicated or indirect these relations may be, the final goal of the science of *calculus* is to always deduce the values of the unknown quantities from those of the known quantities. (Comte 1830, Leçon 3, 143–144)

Hence the conclusion:

> And so, owing to its very nature, the geometry of the Ancients will always have a share as a first necessary part, more or less extended, in the total system of geometrical knowledge. It constitutes a rigorously indispensable introduction to *general* geometry. But it must be reduced to this in a completely dogmatic exposition. (Comte 1830, Leçon 10, 395)

The problem then is to precisely delimit the domain of that "Geometry of the Ancients" when conceived as such an indispensable "preliminary basis" for modern geometry. Comte comes to grips with this problem in his eleventh *Leçon,* and reaches the conclusion that its perimeter encloses the study of the straight line, the quadrature of polygons and the cubature of polyhedra, to the exclusion of any other topic. The option of including the study of the circle and conic sections is discussed, but eventually rejected as ultimately unnecessary. Descriptive geometry, however, offers an opportunity for an interesting digression on Monge which, by contrast, will help cast light on Chasles' own take on the relationship between ancient and modern geometry.

Comte defined the study of the straight line, one of the topics to be ranked in the realm of ancient geometry, as the problem of determining the various parts of an arbitrary rectilinear figure from one another, a problem which ancient geometers solved by graphical means. When the original figure was not planar, projections were needed. In this connection, Comte alluded to Vitruvius' and Ptolemy's *analemnata*, which were systems of projections they devised in their dealings with problems pertaining to the celestial sphere. Allegedly, this supported the view that "the Ancients knew the elements of descriptive geometry, although they did not conceive them in a distinct and general way" (Comte 1830, Leçon 11, 411). More generally, Comte noted,

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descriptive geometry should be granted an intermediary status between the Geometry of the Ancients and the Geometry of the Moderns not only from the viewpoint of the historical order, but also from that of the dogmatic one. While being akin to the former by the genre of its solutions, namely graphical ones, descriptive geometry gets closer to modern geometry by the nature of its questions.

However, in spite of all this, in Comte's view, descriptive geometry should not be included in the "preliminary basis," for essential reasons involving the way in which its true philosophical significance should presumably be assessed.

> It is very important not to be mistaken about the real meaning which belongs to it [*viz. descriptive geometry*] so specifically, as have done, particularly in the immediate aftermath of this discovery, those who saw in it a means to expand the general and abstract domain of rational geometry. As it turned out, these ill-conceived expectations were by no means met. Is it not obvious, indeed, that descriptive geometry only has a special value as a science of application, as constituting the true proper theory of the geometrical arts? Considered from the abstract viewpoint, it could not introduce any truly distinct order of geometrical speculations. (Comte 1830, Leçon 11, 414–415)

This was a serious point of contention for Comte with Poncelet and Chasles. When contrasted with Comte's above statement, Chasles' well-known praise of Monge's geometrical style "without figures" strikes one in hindsight as being, at least partly, an elegant remonstrance to his philosophically minded comrade.

> Monge's descriptive geometry is a source of good doctrines which has not dried up yet. After having recognized therein the germ, more or less developed, of several methods which increase the power and extend the domain of Geometry, we also see in it the origin of a new way to write and to speak this science. Style, indeed, is so intimately connected to the spirit of the methods, that it must advance with them; as well as, if it takes the lead, it must also have a powerful influence on these methods and on the general progress of science. This is undeniable and does not need to be proved. (Chasles 1837, 207–208)

Chasles thus decidedly distanced himself from Comte's view of Monge, but at the same time he adopted his lexicon and his semantic oppositions—namely, abstract/concrete, general/special, dogmatic/historical—as for instance when characterizing ancient geometry a few lines below.

> Ancient geometry is spiked with figures. The reason for this is simple. Since general and abstract principles were lacking at the time, each question could only be dealt with in a concrete state, upon the very figure which was the object of the question, and whose sight alone could bring one to discover the elements needed for the sought demonstration or construction. (Chasles 1837, 208)

Chasles gathered these different threads in an original way. Like Poncelet, he regarded Monge as the initiator of the current of research intent on opening paths to implicit reasoning in pure geometry by granting new freedom with respect to figures. But unlike him, Chasles denied that the Ancients had had projective properties of their own, however embryonic or prototypal. Unlike Comte, he fully perceived how Monge's geometry paved the way to "a truly distinct order of geometrical speculations," while he gainsaid that modern geometry should only amount to mere analytic Geometry. Like Comte, however, Chasles thought of the relationship between the "Geometry of the Ancients" and the "Geometry of the Moderns" as one that implied a measure of reworking or rewriting, so that historical continuity and the growth of knowledge could only be correctly assessed if diffracted through the prism of a suitable balance between the historical and

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the dogmatic modes. In following Comte's above-mentioned suggestions, Chasles knew that any rational classification of the sciences would involve "something, if not arbitrary, at least artificial," but that having recourse to such an artificial element would not downgrade the exposition to a mere motley collection of ancient and modern topics. On the contrary, provided it was suitably chosen, it would permit one to achieve a fully rational exposition of geometry, both ancient and modern, without cancelling the historical distance separating them, or projecting unwarrantedly modern concepts onto ancient practices.

Comte had eloquently made clear that there would always be a seam between the "Geometry of the Ancients" and the "Geometry of the Moderns," an inescapable fact it was both useless and illusory to try to conceal. The best one could do, and this would be no small achievement, was to figure out how both Geometries should best be stitched together so that the true meaning of their respective methods be fully grasped and a perfectly rational exposition of the whole of Geometry be possible. In his third *Leçon*, after explaining what "abstract mathematics" was, Comte significantly added: "so as to conceive sharply the true nature of a science, one always has to suppose that it is perfect" (Comte 1830, Leçon 3, 144). In other words, the "true nature of a science" can only be revealed by an intervention on the expositor's part. No such disclosure is possible unless one makes a strong assumption, whatever artificial devices may be needed to implement it. One indeed must assume that the science one deals with has been brought to completion, and is hence perfect from the dogmatic viewpoint, although it may still prove to be in an imperfect state from the historical one (Macherey 1989, 92). In the case of geometry, both ancient and modern, Chasles had recourse to such an artificial device, which would qualify as an interpretive tool and an instrument for further research, and thus make it possible to embrace the whole of geometry from an assumedly privileged standpoint: the concept of the anharmonic ratio. In contrast to his seventeenth-century predecessors, Chasles could read Pappus' lemmas on Euclid's lost *Porisms* through the lens of a notational technology of his own devising, supposedly connecting ancient and modern methods. But here again, he drew on previous work.

4. A criterion turned into an interpretive tool

In the first chapter of his *Traité des propriétés projectives des figures*, Poncelet had indeed suggested a criterion of projectivity for metric relations, which prepared the ground for Chasles' further step. Poncelet defined the projective properties of a given figure as those which still subsist in all central projections of the said figure.

Among these properties, the "graphic" ones would be those that depend only on the disposition of points and lines, and do not need any criterion, since it is easy to recognize whether or not they are projective, "merely on the basis of their statement, or the inspection of the [*corresponding*] figure" (Poncelet 1822, 5). However, for those "which concern the relations of magnitude and which one can call metric, nothing for sure can indicate *a priori* if they hold in all projections of the figure to which they belong" (Ibid.). Considering an arbitrary spatial figure composed of the points $A, B, C, D, \ldots$ and a center of projection S from which the projecting lines are drawn to the corresponding new points $A', B', C', D', \ldots$ (see figure 1, on the right), Poncelet asks for a criterion which may grant that any property about the original figure $A, B, C, D, \ldots$ would still hold for the projected one $A', B', C', D', \ldots$ Since any metric property amounts to a relation involving distances between some of the points composing the figure, Poncelet observes that two straight lines SA, SB, drawn from the center of projection, determine two different segments AB and $A'B'$ on two arbitrary transversals. Any segment AB may thus be seen as the base of a triangle SAB (see figure 1, on the left) with sides $SA = a$, $SB = b$, and perpendicular p, so that

$$\text{Area of } SAB = \frac{p.AB}{2} = \frac{m.a.b}{2}, \text{hence } AB = m.\frac{a.b}{p}$$

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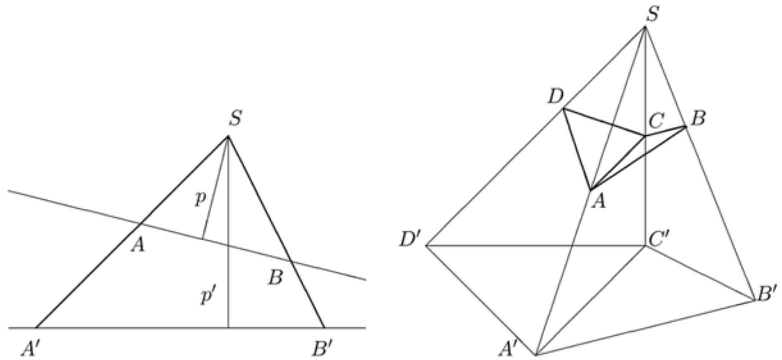

Figure 1. Poncelet's criterion of projectivity for metric properties.

where m is constant, depending on the angle $\widehat{ASB}$ (*viz.* the sine of the projecting angle). Now, if one considers a metric relation involving the segments connecting the points of the figure A, B, C, D, ..., and if one replaces the distances AB, CD, etc. respectively by their expressions in terms of the sides a, b, c, d, ..., the corresponding perpendiculars and constants m, the condition of projectivity imposes that

> these straight lines or distances [*viz.* a, b, ...] must vanish from the result of the substitution, either owing to partial reductions, or as factors common to all terms, so that there would remain only a relation between the constants m, m', ... if one replaces simultaneously the perpendiculars p, p', ... by their values for each particular triangle SAB, $SA'B'$. ... Conversely, if these conditions occur, the relation under survey will necessarily be projective, namely it will hold for all figures $A'B'C'D'$ ... which can be regarded as the projections of the first one. (Poncelet 1822, 7)

However, in certain cases there is a way to know beforehand whether a relation is projective, as stipulated in the following:

> art. 11. There exists a very large class of relations for which the perpendiculars p, p', ... all vanish together with the a, b, ... from the result of the substitution, without one having to replace them by their values in the corresponding triangles, as in the above general assumption. (Poncelet 1822, 8)

Poncelet's criterion of projectivity for metric relations could then be obtained by merely considering their form.

> art. 20. Among the infinitely many projective relations satisfying the particular conditions stated in art. 11, some deserve to be remarked above all by the facility with which their character can be assigned in advance, and, hence, with which they can be recognized on the basis of their mere statement, without it being necessary in any way to carry out any of the prescribed substitutions.
>
> Assuming, indeed, that a relation or an equation of two terms, without denominators, each being composed of the same number of factors merely expressing distances between various points of a given figure; assuming moreover, as the case may be, that one member, or even both, be multiplied by any arbitrary absolute number, it is evident that this relation will satisfy the above-mentioned conditions

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1°. *if the very same letters occur in the linear factors composing both members*;

2°. *if, to any distance belonging to one of the members, there is another corresponding one in the second member, such that the second segment should be in the same direction as the first, or lie in the same straight line.*

For, owing to the first assumption, all the projecting lines a, b, c, ... will vanish from the result of the substitution, and, owing to the second, the same will occur with the perpendicular p, p', ...; so that there will only remain a relation between the quantities m, m', ... which are independent from one another.

Let it be remarked that in order to reach this consequence, it is in no way necessary to have recourse to the principles of algebra; since it is enough to possess the most simple notions of the ordinary theory of proportions or geometrical ratios, for the relation just mentioned can easily be brought back to the equality of two composed ratios, so that the previous reasonings can then be themselves reduced to most simple considerations on these kinds of quantities. (Poncelet 1822, 11)

Poncelet contented himself with giving "a very simple example of this sort of relation," namely the relation of four points on a straight line for which the proportion $\frac{CA}{CB} = \frac{DA}{DB}$ holds, "a property which was known by the Ancients, as Propos. CXLV of the VII$^{\text{th}}$ book of Pappus' *Mathematical Collection* shows" (Poncelet 1822, 12). Remarking that this particular relation belongs to the class of those defined in art. 20, Poncelet characteristically sidestepped the problem posed by a clear-cut delineation of the general class as a whole. While on the one hand, he traced back the above-mentioned particular relation to the harmonic proportion of the Greeks,[31] curiously enough, on the other hand, he more or less conflated it with a more general property first enunciated by the mathematician Charles-Julien Brianchon. In the hands of Chasles, however, this property would later prove best suited to characterize a whole class of properties encompassing by far that of harmonic proportion.

M. Brianchon arrives at this result, [Poncelet observed,] as well as at a few others, in a more or less similar way, in observing that "from four fixed straight lines drawn from one and the same point, under any angles, and met by an arbitrary transversal at A, B, C, D,

$\frac{AC}{AD} : \frac{BC}{BD} =$ *Constante*." (Poncelet 1822, 12)[32]

In contrast to Brianchon, who did not give this last ratio any name and thus failed to identify the corresponding general concept, Chasles made it into a new tool, both interpretive and constructive, the so-called "anharmonic ratio," by means of which he read in a new light the very same Pappus' lemmas on Euclid's *Porisms* to which Poncelet had paid heed. In so doing, Chasles nevertheless shifted the main emphasis from Prop. 145 of Pappus' lemmas to Prop. 129 (see figure 2), now endowed in his view with fundamental significance.[33]

[31]*Ibid.*: "Putting the above relation in this form $\frac{DA-DC}{DC-DB} = \frac{DA}{DB}$, one sees that it amounts to the *harmonic proportion*, as it was defined by the Greeks (Pappus, *Mathematical Collection*, Book III), proportion in which only enter the distances of point D to the three other points A, C, B; this is why the distance of point D to point C, intermediary between A and B, is called the *harmonic means* of the two other DA and DB."

[32]Brianchon's statement is taken from Brianchon 1817, 7.

[33]Chasles observed that Pappus' propositions 136, 137, 140, 142, and 145 are either particular cases or the converse of that main proposition. As for the proposition 145, it concerns the particular case of the harmonic ratio, not the more general case involved in the consideration of the anharmonic ratio. For a translation of Pappus' propositions, see Jones 1986, 268–278.

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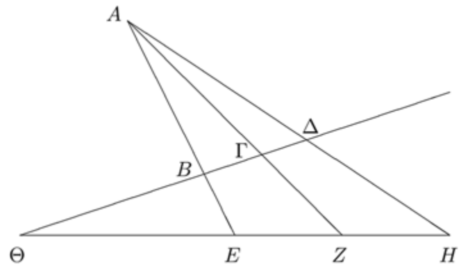

Figure 2. Proposition 129 of Pappus' *Mathematical Collection*.

> The 129th proposition shows that *when four straight lines are drawn from one and the same point, they form on any transversal, arbitrarily drawn in their plane, four segments having among themselves a certain constant ratio, whatever the transversal.* Hence *a*, *b*, *c*, *d* being the points at which the four straight lines are intersected by an arbitrary transversal, and *ac*, *ad*, *bc*, *bd*, the four segments, the ratio $\frac{ac}{ad} : \frac{bc}{bd}$ will be constant, whatever the transversal. . . . Until now, this proposition seems to have hardly fixed the attention of the geometers. However, we think it is susceptible of many applications, and we regard it as one which may become one of the most useful and the most fruitful of Geometry. . . . For this reason, we feel, right now, the need to give a name to the ratio of the four segments therein considered. This ratio being called *harmonic* in the particular case in which it is equal to the unity, in the general case, we will call it an *anharmonic ratio* or *function*. (Chasles 1837, 33–34)

Yet Chasles' interpretation of Proposition 129 would definitely imply a significant amount of distortion, for Pappus' lemma was originally formulated as a proportion between rectangular areas.

> Let two straight lines Θ*E*, ΘΔ be drawn onto three straight lines AB, ΓA, ΔA. That, as is the rectangle contained by Θ*E*, *HZ* to the rectangle contained by ΘH, ZE, so is the rectangle contained by ΘΔ, BΓ to the rectangle contained by ΘΔ, BΓ. (Jones 1986, 262)

Chasles thus couched the ancient geometrical proportion in another language so as to transform it into an equality of anharmonic ratios, that is

$$\frac{\Theta E.HZ}{\Theta H.ZE} = \frac{\Theta B.\Gamma\Delta}{\Theta\Delta.\Gamma B}, \text{ hence } \frac{\Theta E}{\Theta H} : \frac{ZE}{HZ} = \frac{\Theta B}{\Theta\Delta} : \frac{\Gamma B}{\Gamma\Delta}$$

Besides, no less than eight different configurations are distinguished in Commandino's Latin translation of Pappus, each case being illustrated by a different diagram, whereas Chasles' treatment claimed to be uniform and carried out in one stroke. It is further to be noted that neither Poncelet in his 1822 *Traité*, nor even Chasles at the stage of the *Aperçu historique* took into account the difference in the orientation of the segments when considering quotients of ratios being equal for four segments on a transversal. It will be seen below that Chasles' later notational technology would remedy this dearth of expressive means.

Fully aware that Pappus' lemmas, when rewritten in terms of anharmonic ratios, would inevitably have an artificial ring to them, Chasles nevertheless insisted that having recourse to such a contrived device was essentially justified because of the extent to which it would simplify and unify geometry.

> Here naturally an observation comes up which may justify the importance that we have sought to give to Proposition 129 of Pappus, and to the notion of an *anharmonic ratio*. Namely that all the theorems that we have just extracted from the VIIth book of the

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> *Mathematical Collection*, that on the deformation of a polygon and that *ad quatuor lineas* included, and several other theorems on the involution of six points to which we shall come later, all these theorems being among the most general and the most useful in recent Geometry, can all derive, as from their common source, from this only property of the *anharmonic ratio* of four points. And this way of presenting them will be as simple as possible, for it will need, so to speak, no proof.
>
> We will add that, after having recognized that most of Pappus' lemmas, which seem to pertain to the first book of Euclid's *Porisms*, could be inferred from the proposition in question, we thought that this proposition could also very well be the key to all this first book on *porisms*, and lead to an interpretation of the statements that Pappus left us. (Chasles 1837, 38–39)

However, this viewpoint was not unanimously shared. Poncelet would later consider the choice by Chasles of the epithet *anharmonic* as misguided for both linguistic and mathematical reasons. "What logical and grammatical meaning can be attached, [he] wonder[ed], to those words *anharmonic*, *homographic*, of which M. Chasles has so skilfully taken advantage" (Poncelet 1866, §IV, 435). Since the negative prefix a- as in *anarchy* or *anhydrous* would not make sense in this case, Poncelet repeatedly insisted that he would have definitely preferred alternative labels such as *surharmonic ratio* or, to meet the German school half-way, *ratio of double section*, which would correspond to Möbius' *Doppelschnittsverhältniss* (Poncelet 1866, 360, 408, 411, 435).[34]

But there were more compelling reasons underlying Poncelet's discomfort with Chasles' decision to confer a central importance to this notion. Not only did he admit that the whole *Aperçu historique* was "one of the books whose reading cost [him] the most, because of the assessments therein contained and the false interpretations to which it gave rise of [his] own most earlier works" (Poncelet 1862, 493), but he also adduced a few remarks in support of his reluctance to accept the new terminology. By deliberately giving pride of place to the conservation of the anharmonic ratio upon which the whole of the new geometry was intended to be built, Chasles would presumably limit himself, Poncelet retrospectively claimed, to a restricted class of projective properties in the meaning he himself set for this term in his *Traité*:

> It is evident that the *anharmonic* and *homographic* relations are, in their consequences, of an essentially projective nature or can become so by transformation. For, according to Chasles' own remarks and demonstrations (*Géométrie supérieure*, n°563), *any two homographic figures can be placed so as to be homological or in perspective with respect to one another*; which supposes a mere relative displacement as for the figures being similar but not similarly placed in space or in the plane, while offering certain advantages, as I pointed out elsewhere (Applications d'Analyse et de Géométrie, vol. 1, footnote to p. 493). As one sees, homography is nothing but a geometrical transformation limited to the simplest *projective* metrical relations, whereas ours [*viz.* homology] comprises all those of the *Geometry of the ruler*, of *transversals*, etc. (Poncelet 1866, 435).[35]

The main point here seems to be confirmed by another passage, in which Poncelet later came back to his earlier writings on Euclid's porisms from the winter of 1817 to 1818, so as to reiterate

[34]The French terms proposed by Poncelet are *surharmonique* and *de double section*.

[35]There was a misprint in Poncelet, as the article referred to in Chasles' *Géométrie supérieure* is §563, and not §363.

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what his views were long before the publication of Chasles' *Aperçu historique*. Without claiming any authority as a Greek scholar,[36] he reasserted his conviction that any rewriting of Pappus' lemmas in terms of anharmonic ratios would constrain our understanding of ancient mathematics into an alien format. Referring to some of Pappus' lemmas rewritten in terms of equations involving ratios, he made the following observation that "these ratios being comprised among those to which the Mœbius, the Steiner, the Chasles have, long afterwards, applied diverse denominations, . . . which would not have suited the taste of the Ancients, enemies of this modern neologism much more adapted to restrict than to generalize the use and the understanding of mathematics" (Poncelet 1866, 401). Moreover, Poncelet explicitly pointed out the misrepresentation of ancient mathematics brought about by the notational technology which, as will be discussed below, Chasles would gradually feel compelled to shift to in order to yield a satisfactory account of both Euclid's lost *Porisms* and the modern pursuit of pure geometry.

> I purposely repeat that, for the Ancients, the difficulty to specify the true character of one and the same porism in each kind of statement, consisted in the multiplicity of the possible dispositions of the figures pertaining to that porism – a difficulty to which Pappus himself gives expression and which he seems to conceive in a rather obscure way, if not equivocal, but which all his predecessors had also felt, who, according to Euclid's elementary treatises, were accustomed to see the reasonings change with the positions of the data or the conditions considered fundamental of one and the same figure. For the Greeks had by no means acquired the confidence that we possess today in the *principle of permanence* of geometrical relations, which is essentially based on the law of the signs of position, that some among the Moderns refuse to examine and to recognize under the abbreviating name of *principle of continuity*. (Poncelet 1866, 401–402)

In his 1870 report on the advances of geometry, Chasles, for his part, reaffirmed his opposition to Poncelet's view that the ancient porisms adumbrated to a certain extent the use of perspective in modern geometry. "Actually, one had remarked," Chasles conceded, "in Pappus' lemmas pertaining to the Porisms, some traces of the theory of transversals, such as a few properties about the *anharmonic* ratio of four points, and a relation of involution in a quadrilateral cut by a straight line" (Chasles 1870, 240). And here Chasles explicitly mentioned in a footnote Poncelet's analyses in the introduction to his 1822 treatise: "But above all it is another proposition, at first overlooked, which forms the stronger link between Euclid's work and modern theories, meaning this property that *the anharmonic ratio of four points is conserved in perspective*. This simple proposition, expressed in other terms, of course, happens to be demonstrated six times, that is in six different lemmas" (Ibid.).[37]

With hindsight, however, Chasles could now assess that the interpretive task that he had set for himself in the *Aperçu historique* actually required some preliminary work so as to build the appropriate frame to carry it out. "Thus, the study and the development of the elementary theories of modern Geometry were necessary to achieve the restoration of Euclid's three books on p orisms. This is what stopped the geometers of the last century from fulfilling the work of which Simson had established the principle" (*Ibid.*). Indeed, in 1837, on the basis of the mere expressive means available at the time of the *Aperçu*, Chasles could only roughly delineate the goal to achieve and

[36]Poncelet denied any intention to settle the scholarly dispute on the proper interpretation of Euclid's *Porisms* that Chasles' work would arouse on the long run, involving, among others, Alexandre-Hydulphe Vincent, Charles Housel and Paul Emile Breton de Champ (see section seven). In order to distance himself from this debate, he mockingly quoted a verse by Virgil about Palaemon's verdict concerning Damoetas's and Menalcas's singing contest, from the third eclogue of the Bucolics: *Non nostrum inter vos tantas componere lites* ("It is not for us to end such great disputes").

[37]Chasles here referred to the above-mentioned Pappus' lemmas, namely the string of propositions 129, 136, 137, 140, 142, 145.

offer a few hints as to the direction this interpretive work should take and the way it may proceed on a middle ground, so to speak, between ancient and modern mathematics.

> For want of sufficient documentation [Chasles thus suggested,] so as to restore the complete doctrine of porisms by the analytical way, one must somehow recompose this doctrine in an *a priori* way, by pure synthesis. It is a system one has to form, and to submit to all the questions and tests to which the fragments that have come down to us may give rise. (Chasles 1837, 275)

These guidelines were to become effective in later years when geometrical resources were augmented enough with the shaping of a new notational technology.

5. Porisms as geometrical equations: A working hypothesis

Chasles published his first interpretation of Euclid's lost *Porisms* in Note III of the *Aperçu historique*, which he wrote in 1835. At this stage, his interpretation did not entail an exhaustive survey of Pappus' propositions, a task to which he would turn in the late 1850s when preparing his book on Euclid's porisms. Instead, Chasles sought to tackle the questions which he deemed unanswered by Simson's previous restitution of the meaning of Pappus' fragments, namely those of "the doctrine of porisms, its origin, or the philosophical thought which created it, its destination, its uses, its applications, and its transformation in modern doctrines" (Chasles 1837, 275). Chasles' putative answer to this series of questions, which at this stage he could not or would not support with a complete discussion of the extant sources, was to frame Euclid's porisms as "the *equations* of curves;" and, more precisely, as a doctrine whose chief purpose was to introduce in (pure) geometry "changes of coordinates" (Chasles 1837, 276). In other words, for Chasles, the doctrine of porisms was the art of passing from one mode of description of a figure to another. The porisms thus interpreted, Chasles claimed, therefore constituted a crucial junction between ancient geometry and modern methods.

Indeed, according to this description, all of Descartes's geometry consists of "one continual porism" (Chasles 1837, 278), expressed in the language of algebra. As an example of this claim, Chasles considers the following (Cartesian) equation:

$$x^2 + y^2 + ax + by = c^2$$

defined along two perpendicular axes, and from one single origin. This equation can just as well be "translated" into geometrical language as the following local problem:

> One can find two straight lines a, b and a square c^2 such that the squares of the distances of the point being sought to two axes drawn in the plane of the figure, plus the products of these distances with the two lines a, b respectively, form a sum equal to the square c^2.

Using a simple change of coordinates,[38] that is to say a transformation of the axes and the origin by means of which the locus was theretofore defined, the former equation can be transformed into

$$X^2 + Y^2 = A^2$$

This equation, in turn, can also be translated into the language of pure geometry: it means that "there exists a point in the plane, which can be determined, and which is always at a constant distance, which can also be determined, from all the points being sought" (ibid.). Through this description, one recognizes easily that the locus whose description has been transformed was a circle all along—a conclusion to which one might have also arrived by means of the algebraic

[38]It suffices to take $X = x + \frac{a}{2}$, $Y = y + \frac{b}{2}$, and $A^2 = c^2 + \frac{a^2+b^2}{4}$.

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transformation of the first equation. In particular, the point mentioned in the second description turns out to be the center of the circle, and the constant distance is its radius.

For Chasles, Euclid's doctrine of porisms was the art of navigating between these modes of description of loci without the help of Descartes's instrument. What precisely this navigation might have entailed for ancient geometers remains, at this stage, partly unclear in Chasles' exposition. However, Chasles seems to have had in mind (among other things) geometrical transformations of the components involved in the description of a locus (such as the squares and straight lines involved in the above description of the circle).

Of course, Chasles acknowledged that the generality and uniformity of Descartes' geometry was absent from Ancient geometry. Where, for Euclid or Pappus, two propositions which differ only in that a point is either to the left or to the right of another point require two different proofs, modern geometry can identify one single theorem, and prove it in "a single stroke of the quill" (Chasles 1837, 143). Furthermore, the transformation of an algebraic equation (for instance through a change of coordinates) was deemed much more systematic and uniform than the means of transformation of geometrical figures at the disposal of ancient geometers—a state of affairs which, in Chasles' narrative, changed dramatically at the onset of the nineteenth century, in large part owing to Monge's and Carnot's contributions.[39] And yet, in Chasles' reconstruction, the scope of the doctrine of porisms goes considerably beyond Descartes' analytic geometry proper. The example above shows how Descartes' conceptions could have arisen from what Chasles took to be Euclid's porisms, had the Greeks possessed Algebra.[40] In fact, Cartesian equations at large were only one of the many by-products of the doctrine of porisms in Chasles' understanding thereof.

This appears clearly in Chasles' presentation of "two very general propositions," whose many corollaries were deemed to include Pappus' list of propositions (Chasles 1837, 279). The first of these two general propositions is the following (see figure 3):

> First porism. Taking two points P, P' in a plane, and two transversal lines which cross the straight line PP' at points E, E'; and taking, on these transversal lines, respectively, two fixed points O, O';
>
> If, from each point of a given straight line, one draws two straight lines to the points P, P', which will intersect respectively the two transversal lines EO, $E'O'$ at two points a, a';
>
> One will be able to find two quantities λ, μ such that one will always have the relation:
>
> $$\frac{Oa}{Ea} + \lambda \frac{O'a'}{E'a'} = \mu$$

Furthermore, Chasles noted, the converse statement is true. If, drawing the straight lines joining the points m of a given locus to P and P', one intersects EO and $E'O'$ at two points a, a' which satisfy the equation above, then said locus is a straight line.

[39]In one typical passage, Chasles notes that "the Ancients [by contrast with Descartes] did not possess such a general and uniform means of investigation [as the application of algebra to the theory of curves]" (Chasles 1837, 277). Monge, in turn, took a decisive step toward bringing pure geometry to a similar level of uniformity by "reducing to a small number of abstract and invariable principles . . . all of the geometrical operations which may arise in stone-cutting, perspective etc." (Chasles 1837, 189). Carnot would be brought to the same level of historical importance in Chasles' later narratives (see for instance Chasles 1847, 30).

[40]For Chasles, "only the use of Algebra was missing for Euclid to create Descartes' systems of coordinates" (Chasles 1837, 276). Zeuthen, a student of Chasles, would famously contend that the Greeks did in fact possess something like Algebra, thereby putting forth a much-debated historiographical thesis on the existence of a so-called geometrical algebra (Zeuthen 1886, 44–53).

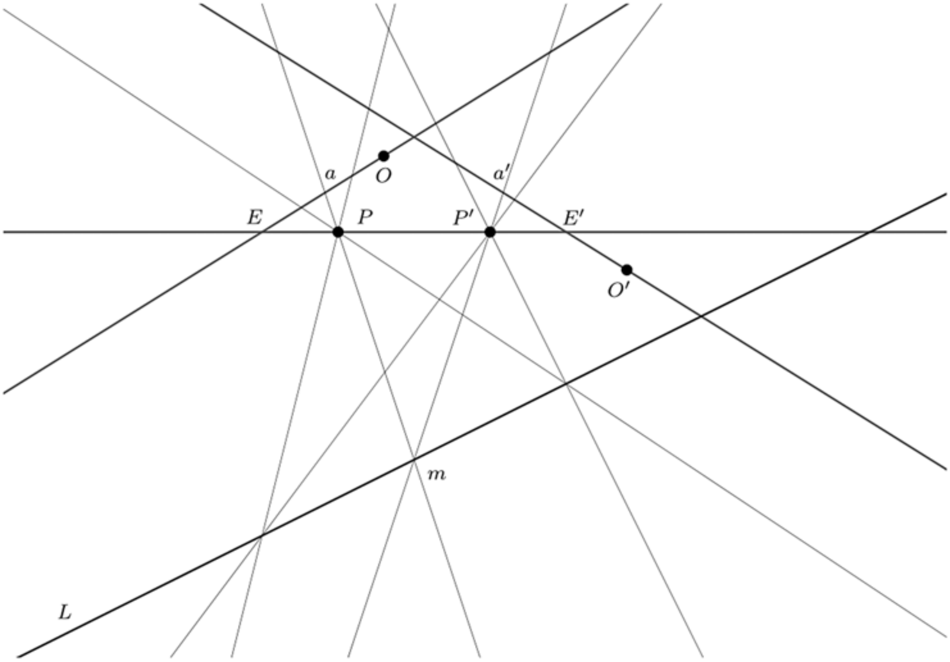

Figure 3. Chasles' first porism (general case).

While Chasles did not provide any proof for either part of this proposition, we shall see below that it derived easily from his work on the theory of homographic divisions—a theory itself based on the aforementioned concept of the anharmonic ratio (see section 6). In the course of his 1837 analysis of the porisms, however, Chasles made no mention of this ratio, of which he would later say that it was the keystone of Euclid's lost doctrine (Chasles 1870, 239–240). In Note IX of the *Aperçu historique*, devoted entirely to this mathematical concept, Chasles put forth certain equations which make the connection with the general porism more palpable. Remember that the anharmonic ratio of four aligned points A, B, C, D was defined by Chasles as the quantity:

$$\frac{AC}{AD} : \frac{BC}{BD}$$

For Chasles, the characteristic feature of this quantity as a function of the segments formed by four points was the one he had read in Proposition 129 of Book VII of Pappus' *Mathematical Collection* (see section four above). However, Chasles noted, one could just as well have considered the quantities $\frac{AD}{AB} : \frac{CB}{CD}$ or $\frac{AD}{AC} : \frac{DB}{DC}$, which are also functions of the segments formed by A, B, C, D and which could serve a similar role in Pappus' lemma. These two quantities, which Chasles defined as the second and third anharmonic ratios (of the same four points) are in fact the only such functions satisfying this criterion, and they are easily expressible in terms of the first ratio. For instance, denoting r_1 and r_2 the first and second ratios, Chasles showed that, for instance,

$$r_1 = 1 - \frac{1}{r_2}$$

Therefore, Chasles noted, if the first anharmonic ratios r_1 and $r_1{}'$ of two systems of four points A, B, C, D and A', B', C', D' are equal, then so are their second and third ratios.

This was particularly useful to Chasles because it implied that one could express the equality between anharmonic ratios r_1, $r_1{}'$ in the following manner:

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$$r_1 = 1 - \frac{1}{r_2{}'}$$

that is to say:

$$\frac{AC}{AD} : \frac{BC}{BD} + \frac{A'B'}{A'D'} : \frac{C'D'}{C'B'} = 1$$

This is what Chasles called a three-term equation for the equality of two anharmonic ratios (Chasles 1837, 304).

Chasles' first general porism above derives from precisely such an equation. Let us denote L the straight line being described by the porism; and, to any point a on OE, let us associate the point m on L obtained as the intersection of L and aP. In such a manner, we can also associate to the point a a unique point a' on $O'E'$, namely the intersection of $O'E'$ and mP'. Conversely, to each point a' on $O'E'$, we can associate, in the same way, a unique point a on OE, by forming m', the intersection of $a'P'$ and L, and then a, the intersection of $m'P$ and OE. Note that the points E and E' correspond to one another via this construction, and let us denote Q and Q' the two points (on OE and $O'E'$ respectively) corresponding to O' and O. The key property of this correspondence between the points of OE and $O'E'$, in connection with the discussion above, is that it "preserves the anharmonic ratio." In other words, for any variable point a on OE, the anharmonic ratios of a, E, O, Q and a', E', O', Q' are always equal.

Indeed, the first anharmonic ratio can be viewed as formed on OE by a pencil of rays drawn from P. But the rays Pa, PE, PO, PQ also intersect L at four points m_a, m_E, m_O, m_Q. The anharmonic ratio of the four points thus defined on L is equal to that of the four points on OE, precisely owing to Pappus' lemma. The same can be said of the anharmonic ratio of a', E', O', Q', which is defined on $O'E'$ by a pencil of rays drawn from P'. However, the points m_a and $m_{a'}$, as well as the points m_E and $m_{E'}$, coincide per construction. Furthermore, $m_O = m_{Q'}$ and $m_Q = m_{O'}$, per definition of Q and Q'. Thus, the anharmonic ratios on OE and $O'E'$ are both equal to the same anharmonic ratio on L, and so are equal to one another. Using the three-term equation for the equality of the corresponding anharmonic ratios, one can therefore write:

$$\frac{Oa}{OQ} : \frac{Ea}{EQ} + \frac{O'a'}{P'Q'} : \frac{E'a'}{E'Q'} = 1$$

which is exactly the equation associated to Chasles' general porism, with $\lambda = \frac{OQ}{EQ} \cdot \frac{E'Q'}{O'Q'}$ and $\mu = \frac{OQ}{EQ}$.

We can now understand the analogy which Chasles drew between the components of this proposition and the terminology of Cartesian geometry. In this proposition, for Chasles, the points O and O' therefore act as two origins, and the straight lines OE, $O'E'$ as axes; such that the abscissa and ordinate of a locus (in this case, a straight line) can be defined as the ratios $\frac{Oa}{Ea}$ and $\frac{O'a'}{E'a'}$. Note that they might as well have been defined simply as the magnitudes Oa and $O'a'$, but the introduction of E and E' in the equation gives Chasles more flexibility in future transformations of this description of the straight line. For instance, one can consider particular descriptions of a locus in which E and E' coincide or lie on another given curve. The points P and P' serve to define the way in which points of the locus are projected onto the axes. Denoting these ratio x and y, this equation states merely that a straight line is a locus whose abscissa and ordinate are in a linear relation, and conversely. This equation (and the geometrical setting from which it derives) does indeed constitute a general equation of the straight line, in the same way as the Cartesian equation $ax + by = c$ does.

The epistemic strength of Cartesian equations, for Chasles, was that one could submit them to all sorts of transformations, thereby obtaining all possible properties of the figure they represent. The same can be said of this first porism: by applying to it various kinds of transformations, one can obtain as many descriptions of the figure it represents, namely the straight line. For instance,

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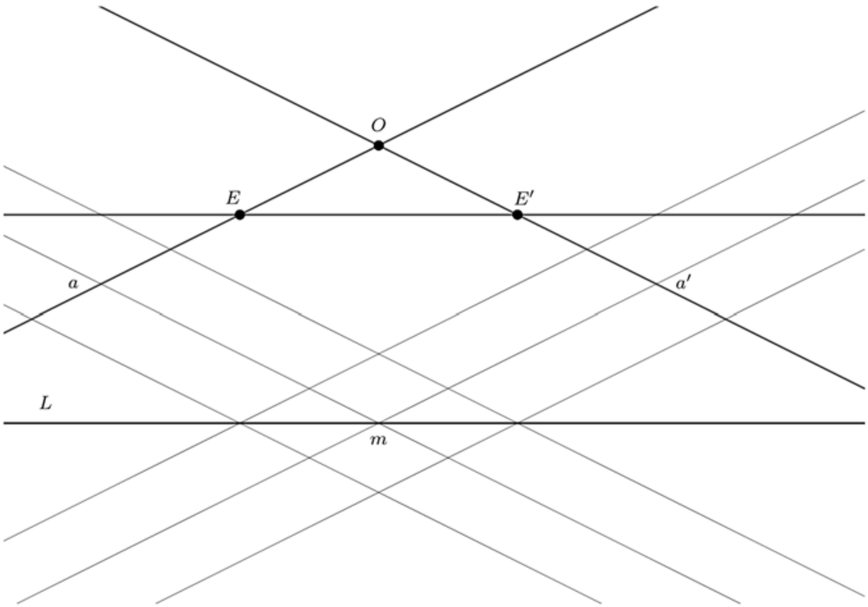

Figure 4. Chasles' first porism (special case).

by taking O and O' to coincide, and by sending P and P' at infinity on the transversal lines OE and OE', one obtains the Cartesian equation of the straight line. Indeed, the two lines OE and OE' serve as two axes (i.e. as the abscissa and the ordinate), and O as the origin. Since P and P' are at infinity, the straight lines drawn from m are the parallel lines to OE and OE' (cf. figure 4). More generally, if the equation above is of degree m in $\frac{Oa}{Ea}$ and $\frac{O'a'}{E'a'}$, then the locus is a curve of order m.[41] Chasles takes note of this fact, but contends that Euclid's porisms were only used to describe rectilinear figures (as well as circles).

For this reason, Chasles claimed that from this single general porism, "one will draw a multitude of porisms, whose number could rise to two or three hundred without exaggeration" (Chasles 1837, 281). This assertion echoes many others made by Chasles throughout the *Aperçu historique*, such as his praise of Pascal's *mystical hexagram*: in a single theorem, prone to take many forms, "an immense number of properties of conics were comprised" (Chasles 1837, 73). This sort of geometrical achievement was the preserve of the Moderns. The general porism, in that sense, could not possibly be one of the Ancients' porisms proper, but rather a more general theorem, encompassing all of them.

Chasles' interpretation of Euclid's porisms also supplied him with an alternative model of modern geometrical practice to that of analytic geometry. For all its fecundity and efficiency, the use of Cartesian equations had one major drawback for Chasles: geometrical knowledge derived from the transformations thereof rested on "an auxiliary and artificial system of coordinates," whereas truths obtained by pure geometry alone always required "a more direct reason, borrowed from the very nature of the thing [under study]" (Chasles 1837, 119). In other words, modern analytic geometry could let practitioners bypass some of the intermediary truths between data (of a problem or a theorem) and conclusion by hiding them amidst uniform, algebraic computations. Practitioners could thereby be rewarded with speed and efficiency, but only at the cost of full knowledge of "why and how a thing is true, [and of] its place in the order of truths to which it belongs" (Chasles 1837, 114). Chasles famously rejected such a trade-off on the grounds that it was

[41]In the case of a curve of order m, the correspondence between the points a and a' is such that to each point a', there correspond m points a', and conversely. Therefore, the polynomial in Oa and $O'a'$ which one must form is of degree m.

possible to elevate geometrical methods to the same level of simplicity and generality as that of analysis without reneging on epistemic completeness and clarity (Michel 2020, 125–129).

We may now begin to grasp the importance of this interpretation of the doctrine of porisms for Chasles. If these forgotten statements acted as equations of loci, and if their doctrine taught us how to transform such equations (*qua* modes of description of a locus), they could accomplish the same task as Descartes' system whilst retaining geometrical clarity. Chasles' selection of examples clearly indicates that, indeed, the Cartesian equation is but a special form of the porism. Moreover, the system of coordinates from which a locus is described in a porism is totally undetermined. One could select, transform, tailor the choices of the origins and axes to a specific problem or proof, and thus enshrine it into the "chain of truths" without introducing artificial auxiliaries, foreign to the question at hand.

However, within the *Aperçu historique,* this tentative reading of the porisms and of their epistemic importance was but one of many attempts by Chasles to characterize the form that modern geometrical theories should adopt. One, in particular, deserves a mention here. In his retelling of the modernization of geometry—that is to say in the second stage or "epoch" of his narrative for the development of geometrical methods—Chasles had highlighted the importance of Desargues's involution theorem for the theory of conics. Like Pascal's aforementioned mystical hexagram, Desargues's theorem could yield a large number of other propositions via simple transformations. However, Chasles saw a greater purpose in this theorem: by rewriting it so as to highlight the role of the anharmonic ratio therein, he claimed that it could be made into the adequate foundation for a properly geometrical, yet fully general theory of these curves (Chasles 1837, 77–81).[42] In Note XV, which was written at a later date, Chasles went back to this point, and sought to derive the basis for a methodical theory of conics from these historical analyses. The modified version of Desargues' theorem which Chasles thought so promising stated the following:

> If, in two pencils of four straight lines in a one-to-one correspondence, the anharmonic ratio of the first four lines is equal to that of the four corresponding lines, then the lines of a pencil with meet their corresponding lines, respectively, at four points, which will be on a conic passing through the two points which are the centers of these pencils. (Chasles 1837, 335)[43]

In the rest of this note, Chasles showed how all classical properties of conic sections (including Pascal's mystical hexagram, Newton's organic description, or Pappus' *ad quatuor lineas* problem) could be obtained by simply transforming this general proposition, for example by moving the pencils around or specifying their correspondence further. Since all truths pertaining to conics can be obtained at the end of a short "chain" of propositions starting from it, Chasles had effectively shown that this theorem ought to be taken as the "center" of the theory of these curves (Chasles 1837, 338).[44]

[42]This theorem states the following: let C be a conic, $ABCD$ be a quadrilateral, and L be a straight line, all in the same plane. Suppose that L intersects C at two points O, O'; and draw the lines AO, AP, CO, CP. These four lines intersect L at four points M, N, M', N' such that the three pairs of homolog points (O, O'), (M, M'), (N, N') are in involution, that is to say that the anharmonic ratio of any four of these six points is equal to that of the four homolog points.

[43]A dual version of this proposition is given and discussed in Note XVI.

[44]Remarkably, the centrality of a proposition for Chasles is not tied to its being logically fundamental, but rather to its expressive power. In fact, Chasles arrived at this central proposition by transforming another, equivalent theorem, *viz.* Desargues's theorem itself. And yet, to this latter result, Chasles ascribed a lesser generality in spite of their logical equivalence. Karine Chemla comments this passage in the following terms: "In other words, equivalent formulations of the same fact—in Chasles' words, different 'expressions' of the same theorem—do not have the same generality. This statement contradicts common beliefs about generality and is thus worth pondering. In the case of Desargues's theorem, the key feature of the reformulation is that it highlights *how* the property relates to the anharmonic ratio. In Chasles' understanding, the important feature of the new 'form' is that it allows practitioners to derive from it, as 'corollaries,' a wider range of propositions than its earlier form could apparently reach. As a result, a greater number of theorems will thereby appear to derive from the same 'source' and will thereby be connected. The ease brought into proof plays a key part in the *dispositif*" (Chemla 2016, 79–80).

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Whilst Chasles makes no explicit connection between the two, this proposition has much in common with the first general porism described above.[45] Both describe a locus as the intersection of corresponding elements of two pencils of straight lines satisfying a specific property, which Chasles would later define as the homography of the two pencils (see section four). Furthermore, both are called to play a similar role in the organization of a geometrical theory as a general proposition, involving as many indeterminate elements as possible, to which one can apply a variety of transformations (such as those given by the principles of duality and homography) and specifications, thereby producing the entire body of properties of these loci in a systematic and effortless manner. And in both cases, the centrality of these propositions is linked to their intimate connection with the concept of the anharmonic ratio. This connection is such that the equation defining the first porism (i.e. the homographic description of the straight line) is reproduced *verbatim* in Notes XV and XVI, albeit with different interpretations: in these two notes, this same equation now characterized the (anharmonic) correspondence between two pencils whose intersection generated a conic instead of a straight line (Chasles 1837, 339, 345).

These interpretative strands, by 1837, remained largely untied. The center of the theory of conics was not thought of as an equation of these curves, and the role of anharmonic ratios and homographic correspondences in the doctrine of the porisms was not made explicit. In what follows, we shall see that this would change considerably when, throughout the 1850s, Chasles would return to the porisms from the vantage point provided by his new language for the writing of geometrical propositions.

6. On the need to rewrite the propositions of Greek geometry

Chasles' *Aperçu historique* was explicitly geared towards epistemological ends as much as historical ones. Through the study of past methods and theories, Chasles aimed to understand what had enabled geometers to develop ever more general methods and results, and to further these investigations on that very same path of increasing generality. But this was not the final form to which Chasles destined his geometrical research. Borrowing from Comte's terminology, Chasles stressed in the *Aperçu historique* already that this historical exposition was to be completed by a "dogmatic exposition," wherein he would "coordinate these partial and isolated truths [which rational Geometry until then had produced], to make them all derive from only a few of them, taken amongst the most general" (Chasles 1837, 234).[46] Such was the project he took up starting in 1846, as a chair of higher geometry was created for him at the Faculté de Paris.[47]

In both the inaugural lecture of the very first year of this course's existence, delivered on 22 December 1846, and the opening lesson for the following academic year (1847–1848), Chasles came back to his historical studies in order to present and motivate the content of the courses of the year to come.[48] In both texts, Chasles framed his upcoming lectures as the continuation of "the Methods of the Ancients," but through the lens of new notational *dispositifs* and methods created

[45]In his 1860 book on porisms, however, Chasles would reconnect these notes: "I gave, in the *Aperçu Historique* (279), two general Porisms whose many consequences include a very large number of Euclid's porisms on rectilinear figures. Shortly thereafter, I showed that there are also, in the theory of conics, and later in that of circles, two very similar propositions which constitute the most fruitful properties of these curves (*Aperçu*, Notes XV and XVI)" (Chasles 1860a, 73).

[46]Chasles' instrumental use of historical surveys of past principles, which are then to be subsumed within methodical theories centered around a minimal set of principles, is not an idiosyncrasy. On the contrary, it bears the mark of a scientific culture which one can trace back to the École Polytechnique, and in particular to Lagrange's influential teaching there (see for instance Dahan-Dalmedico 1992; Wang 2022). Other contemporary books which resort to similar expository and methodological devices include Lacroix's 1797 *Traité du calcul différentiel*, Delambre's 1814 *Astronomie théorique et pratique*, and Fourier's 1822 *Théorie analytique de la chaleur*. Comte was equally steeped in this tradition.

[47]On Chasles' teaching at the Faculté des Sciences, notational innovation, and the role of historiography therein, see Michel and Smadja 2021a.

[48]The opening lesson for the year 1847–1848 is only extant in the form of a handwritten manuscript preserved in Chasles' scientific archives. We shall come back to it in section 9.

by Monge, Carnot and other "Moderns." The works of the Ancients were not devoid of interest because they were somehow outdated or obsolete, Chasles argued, but their study alone would be insufficient if it was not turned into a unified body of doctrine and set of methods (Chasles 1852, XXXVII). The first step in that direction, in Chasles' teaching, would involve the construction of a new technology for the writing of geometrical propositions.

In a particularly telling passage of his 1846 lecture, Chasles described the chief purpose of Carnot's works as "the extension of the Geometry of the Ancients," but expressed in the garb of the theory of transversals and of the principle of correlation.[49] Carnot's methods, Chasles continued, lead to "easiness and strength, as well as brevity and generality"—in other words, the core epistemic values which structured the main narratives of the *Aperçu historique*. The main factor behind this virtuous geometrical practice, Chasles went on, lay in Carnot's "way of writing Geometry," a way which he claimed had come to "characterize Modern Geometry." Chasles explained that one underlying reason for the benefits of Carnot's notational innovations is that they serve to represent curves in a variety of ways, and to move from one such representation to another. Connecting this diagnosis of modern successes with his interpretation of ancient techniques, Chasles commented that, as such, Carnot's "beautiful research . . . belongs essentially to the doctrine of Euclid's porisms" (Chasles 1847, 34–35). In keeping with this tradition, Chasles set out to construct a new technology for the expression of geometrical propositions—that is to say, a "way of writing geometry"—partly modelled on his understanding of the nature of Euclid's porisms. In doing so, Chasles would deepen his working hypothesis on the equation-like nature of these ancient propositions by centering them more explicitly around the concept of anharmonic ratio, thereby finalizing the construction of an edifice from which Pappus' text could be read anew.

Chasles' rewriting of geometry begins with the introduction of what he called the "principle of signs," with which letters are used to denote segments (or angles) instead of the numerical values of their lengths (or magnitudes).[50] For any two points a, b on a straight line, if one denotes the segment delineated by these points as ab, then a will be taken to be the "origin" of the segment; conversely, b is the origin of the segment ba. These two ways of denoting what seems to be the same segment naturally correspond to two different directions; and in the course of computations, these directions will be denoted by + and – signs.[51] This is obviously not to say that Chasles assumes direction to be an absolute property of segments, but rather that when multiple segments occur in a single proof or proposition, one will merely have to arbitrarily select one as the basis for comparison with all others. For this reason, the following equation always holds:

$$ab = -ba$$

This equation merely translates the fact that, if one takes the direction of ab as a basis for comparison, then ba is obviously a segment of the opposite direction, and therefore should be denoted with a "minus" sign next to it. More generally, for any three points a, b, c on a straight line, the following holds (and, *mutatis mutandis*, for any n points on a straight line):[52]

[49]In his *Géométrie de position* (1803), Carnot tackled the decade-long problem of introducing negative quantities in geometry. His method was the following: given a geometrical problem, he would list the figures composing it, consider the "primitive system" they form, and then write the so-called correlations between the elements of the primitive system and those of a "correlative system" (that is to say, a system obtained by applying a continuous transformation to the primitive system). He would then write the equations pertaining to the primitive system, and establish a "tableau de correlations," indicating how equations between the elements of the first system ought to be changed (by the addition of signs) in order to yield equations between elements of the correlative system. For an analysis of this method, see Chemla 1998.

[50]The introduction of negative quantities in geometry had been a major topic for dispute amongst mathematicians and philosophers, especially over the last third of the eighteenth century, and one on which Poncelet extensively wrote as well (see Schubring 2005, 99–113; 353–364).

[51]In what follows, we shall restrict ourselves to notations, propositions, and proofs pertaining to segments and not angles, for the sake of brevity. However, due to the principle of duality, everything we present here can be also applied to angles.

[52]Note that this relation is not about vectors, but only about aligned points (or angles turning about one fixed point).

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$$ab + bc + ca = 0$$

Among the many reasons for the introduction of this principle, Chasles stressed that it allowed one to write, in a single formula, the "abstract form" of a theorem, thereby encompassing what could otherwise appear as many "concrete expressions" (Chasles 1852, 17–18). One crucial example is the following relation between four points a, b, c, d on a straight line, or rather between the six segments that they determine:[53]

$$ab \cdot cd \; + \; ac \cdot db + ad \cdot bc = 0$$

If this equation is read as a relation between the lengths of six segments (or the areas of the three rectangles they determine), then its geometrical meaning varies with the relative configuration of the four points. Each permutation of the four points yields a different property of these rectangles. All of these properties, however, essentially derive from the same unique theorem, which the principle of signs brings to the fore as a general property of six points on a line. Besides the use of Comte's semantic opposition (concrete/abstract) to frame the modernity brought about by his notational innovation, Chasles here stressed a virtue of his mode of representation that is crucially absent in Ancient geometry. The propositions of the Greeks, Chasles had argued at length in the *Aperçu historique*, were limited in that they could not deal with the multiplicity of cases generated by the various configurations of a same figure (here for instance, the "figure" being the system of six points on a straight line). Through the principle of signs, Chasles brought into geometry the same sort of generality and abstraction that Descartes brought through the introduction of algebraic equations, but in a different manner.

The *Géométrie supérieure* was to be built upon three central theories, namely that of the anharmonic ratio, the homographic division, and the involution of segments. The last two theories, as we shall see, are in many ways correlative to the first one. Chasles had already contended, at many points in the *Aperçu historique*, that anharmonic ratios could serve as the appropriate basis upon which to found a large part of modern (pure) Geometry (Chasles 1837, 35, 72, 255). In his courses, he did turn them into the main resource through which to express relations between figures (or elements thereof). Furthermore, anharmonic ratios would now be written using the principle of signs, thereby gaining new features in terms of generality and independence from specific configurations that they did not have in Chasles' earlier works. More specifically, the anharmonic ratio of four points a, b, c, d on a straight line could now be defined as the expression:

$$\frac{ac}{ad} : \frac{bc}{bd}$$

the segments involved in this expression being oriented in the way described above (Chasles 1852, 7).

Amongst the many properties of this notion which Chasles stated and proved, let us mention the fact that given a number λ and three points a, b, c on a straight line L, it is always possible to construct a fourth point d on L such that the anharmonic ratio of a, b, c, d is equal to λ. Chasles' construction also has the important merit of being general, a term which here means that the operations stipulated in the procedure are the same regardless of the positions of the given points and given ratio. This generality also extends to configurations that are absolutely foreign to ancient geometry. An important example is the case of points at infinity, which Chasles was well equipped to tackle at this stage. Suppose, indeed, that the point a is at infinity on the straight line on which lie b, c, d, then the anharmonic ratio defined above is equal to $\frac{bd}{bc}$. More broadly, the

[53]In fact, this equation appeared already in Chasles' Note IX on the anharmonic ratio in the *Aperçu historique*, albeit without the help of the principle of signs (Chasles 1837, 305).

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arithmetic of segments introduced by Chasles allowed for the cancellation of ratios of "infinite segments."[54]

From the anharmonic ratio, Chasles derived his second fundamental notion, namely that of "homographic divisions" (*divisions homographiques*).[55] These are defined as two divisions of two straight lines in a one-to-one correspondence such that the anharmonic ratio of any four points of one division is always equal to that of the four corresponding points (Chasles 1852, 67). Rather than an abstract object to be studied in its own right (like a family of mappings), homographic divisions serve as the basic grammar for Chasles' geometrical language. Such correspondences between series of points occur often in geometrical practice, as Chasles had noted. For instance, the fact that the anharmonic ratio is projective means that the transversal lines of a pencil intersect any two fixed straight lines L, L' respectively at points a, b, c, … and a', b', c', … in a way that forms a homographic division. Another typical example is the following: consider a ray of straight lines turning about a fixed point and intersecting a fixed conic section at two (variable) points P, Q. The tangent lines to the conic at P, Q will intersect an arbitrary fixed straight line L in the plane, at two (variable) points a, a'. This defines two divisions of the line L, among which the construction previously described yields an obvious one-to-one correspondence. One can easily show that this is in fact a homographic correspondence. In fact, given a division a, b, c, d, … of any straight line L, one can construct an infinity of homographic divisions of any other straight line L', by choosing arbitrarily three points a', b', c' as the homologues (that is to say, corresponding points) to a, b, c. The other points d', e', … of the second division will then be uniquely determined, and can be constructed as the points whose anharmonic ratios with a', b', c' are respectively equal to the (given) anharmonic ratios of a, b, c, d; of a, b, c, e; and so on.

Upon defining this concept, Chasles immediately explored its uses with an example that would have sounded familiar to anyone who had read his previous discussions of Euclid's porisms. Indeed, Chasles showed how homographic divisions could serve as the basis for the writing of a geometrical equation of the straight line.

Chasles had noted that if the rays turning about two points intersect at points on a straight line, these rays form two homographic pencils, with the line joining the centers of both pencils being its own homologue.[56] Conversely, if two homographic pencils are such that the line joining their centers is its own homologue, the intersection points of homologous rays will all lie on a line (which they entirely describe).[57] This provided Chasles with the following "general mode of description of a straight line" (see figure 5):

[54]Chasles also introduces imaginary points in his system (see Chasles 1852, 56–66).

[55]This notion was already introduced toward the end of Chasles' memoir on the principle of homography at the end of the *Aperçu historique* (Chasles 1837, 832). However, this notion did not occupy such a central place as it would in Chasles' lectures and later treatises. This is partially due to the fact that, in 1837, Chasles mostly viewed the principle of homography as a geometrical transformation, used for instance in the study of second-order surfaces.

[56]Chasles' proof goes as follows: take any four rays of the first pencil, and denote a, b, c, d their respective intersections with the four corresponding rays in the second pencil. By hypothesis, these four points are aligned, and their anharmonic ratio can be defined. This anharmonic ratio is also that of the first four rays, and of the four corresponding rays. Therefore, the pencils are homographic.

[57]Chasles' proof relies on the following lemma, from which the proposition clearly follows: if two pencils of rays are homographic, and if two corresponding rays in these pencils also coincide, then all the other corresponding rays intersect along a straight line. Indeed, take four rays Oa, Ob, Oc, Od of the first pencil and four corresponding rays $O'a$, $O'b$, $O'c$, $O'd$ in the second pencil, and suppose that Oa and $O'a$ coincide. b and c denote the intersections of the corresponding rays Ob, $O'b$ and Oc, $O'c$. We can assume without loss of generality that the straight line bc intersects OO' at point a (since a can denote any point on OO'). This straight line intersects the rays Od, $O'd'$ at (respectively) δ, δ'. The anharmonic ratios of a, b, c, δ and a, b, c, δ' are equal, because the pencils of rays Oa, Ob, Oc, Od and $O'a$, $O'b$, $O'c$, $O'd$ are homographic. Therefore, δ and δ' coincide (because there is only one fourth point which forms a given anharmonic ratio with a, b, c), which means that Od and $O'd'$ intersect on bc.

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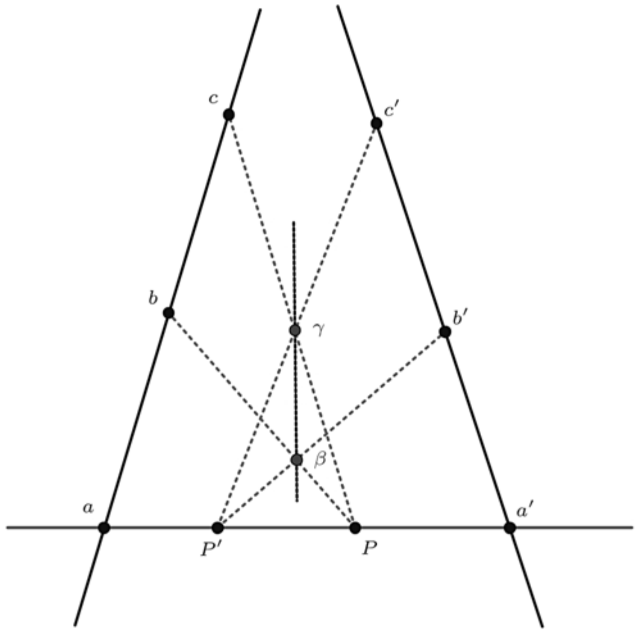

Figure 5. The general mode of description of a straight line (adapted from Chasles 1880, 68).

> When two lines are homographically divided in the points $a, b, c\ \ldots$ and $a', b', c'\ \ldots$ in a 1-to-1 correspondence, taking any two fixed points P, P' on the line aa', the lines $Pb, Pc\ \ldots$ will intersect the lines $P'b', P'c', \ldots$ at points $\beta, \gamma\ \ldots$ on a straight line. (Chasles 1852, 71–72)

While Chasles did not draw this connection explicitly in his dogmatic exposition of his geometrical methods, this mode of description of the straight line is strongly connected to the general porism he had given in 1837, as the points P and P' serve to project the points of a straight line on two axes, in a way that results in the corresponding points having a certain relationship (here, them forming a homographic division). This connection appears even more clearly in the following paragraphs of Chasles' *Traité*, as he examined the different ways in which homographic divisions can be expressed. To that end, Chasles formed several kinds of equations by introducing variable (or, rather, indeterminate) as well as fixed points on the two lines that are homographically divided.

For instance, given two straight lines, the equation for a homographic division thereof can be obtained by fixing three points a, b, c on the first one, and three points a', b', c' on the second one. For any point m on the first line, its homologue m' is determined by the two-term equation:

$$\frac{am}{bm} : \frac{ac}{bc} = \frac{a'm'}{b'm'} : \frac{a'c'}{b'c'}$$

which can be rewritten as $\frac{am}{bm} = \lambda \frac{a'm'}{b'm'}$, where λ is a constant quantity, depending only on a, b, c, a', b', c'. Conversely, two variable points m, m' on two lines ab, $a'b'$, which satisfy an equation like this one (for any fixed λ), form homographic divisions on these lines. Chasles then noted that λ has a "very simple geometrical expression," which he obtained by sending m' at infinity. If I is the homologue (in the first division) of the point at infinity, then $\lambda = \frac{aI}{bI}$. This, in turn, allowed Chasles to choose a, b, a', b' such that $\lambda = -1$, and to simplify the equation into $\frac{am}{aI} = \frac{a'm'}{J'm'}$, where J' is the homologue of the point at infinity (viewed as a point of the second division).

The equations we have discussed so far are what Chasles called "two-term equations" (*équations à deux termes*), in a transparent analogy to the equations relating the various anharmonic ratios of the same four points (Chasles 1852, 81). Chasles also showed how to obtain three-term equations by substituting to an anharmonic ratio r the second or third such ratios, which, as we saw previously, are simple functions of r. Subsuming the fixed quantities depending only on a, b, c, a', b', c' into two constants, he obtained, for instance, the following equation for homographic divisions (Chasles 1852, 87):

$$\lambda \frac{am}{bm} + \mu \frac{c'm'}{b'm'} = 1$$

Then, in a similar fashion, he derived the following four-term equation (Chasles 1852, 93):

$$am \cdot b'm' + \lambda am + \mu b'm' + \nu = 0$$

In fact, Chasles even showed how to create an equation of an arbitrary number of terms, even though uses of equations with more than five terms are scarce in his entire body of work. What must be stressed here is that these equations are all equivalent, in the sense that they all represent the same fundamental configuration. This work on the many possible equations representing one identical configuration both illustrates the fruitfulness of the concept of anharmonic ratio and constitutes a reservoir of equations, from which the most suitable will be picked by Chasles when actively solving a problem—including that of "restoring" the content of Euclid's lost *Porisms*.

At this stage, one can understand how Chasles' 1837 general porism for the straight line easily derives from the fundamental theories of higher geometry. Indeed, the equation at the heart of the general porism was merely a specific kind of three-term equation: one way of describing the configuration in which two homographic pencils of rays intersect along a straight line. This connection between homographic divisions and Euclid's lost porisms would, in the years following the publication of the *Traité*, run deep. In fact, the very statements that Chasles had given in 1837 are reproduced in the third section of this book, without any reference to the *Aperçu historique* (Chasles 1852, 339–344). In his 1870 *Rapport*, Chasles would explicitly state that all of Pappus' porisms are cases of this general equation (Chasles 1870, 239). And, in his 1860 book on porisms, in order to classify and interpret Pappus' propositions, Chasles explicitly mobilized this notational technology, which he had crafted for the description of homographic divisions.[58]

Before doing so, and in keeping with his theses on the limitations of Ancient geometrical methods (in particular in terms of their generality), Chasles had to carry out a meticulous adaptation of his abstract equations to the multiplicity of concrete cases with which he imagined Euclid had busied himself. The scope of this task, however, would be transfigured throughout the 1850s, as Chasles adapted his notational technology not only to his anterior interpretation of the porisms, but also to the dissenting opinions of a growing community of classicists.

7. Conflicting norms

While Chasles delayed publishing the thorough analysis of Euclid's *Porisms* he had heralded in the *Aperçu historique* until the very end of the 1850s, the so-called "question of porisms" was the focus of heated scholarly debates throughout the decade. In particular, a string of controversies raged about the proper way in which a few sentences by Pappus should be read. A microhistorical analysis of this dispute reveals how the expectations of the academic community became increasingly polarized by conflicting norms within a gradually splintering scientific field, in which classicists and mathematicians would eventually struggle to find common ground.

The main controversy burst into the open as a priority claim in 1859. It set the towering Michel Chasles, Academician and Professor at the Sorbonne, against Paul Émile Breton de Champ (1814–1885), former *Polytechnicien* (1834), *Ingénieur des Ponts et Chaussées*, and a minor figure

[58]We shall return to the specific labor involved in tailoring modern equations for ancient configurations in section 8; see in particular figure 8–10.

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occupying a rather peripheral position in the Paris scientific community. This controversy had been smoldering, however, since Breton's initial clashing claims about ancient porisms were released in a series of publications, first in the *Comptes rendus* and then in Liouville's journal (Breton 1849, 1853, 1855). These immediately elicited skirmishes on two distinct fronts. Charles Pierre Housel (born in 1817) and Alexandre Joseph Hydulphe Vincent (1797–1868) were both trained in mathematics at the *École Normale Supérieure*, albeit twenty years apart. They occupied different positions in the scholarly field, and therefore read and questioned Breton's contributions from different perspectives. With hindsight, both these minor quarrels appear to have been nested within the main structuring controversy, as side-effects of the fracture line that was progressively setting apart two distinct kinds of agendas. Thus, the controversy casts light on the process of social differentiation that underlay the growing estrangement between two different groups of actors, who were embracing different scholarly methodologies.

Ever since his 1849 note, Breton had suggested that Simson had been wrong to replace the original definition of porisms given by Pappus, however obscure and intractable, with his own made-up formulation:[59] an utterly different phrasing, allegedly designed to match Pappus' incidental comment that porisms were neither theorems nor problems, but instantiating some sort of mean between them. In Breton's view, Simson had misconstrued external features in the porisms' statements as the elements of their possible definition. However, Breton only offered his alternative reading of Pappus in 1855. He fully acknowledged that Simson's recast definition of porisms, as well as his reconstruction of the only two complete propositions that came down to us in Pappus' text as instances of Euclid's porisms,[60] made perfect mathematical sense. But he also pointed out that the whole approach was entirely misguided, for it was not based upon an accurate and updated account of the original sources themselves.

By contrast, Breton insisted that one should start with the Greek text, which he set himself the task of editing and translating on the basis of new and enriched evidence. He thus went back to Halley's 1706 printed edition (Apollonius 1706, XXXIII–XXXVI), which he amended by comparing it with two different manuscripts he could consult at the *Bibliothèque impériale*.[61] His primary goal was to provide a more faithful edition than Halley and a better translation than either Commandino or Simson. His main contention concerned the difference between two kinds of utterances, namely between Pappus' twenty-nine *statements,* which were listed *verbatim* in the Greek text, and Euclid's 171 lost *porisms*, of which only two are extant and explicitly enunciated in the original. Pappus' statements are all framed after the same pattern and phrased in the form of an object clause introduced by the Greek preposition ὅτι, as for instance in "*that this point touches a line given in position*," or "*that the ratio of this (line) to this (line) is given*" (Jones 1986, 100).

[59]As is well known, Pappus gave two distinct definitions of *porism* in Book VII of the *Mathematical Collection*. The first one which he imputed to the Ancients amounted to a terse statement among a series of three similar ones, all cast from the same mold for *theorem*, *problem*, and *porism:* "a theorem is what is offered for proof of what is offered, a problem what is proposed for construction of what is offered, a porism what is offered for the finding of what is offered" (Jones 1986, 96). The second definition, by contrast, which Pappus ascribed to the Geometers that came after Euclid, for they presumably could not grasp the full import of the ancient definition, enunciated that "a porism is what is short by a hypothesis of (being) a theorem of a locus" (ibid.*).* Simson overstated Pappus' suggestion that this second definition was based on a mere accidental trait and declared it faulty. But the first definition proved too cryptic to be helpful, or, in Simson's terms, "exceedingly general" (Tweddle 2000, 17). Simson then gave an alternative definition, which Chasles took as a starting point.

[60]Namely the two propositions now referred to in the literature as the *Hyptios*-porism (Jones 1986, 98, 556); and Euclid's First Porism (Jones 1986, 100, 554). For a synthetic presentation of Simson's reconstruction of both propositions, see also Hogendijk 1987.

[61]Breton used the manuscripts now respectively referred to as manuscripts B (*viz. Parisinus* 2440) and C (*viz. Parisinus* 2368). On the identification of the manuscripts and the progressive establishment of the *stemma codicum*, see Jones 1986, 64. The sigla for the manuscripts, introduced by Friedrich Otto Hultsch, were taken over by later scholars (see Hultsch 1876–1878, Vol. I, *Praefatio*). Halley thus used the manuscripts labeled O and P, Simson used Halley, B and C, while Breton also independently used B and C. Breton's variant readings are duly registered in the critical apparatus of nineteenth- and twentieth-century authoritative editions of Pappus' text, by F. O. Hultsch and Alexander Jones.

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Obviously, these were incomplete statements and the whole point was to understand the exact way they would connect with Euclid's 171 supposedly complete propositions.

In a much commented upon passage, Pappus seemed to offer a hint with respect to this interpretive problem, by indicating that various hypotheses may be adjusted to one and the same ending, stipulating the result and that which is sought.[62] The common view held by generations of scholars from Halley and Simson to Chasles (at least in the 1830s) was that there were lacunae in Pappus' original Greek text, an assumption which was reflected, from Halley's Latin translation onwards, by the presence of typographical ellipses in the way Pappus' statements were rendered.[63] Breton strongly denied the reality of these lacunae, which in turn made it compelling to interpret Pappus' text as it stood, that is, to acknowledge each of these statements as constituting a *porisma* in its own right, as something to be discovered or invented in the context of various hypotheses. Breton, moreover, claimed that Simson, and Chasles after him, had failed to ascertain the exact nature of Pappus' statements, for they had incorrectly thought of these as defective propositions, whereas, in Breton's view, they were neither defective nor propositions. Besides, Simson and Chasles would frequently jumble together the two kinds of utterances, namely Pappus' statements and Euclid's porisms proper, which Breton insisted should be strictly distinguished as being different types of linguistic expression.[64]

The controversy emerged publicly in 1859, when Chasles presented his forthcoming book on porisms to the Paris Academy. In a note published in the *Comptes-rendus*, which would be reprinted in the book as §§ I–II of the introduction, he sounded much more specific with respect to the textual particulars than in the *Aperçu historique*. The difference between Pappus' statements and Euclid's porisms was now made explicitly. In contrast to the latter, assumed to be of a propositional nature, the former were now thought of as *genera*, that is to say, the "twenty-nine statements, transmitted by Pappus, which, in their concise and enigmatic style, summarize Euclid's many propositions" (Chasles 1859, 1038; Chasles 1860a, 8). Chasles only acknowledged his debt to Simson and expressly warned that he would leave aside any work on porisms later than 1835, when his ideas came to be fixed on this topic. Hence, he would "not [even] allude to the most recent research, especially that which gave rise to an ongoing polemic" (ibid.). This was enough to infuriate Breton, who claimed priority for introducing this very notion of "summarizing" so as to highlight the way the relationship between the two kinds of utterances should be understood. Thus, most of the dispute hinged on words, and Chasles soon had the upper hand by dismissing Breton's priority claim as mere quibble about nothing, a vain equivocation about phrasing and commas. As Breton reiterated and substantiated his priority claim in a series of notes submitted to the Academy, Chasles eventually delivered a detailed answer.[65] With the icy tone of outraged pride, he reaffirmed the consistent continuity of his own independent train of thought, from the *Aperçu historique* to the 1860 book. To settle the case, a committee was appointed, which was to include Bertrand, Serret, Lamé and Chasles himself. The last two declined, and in the end the report was written by Serret alone. After having dissected Simson's Latin to decide whether it meant what Breton and Chasles contradictorily imputed to it, Serret concluded that, by and large,

[62] Jones 1986, 98: "Their classes should be defined, not by the various hypotheses, but by the various things that result in them and are sought in them. All the hypotheses differ from each other, being very individual, but each of the results and things sought turns up exactly the same in many different hypotheses."

[63] It is worth noting that the ellipses do not appear in Commandino's Latin translation, which reinforces Breton's point that the notion of Pappus' text being lacunary starts with Halley and should be connected with his avowal that the sources remained unintelligible to him in this critical juncture.

[64] As evidence of Chasles' confusion, Breton cited the passage, partly quoted above, from the *Aperçu historique* (Chasles 1837, 12): "these statements [*viz.* Pappus's] are so terse and have become so defective because of lacunae and the absence of the relevant figures, that the famous Halley so deeply versed in ancient Geometry, confessed that he understood none of it." By contrast, Breton repeatedly hammered that nothing ever missed in these passages, not even the figures which never existed, for Pappus' statements were about general geometrical facts common to whole ranges of hypotheses.

[65] For the whole sequence of exchanges marking out in print the stages of the controversy between June 1859 and the end of 1860, see, in order of publication: Chasles 1859; Breton 1860a; Breton 1860b; Chasles 1860a; Breton 1860c; Chasles 1860b; Serret 1861.

Breton's claim was to be rejected, and only conceded an imprecise use of language on Simson's and Chasles' part.

However, one aspect of this controversy is of interest for our present concern, as it sheds light on the contrasting ways in which the investigation's very goal was to be conceived on both sides. In Breton's view, solving the question of porisms solely amounted to establishing Pappus' text so as to make it "explicit enough" (Breton 1860b, 996). What he dubbed the "conjectural restitutions of Euclid's one hundred and seventy-one propositions," could not in the least pretend to solve the question, since setting up reconstructions was essentially secondary to the primary task of providing a satisfactory edition and translation of the text. Moreover, it would prove an open-ended endeavor, as "the field open to restitutions is inexhaustible" (ibid.). Therefore, Breton underlined, the priority claim did not encroach upon Chasles' reserved area—the mathematical content of the presumably reconstructed porisms—but only on matters pertaining to the text itself and the way Greek locutions should be understood.

Although trained as a mathematician, Breton concurred here with another group of scholars, those who gathered at the Académie des Inscriptions et Belles Lettres rather than the Académie des Sciences, such as the well-known lexicographer Émile Littré, with whom Breton was in contact.[66] Originally trained as a physician, Littré entered the former of these *Académies* in 1839, authored monumental translations of Hippocrates and Pliny the Ancient, and devoted most of his life to the famous *Dictionnaire*, in which, significantly enough, the entries *Philologie* and *Philologues* both referred to Charles Rollin's definition of this subgenre within the *Belles-Lettres*:

> One calls *philologists* those who worked on ancient authors, to examine them, correct them, explicate them, and bring them to light: those who embraced this universal literature extending over all sorts of sciences and authors, and which of old made the most beautiful part of grammar. By *philology*, one thus understands a kind of science composed of grammar, rhetorics, poetics, antiquity, history, philosophy, and sometimes even mathematics, medicine and jurisprudence, without dealing with any of these matters thoroughly nor separately, but only touching upon them all or partly. (Rollin 1817, 564–565)[67]

The term "philology" thus had a different resonance in France than in Germany at the time. Associated with the tradition of the *grammaire générale* and rhetorics, the philological practice was never imbued with the overbearing social and cultural significance it acquired in the German context.[68] In return, the value attached to the kind of composite and universal knowledge required of those engaging with ancient texts may account for greater social porosity, allowing newcomers to cross apparent boundaries between disciplines and draw on their original training to contribute to the common knowledge of our literary past. Since the turn of the century, a specificity of the French higher education system was that the preparatory training to the "*grandes écoles*" should be entrusted to the *lycées* and *collèges royaux*, most of which, "for lack of a proper statutory setting, improvised locally, on the fringes of the classical curriculum, a special program for these 'mathematical pupils'" (Belhoste 2001, 9). This provided a distinct social context for the shaping of a public interest in combining classical and scientific backgrounds. For all these structural reasons, Breton could claim to consistently occupy a middle ground between mathematics and classical philology, despite not being a professional classicist.[69]

[66]In a footnote to his edition of the Greek text of Pappus, Breton indicates that he was indebted to Littré for one of the alternative readings supposedly emending Halley's corrupted version (see Breton 1855, note *f*, 214).

[67]The passage selected by Littré is excerpted from the *Histoire ancienne* (1738) by the French historian Charles Rollin (1661–1741), also famous for his pedagogical treatise codifying the French tradition of the *Belles-Lettres*.

[68]The complex relationship between history of mathematics, mathematics and philology thus presents itself differently in these different national traditions (cf. Smadja 2015).

[69]The whole controversy between Breton and Alexandre Joseph Hydulphe Vincent, an honorary member of the *Académie des Inscriptions et Belles-Lettres*, notorious for his scholarship on the Greek musical scale and harmonies, revolved around

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In his reply to Breton before the Paris Academy, Chasles proposed a counter-definition of the so-called "question of porisms" by putting a premium on understanding the mathematics of the Greeks, rather than on faithfully translating the language in which it was (supposedly) originally couched.

> The difficulty did not consist, as [*Breton*] believes, in the translation of Pappus, after those of Commandino, Halley and Simson, which were more than enough and do not stand in any respect behind Breton's own. … The best translation could not prevent the question of porisms from remaining an enigma. The difficulty was to discover what were the theories or the families of propositions contained in Euclid's work, and to which Pappus' statements referred. It was a prolonged piece of mathematical work that was required, and not the work of a translator. (Chasles 1860b, 1050)

The whole point was then to frame a system of mathematical propositions which would help bringing out the mathematical content of Pappus' statements. "What should be understood by this word *restoring*? Obviously, [Chasles insisted,] what Simson himself understood, namely finding hypotheses that could be applied to Pappus' statements, while considering these not as individual propositions, defective and mutilated in the manuscripts, but as only expressing assertions emanating from Pappus himself" (Chasles 1860b, 1058–1059). Chasles did not conceal the trial-and-error nature of this interpretive process.

> Whatever the system adopted, in this kind of work, one cannot dispense with verifying and demonstrating its aptness: which can only be done by submitting this system to practical experiment. And here this experiment consists in forming … a systematic set of propositions, distinct in certain respects from both theorems and problems, matching Pappus enigmatic statements and his sayings on the importance and the usefulness of Euclid's work. (Chasles 1860a, 10)

Chasles' and Breton's respective methodologies were thus presented on both sides as being incompatible. The unfolding of the controversy indeed implied a hardening of the positions, in which each protagonist became ever more entrenched. A closer inspection, however, reveals the situation to be more nuanced.

In 1856, Charles Housel contributed a paper to Liouville's *Journal*, in which he attempted to conciliate, to a certain extent, both approaches. Favorably impressed by Breton's conjecture that Euclid's porisms were about geometrical configurations liable to variation with respect to their form, he tried to flesh out this suggestion from the mathematical point of view. Fully endorsing Breton's point about the distinction between Pappus' statements and Euclid's propositions, Housel held that, while the latter comprised both a *hypothesis* and a *result* interlocking into one

---

linguistic issues (see Vincent 1857a; Vincent 1857b; Breton 1858; Vincent 1859). In a series of papers, Vincent and Breton thoroughly discussed matters of translation, which the former had found faulty in the latter's rendering of Pappus. Among other things, Vincent had contested Breton's substitution of a modern lexicon ("*fixed,*" "*variable*") to what was originally expressed in the Greek language of the given, a choice Breton nevertheless vindicated as being justified from the mathematical point of view. In some instances, Vincent advocated that the translation be closer to the original, as in the case of the *Hyptios*-porism, where Vincent resisted the paraphrase Breton, like Simson, had favored. However, Vincent suggested, "the meaning of the words ὕπτιον and παρύπτιον does not seem questionable to me. They represent the situation of the complete quadrilateral whose main diagonal would be more or less horizontal … E ὕπτιον is an E lying in this way ш. Hence the complete quadrilateral can be compared to an A ὕπτιον" (Vincent 1857a, 10). Whereas Breton strongly put the emphasis on a word-for-word translation that would at the same time keep track of the "sense of geometrical things," Vincent countered by insisting that the sense of Greek language would sometimes preclude a too pointillist translation, "similar to those portraits done from life by means of the Daguerreotype or the physionotype," a kind of device then in vogue which, for all its minute faithfulness, nevertheless failed to grasp the liveliness of physiognomy (see Vincent 1857b, 3).

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another, the former focused on the *result* alone (namely the *porisma* proper, according to Breton). In other words, Pappus' statements focused on what was to be sought, constructed or determined from what was given at the outset, so that the whole proposition could then be demonstrated. More specifically, the *hypothesis* would by and large contain the fixed parts on which the moving figure would hinge, together with its "conditions of variability or mobility" (Housel 1856, 196–197).[70] Having done so, Housel was able to perceptively re-interpret Euclid's so-called *Hyptios*-porism, by adjusting it to the format in which the *result* would be separated by an object clause as in Pappus' statements:

> *If, in a system of four straight lines pairwise intersecting in six points, the three points lying on one of these lines are fixed, and if two of the three remaining ones are each subjected to move on a fixed straight line,*
>
> I say that:
>
> *the last vertex also moves on a straight line.* (Housel 1856, 206, italics in original)

Furthermore, Housel made the connection between Euclid's porisms, which he interpreted after Breton, and Newton's organic description of conics by means of rotating angles, which he offered alongside Euclid's *Hyptios*-porism as another telling example of porism, albeit one taken from "outside Pappus and Euclid" (Housel 1856, 197).

Although both examples were instances of the generation of curves by means of homographic pencils, it is significant that Housel did not attribute this parallel to Chasles, whose printed views on porisms, he pointed out, amounted only to characterizing them as playing, in ancient Geometry, a role analogous to that of Cartesian coordinates in modern Geometry. Indeed, it was a long way from the *Aperçu historique* to the 1860 book on porisms. Despite Chasles' claim that his ideas were already fixed on this score in 1835, it seems that much remained implicit and needed to be patiently unpacked and delineated.

With hindsight, however, Chasles retraced the path he had trodden while allegedly unfolding an unequivocal thread:

> Let me add a few words about the work on porisms contained in Note III of the *Aperçu historique*.
>
> I did not limit myself to announce the restitution of Pappus' 24 statements that had remained until then a sealed letter. I gave on the spot in the form of porisms two general propositions whose consequences were to embrace the 15 statements of [Euclid's] Book I. To get there, I said that one had to transform the equations that these propositions implied into other ones with 2, 3, or 4 terms, each of which would be the expression of one of Pappus' statements. And to complete this work, which constitutes the divination of the enigma of Porisms, and which the current book [*viz*. (Chasles 1860a)] is nothing but the development, I added that each of these equations (consequently each of Pappus' statements) would give rise to several Porisms. (Chasles 1860b, 1059)

Chasles' unpublished drafts reveal that he proceeded in this way so as to frame the equations that would assumedly match Pappus' statements. However, it remains unclear whether he engaged in this kind of "practical experiment," consisting in tinkering with his equations to adjust them to the Greek text, on his own initiative, or because of the controversy. In any case, in the 1860 book, the

[70] As a matter of fact, Housel split the hypothesis in two parts, corresponding severally to the fixed parts and the conditions of variability. He also attempted to find some way of connecting this distinction with Pappus' textual particulars, which Breton found inadmissible (see Breton 1857, and Breton 1858, 192).

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emphasis would shift from coordinates to anharmonic ratios as yielding the key to the restoration of porisms.[71]

8. A "practical experiment"

Archival evidence sheds light on how Chasles endeavored to bridge the gap between his 1837 "first (and second) porism(s)," and the wealth of porisms adjusted to the variety of cases that Greek geometry supposedly required. Among the many drafts on ancient porisms that have been preserved in the Chasles' archive at the Paris Académie des sciences, we select here the sheaf titled "*Porismes généraux*,"[72] which best illustrates the kind of "practical experiment" Chasles alluded to in his controversy with Breton, and which would admittedly buttress his own attempt at restoring Euclid's lost porisms. Although undated, these drafts do arguably document the process through which Chasles would transform the consequences drawn from his 1837 general propositions into two-, three- and four-term equations, each of which would presumably fit one of Pappus' statements.

Starting from the general configuration, corresponding to the 1837 "first porism" (see figure 3 above), Chasles aimed to regain the Greek cases by unfolding a similar procedure in various instances. Recall that, in the general case, one dealt with two straight lines, each joining one of two fixed points P and Q and a variable point m sliding along the guiding line MM. These two straight lines would in return intersect two given transversals in respectively two points a and a'. Since the whole configuration allowed for free variation to an extent, one may also consider different cases by making the two points P and Q coincide, by sending them to infinity, or by making the transversals themselves coincide into one and the same straight line, which may, or may not, be assumed parallel to the guiding line or to the straight line joining P and Q, etc. Chasles methodically enumerated all cases and dealt with them in a parallel way. Here, however, we shall only focus on the generic configuration, consisting of one single transversal SS being parallel neither to the guiding line MM nor to the straight line joining P and Q (see figure 6 and 7).

Further points may then be constructed. Let us denote C the intersection of the straight line PQ and the transversal. Drawing the straight line Pm' parallel to the transversal SS, one determines the point m' on the guiding line, hence the straight line joining the points Q and m' intersects the transversal in point I'. Correlatively, the parallel through Q to the transversal determines the point m'', so that the straight line Pm'' intersects the transversal SS in point I. On the basis of this figure (and a few others), Chasles then offers a series of ten equations, each of which is attributed a number in the margin, namely the number of porisms corresponding to the equation at hand (see figure 8).

Let us explain briefly how Chasles obtained these equations by taking full advantage of the theory of homographic divisions. In the figure above (figure 8), the four straight lines PQ, Pm, Pm', Pm'' meet the four corresponding ones QP, Qm, Qm', Qm'' in four points D, m, m', m'' on the guiding line MM.[73] Since both groups of four lines respectively meet the transversal in the points:

$$C,\ a,\ \infty,\ I, \text{ and } C,\ a',\ I',\ \infty,$$

the anharmonic ratio of the first four points is equal to that of the last four, that is

$$\frac{Ia}{IC} = \frac{I'C}{I'a'}$$

for the segments containing a point at infinity are canceled from the equation according to the practice codified by Chasles in his lecture course, hence,

[71]Both Housel and Breton responded to Chasles' book on porisms, whether in a conciliatory or an uncompromising way (see Housel 1861; Breton 1865, 1867).

[72]This sheaf is kept in Cardboard Box 5, File 10 in Chasles' archive at the Académie des sciences, Archives et patrimoine historique.

[73]Let us indeed assume that the point D is the intersection of the straight line QP and the guiding line MM. This point may be a point at infinity if QP is parallel to the guiding line.

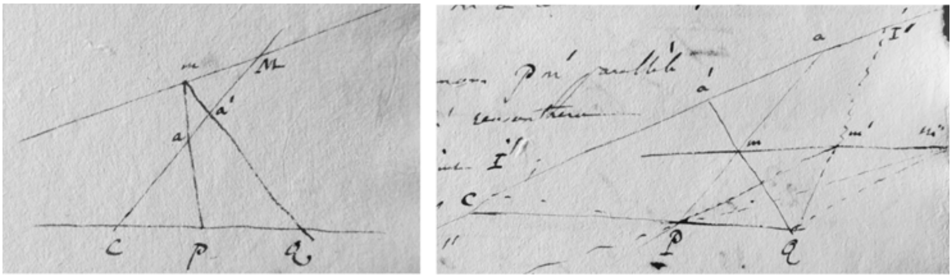

Figure 6. Chasles' diagrams from the sheaf "Porismes généraux," Cardboard Box 5, Chasles archive, Académie des sciences, Paris.

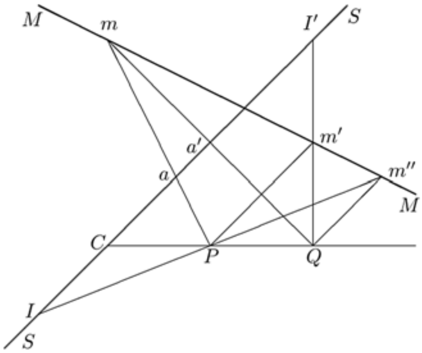

Figure 7. Chasles' generic configuration.

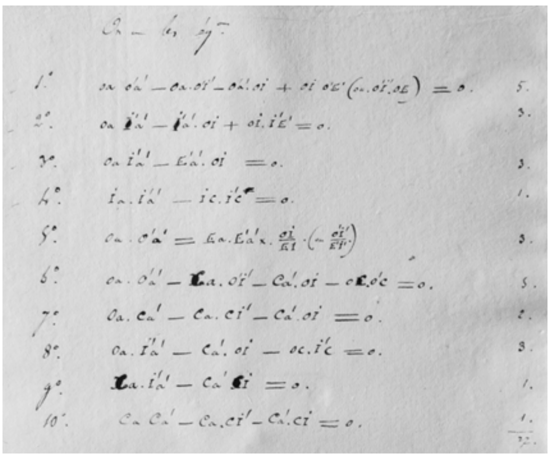

Figure 8. Chasles' list of equations with their associated number of porisms. Sheaf "Porismes généraux," Cardboard Box 5, Chasles archive, Académie des sciences, Paris.

$$Ia \cdot I'a' = IC \cdot I'C$$

which is the fourth equation in Chasles' list (see figure 8), allowing in return for only one porism, namely in Chasles' own terms: [Given the generic configuration with the transversal being parallel neither to the guiding line nor the straight line PQ,] "one will always be able to find two points I and I' on the transversal so that the product $Ia \cdot I'a'$ be constant."

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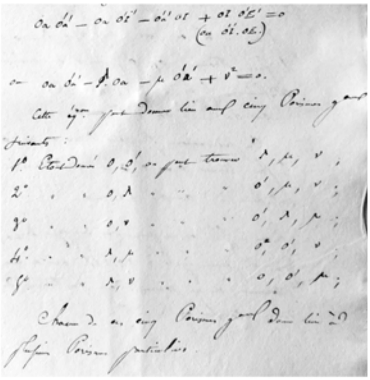

$$Oa \cdot O'a' - Oa \cdot O'I' - O'a' \cdot OI + OI \cdot O'E' = 0$$
$$(\text{or } O'I' \cdot OE)$$

or

$$Oa \cdot O'a' - \lambda \cdot Oa - \mu \cdot O'a' + \nu^2 = 0,$$

This equation can give rise to the five general porisms that follow:

1°.	Given	O, O',	one can find	λ, μ, ν
2°.	… …	O, λ,	… … …	O', μ, ν
3°.	… …	O, ν,	… … …	O', λ, μ
4°.	… …	λ, μ,	… … …	O, O', ν
5°.	… …	λ, ν,	… … …	O, O', μ

Each of these five general porisms gives rise to several particular porisms.

Figure 9. How the five general porisms associated to the first equation are obtained. Sheaf "Porismes généraux," Cardboard Box 5, Chasles archive, Académie des sciences, Paris.

Now one may take into account two further points, O and O', which formerly served as origins on both transversals in Chasles' 1837 first porism. Assuming, at first, that they be given arbitrarily on the now unique transversal SS, one would have:

$$Ia = Oa - OI \text{ and } I'a' = O'a' - O'I',$$

hence the fourth equation above can be rewritten in this way:

$$(Oa - OI)(O'a' - O'I') = IC \cdot I'C$$

hence

$$Oa \cdot O'a' - Oa \cdot O'I' - O'a' \cdot OI + (OI \cdot O'I' - IC \cdot I'C) = 0$$

One may now introduce a further point E' on the transversal SS as the homologue of point O in the above homographic division, that is such that PO and QE' meet in one and the same point on the guiding line MM, and correlatively a point E as the homologue of O'. Then, owing to the fourth equation previously obtained, one may write,

$$IO \cdot I'E' = IC \cdot I'C, \text{ hence } OI \cdot O'I' - IC \cdot I'C = OI \cdot O'I' - IO \cdot I'E' = OI \cdot O'E',$$

so that the above equation can be rewritten as

$$Oa \cdot O'a' - Oa \cdot O'I' - O'a' \cdot OI + OI \cdot O'E' = 0,$$

which is the first equation in Chasles' list (see figure 9).

One may now assume (which is still possible) that the point O coincides with the point C. If one then makes further assumptions so as to particularize the case some more, then one obtains all the other equations in succession.[74] From each of these ten equations, one then obtains the number of the corresponding porisms by considering the various ways in which the given components of the configuration may be distributed in two groups, namely those which are given from the outset and those which are sought so that the property enunciated may hold and which the porism is precisely designed to yield. Owing to his new notational technology, Chasles could now couch the whole configuration into an equation in which the sought components would be represented by Greek letters, playing the role of the indeterminate coefficients of symbolic algebra. Chasles' interpretive practice thus confirmed his theoretical point about the "deep analogy" between Euclid's *Porisms* and his *Data* (Chasles 1860a, 41–44), a notion which was subtly implied in his

[74]If one assumes for instance that O' and I' coincide, one easily derives equation (2) in Chasles' list (see figure 8), and if now $O = C$, one obtains equation (9), while equation (3) stems from considering the homographic correspondence between both series of points O, a, I, ∞ and E', a', ∞, I', and so forth.

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I^er Livre.

Genres.

I. $\frac{Am}{A'm'}=\lambda$ (1).

II. Tel point décrit une droite donnée de position.

III. $\frac{Am}{A'm'}=\lambda$; $\frac{Sm}{Sm'}=\lambda$.

IV. $\frac{Am}{mm'}=\lambda$.

V. Telle droite est donnée de position.

VI. Telle droite passe par un point donné.

VII. $\frac{Am}{A'm'}=\lambda$.

VIII. $\frac{Am}{Mm'}=\lambda$.

IX. $\frac{Am.J'm'}{a.A'm'}=\lambda$. Divisions homographiques sur deux droites.

IX. $\frac{Am.J'm'}{a.A'm'}=\lambda$. Divisions homographiques sur deux droites.

X. $J'm.Im'=\nu+\mu.mm'$. Divisions homographiques sur une droite.

XI. Énoncé incomplet.

XII. $\frac{Am+\lambda.Bm}{Cm}=\mu$, $\frac{Am+\lambda.Bm}{C'm'}=\mu$,

$\frac{Am+\lambda.B'm'}{C'm'}=\mu$, $\frac{Am+\lambda.B'm'}{C''m''}=\mu$.

Divisions en parties proportionnelles sur deux ou sur trois droites.

XIII. $Am.Op=A'm'.O'p'$.

XIV. $\frac{Am+Bm}{C'm'}=\mu$. Divisions en parties proportionnelles sur deux droites.

XV. $Im.J'm'=\nu$. Divisions homographiques sur deux droites.

II^e Livre.

Genres.

XVI. $\frac{Am.B'm'+\nu}{mm'}=\mu$. Divisions homographiques sur une droite.

XVII. $\frac{Am.B'm'}{mm'}=\mu$. Divisions homographiques sur une droite.

XVIII. $\frac{(Am+Bm)(C'm'+D'm')}{mm'}=\mu$. Divisions homographiques sur une droite.

XIX. $\frac{Am(B'm'+\lambda.C'm')+Dm.\lambda_1.E'm'}{mm'}=\mu$. Divisions homographiques sur une droite.

XX. $\frac{Am.B'm'+Cm.D'm'}{Gm}=\mu$. Divisions homographiques sur deux droites.

XXI. $Im.J'm'=\nu$. Divisions homographiques sur deux droites.

III^e Livre.

XXII. $\frac{Am.B'm'}{Cm.D'm'}=\lambda$. Divisions homographiques sur deux droites ou sur une seule.

XXIII. $\frac{\overline{Am}^2}{mm'}=\mu$.

XXIV. $Am.J'm'=\mu.A'm'$. Divisions homographiques sur deux droites ou sur une seule.

XXV. $\overline{Om}^2=\mu.Dp$.

XXVI. $\frac{(Am+Bm)\lambda.C'm'}{mm'}=\mu$. Divisions homographiques sur une droite.

XXVII. Un point duquel on peut mener deux droites (variables) qui comprennent un triangle donné d'espèce.

XXVIII. Un point d'où partent deux droites (variables) qui interceptent des arcs égaux.

XXIX. Un point par où passe une droite faisant avec telle autre un angle donné.

Figure 10. Chasles' 1860 equations matching Pappus' statements (Chasles 1860a, 68–69).

very notation, both in the drafts and in the 1860 book (see figure 10). Some letters (for instance *a*, *m*) denoted variable points, while others (like *O*, *I*, *E*, etc. or *A*, *B*, etc.) denoted fixed points, whether "given actually or virtually." Lastly, Greek letters "λ, μ represent[ed] segments, (rectilinear) areas, or constant ratios, which are also given, actually or virtually" (Chasles 1860a, 68–69). In Chasles' parlance, the phrase *virtually given* would refer here to what, though not being given from the outset, can still be determined, or constructed by means of what is *actually* given—a distinction already at the core of Simson's account, which Chasles nevertheless inventively brought to fruition in unprecedented ways. Knowing how to delicately handle this flexible notational apparatus, Chasles could eventually parse his equations so as to mold them into hundreds of presumably Euclidean porisms, thereby resuming and completing his long-standing restoration project.

Chasles' claim about Euclid's *Porisms* being akin to his *Data* had already been clearly articulated in the *Aperçu historique*. However, adjustments were made in his later book on porisms, and the emphasis shifted significantly.[75] In 1837, Chasles first noted that the very practice

[75]For instance, Chasles first suggested an interpretive track anticipating to some extent the so-called "geometrical algebra" of the Greeks, but soon abandoned it in favor of a different project. In the *Aperçu historique*, he had indeed observed incidentally that "one can deduce from several propositions of the *Data*, that second-degree equations are solvable, which, as far as the Ancients are concerned, can only be found in Diophantus, coming more than 600 years after Euclid" (Chasles 1837, 11). Unsurprisingly, his examples in support of this claim were **Dt** 85 and 86 (cf. Taisbak 2003, 209, 211). Long before Paul Tannery, Hieronymus Georg Zeuthen, Thomas Heath, and Bartel L. van der Waerden elected to give pride of place to these

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of explicitly quoting the propositions of Euclid's *Data,* in addition to those of the *Elements*, and thus using them meaningfully in geometry, had been lost between Newton, for whom it still made sense, and the nineteenth-century geometers. Hence the need to recover the original Euclidean meaning attached to the term "given": namely, in his own rewording, "what immediately results, by virtue of the propositions contained in [Euclid's] *Elements*, from the conditions of a question" (ibid. 10). Chasles illustrated this need with the following proposition from Euclid's *Data*, **Dt 90**: "If from a given point a straight line be drawn tangent to a circle given in position, the straight line drawn will be given in position and in magnitude" (Taisbak 2003, 229). In his Note III on Euclid's porisms, Chasles made his views somewhat more precise, and averred, however tersely, that *porisms* were to *local theorems*, what *data* were to *theorems* (Chasles 1837, 275).[76] This assertion was completed with an interesting footnote about two distinct meanings of the term "given" being associated with only one word in Euclid's usage:

> Let us risk here a remark that we did not dare to allow ourselves when speaking of Euclid's *Data* . . . in this kind of propositions [*viz.* in the porisms] there is something to be found; Pappus designates this thing which is sought by the word *given*, as Euclid did in his book on *Data*, and at the same time, he applies the same word to each and every thing that is given by the hypothesis of the question. Pappus' statements would have been more intelligible, if he had only designated the latter by the word *given*, and the former, that is the things that are to be found, by the word *determined*. This observation applies to Euclid's *Data*; but is is mostly while busying myself with the divination of porisms that the inconvenience of using one and the same word for two different things appeared to me. (ibid.)[77]

Interestingly enough, Chasles found support for his interpretation of Euclid's *Data* in Louis-Amélie Sedillot's translation of the treatise *On Known Things* by the Arab mathematician Ibn Al-Haytham (c. 965–c.1040), based on the so-called manuscript Ms 1104 of the *Bibliothèque royale* (now registered as Ms 2458 at the BNF) (Sédillot 1834).[78] While deeming Al-Haytham's work "an

propositions from the *Data* as key to their interpretation, Chasles had envisioned that very same path, before presumably disregarding it as a dead end (cf. Tannery [1882] 1912, 262; Heath 1921, 423; van der Waerden 1954, 121; Zeuthen 1919, 40, and, more recently, Herz-Fischler 1984, 88; Taisbak 1996; Taisbak 2003).

[76]Recent work on Euclid's porisms may help clarify Chasles' claim, as similar views are conveyed in other words. See, for instance: "A 'porism' is a proposition in which it is required to show that some elements of an assigned geometrical configuration are given. Such 'elements' can be ratios, domains contained by straight lines insofar as geometrical *loci*. The configuration is assigned in the broad sense that some of its elements are given, whereas others are subjected to constraints expressed in terms of given elements. In short, a porism is a geometrical configuration with degrees of freedom, of which one must identify some invariants. As in a theorem of the *Data*, a proposition of the Porisms shows that some object is given. Therefore, the propositions of the *Data* are a subclass of those of the *Porisms* (namely, the ones in which all constraints are rigid), and we must expect that the structure of a proof of a porism and of a proposition of the *Data* be similar. . . . In a *locus* theorem the assigned objects are *all* given. In a porism some objects are allowed to undergo changes, within a class identified by well-defined constraints. . . . Therefore, the idea that 'a porism is what is lacking of a supposition in order to be a local theorem' (this characterization is reported with disapproval by Pappus, *Coll.* VII, 14, because it was formulated 'on the basis of an incidental property') very likely refers to the 'incomplete' assignments relating to some objects present in the configuration of a porism: they are only constrained, not given. We must conclude that also the *locus* theorems are a subspecies of porisms, as Pappus already recognized" (Acerbi 2011, 137–138). See also: "In an (Euclidean) porism one considers a geometrical figure in which certain parts are supposed to be constant, while others are variable. The porism states that a new element defined by means of the variables is constant (or invariant) and can be determined (found) by means of the original constants. Simson used 'given' instead of constant, and 'not given' instead of 'variable', as the Greeks would probably have done. It would be very un-Greek to call any geometrical object 'variable'" (Hogendijk 1987, 94).

[77]Confronted with the same problem, Jan Hogendijk suggests distinguishing both meanings by italicizing the word *given* when (and only when) understood as what is "not assumed in the beginning, but determined (or constructible) by means of the things that are assumed in the beginning" (Hogendijk 1987, 96).

[78]For an edition of the Arabic original, a new translation in French and a commentary of Ibn Al-Haytham's treatise, see Rashed 2002, 393–583.

imitation and continuation" of Euclid's treatise, Chasles probably took his cue for his own distinction between what is "*virtually*" and "*actually* given" from Sedillot's rendering of a corresponding pair of Arabic terms with the French locutions "*connu de fait* (*realiter notum*)" and "*connu virtuellement* (*virtualiter notum*)" (see Sédillot 1834, 9–10).[79] Chasles always remained indebted to Sédillot for calling his attention on the importance of Arabic sources for the history of mathematics,[80] and henceforth became an active supporter of fellow scholars who, like Sédillot and later the young Franz Woepcke (1826–1864), combined the rare skills of a both an orientalist and a historian of mathematics.[81]

In his commentary, Pappus had explicitly averred that Euclid's three books on Porisms contained twenty-nine general statements, thirty-eight lemmas and 171 theorems (Jones 1986, 104). In his 1860 book, Chasles eventually yielded his "restoration" of the ancient doctrine, that is, a complete and systematic presentation of the way in which these three different types of propositions would subtly and precisely interlock. This required an extensive amount of preparatory work of the kind outlined above, so as to gear his abstract equations to the multiplicity of cases characteristic of ancient geometry. Chasles' opus basically comprises a hundred-page *Introduction* and three parts. The *Introduction* authoritatively recapitulates Chasles' own allegedly longstanding take on the various points at stake in the above-mentioned multi-faceted controversy, but without naming any names. It also offers further evidence in support of his claims with respect to the way Pappus' and Simson's texts should be understood. The three subsequent parts unfold the porisms presumably imputed to Euclid in his three lost books, plus forty-eight additional ones, by grouping them by genus. Chasles considered Pappus' twenty-nine general statements as providing many genera, which he listed (see figure 10), and paired with equations couched in his own geometrical notations "so as to fix the meaning attributed to each statement in which enters a relation of segments" (Chasles 1860a, 68). In Euclid's first two books, as reconstructed by Chasles, almost all these relations of segments correspond to geometrical configurations consisting of two series of points forming two homographic divisions. Both these segments may be formed either on two different straight lines (which is mostly the case in Euclid's Book I), or on one and the same straight line (which prevails in Book II). By contrast, in Euclid's Book III, one still considers configurations of the same kind, but the two straight lines turning around two fixed points now meet on a circle, instead of on a straight line as in the previous two books, and the fixed points are also themselves on a circle.

Commenting on his list of equations (see figure 10), Chasles remarks that four of them, namely genera III, IV, VII and VIII, seem to express the same thing, namely that the ratio of two segments is constant. "One may therefore believe at first sight, Chasles continued, that there is some confusion here as a result of some error in the text. But there are significant differences in Pappus' expressions, and he certainly had in view propositions that are not identical, especially with respect to the things sought" (Chasles 1860a, 71). Although the equations are either strictly identical (as for genera III and VII), or almost so (as for genera IV and VIII), the genera are nevertheless to be distinguished, depending on whether one considers segments of which both origins are known and only the ratio is sought (genus III), or—alternatively—segments for which only one origin is known and the other is sought together with the ratio (genus VII). Chasles' use of his notational technology thus affords him enough leeway to connect his equations to the multiple cases of Greek geometry in a both a flexible and insightful way.

[79]See Hogendijk 1987 for a contemporary account of the state of the art about the Arabic traces of Euclid's *Porisms*.

[80]In the main text of the *Aperçu historique*, the bulk of which was written in 1835, Chasles largely ignored the contribution of the Arab mathematicians, who were at best credited for having preserved the works of the Greeks during the long period of stagnation between the end of the Alexandrian school in Antiquity and the Renaissance (see Chasles 1837, 50). However, in the period between 1835 and 1837, when he completed his extensive Note XII, Chasles stumbled upon Sédillot's work and changed his views completely on this score.

[81]On Chasles and his promotion of the work of Sédillot and Woepcke, see Michel and Smadja, 2021b).

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In most cases, Pappus' twenty-nine genera severally give rise to more than one porism, hence the corresponding equations in the above list. The way Chasles proceeds to unpack the multiplicity of cases implied in these genera may be briefly illustrated by an example. The fifteenth genus, for instance, corresponds to a statement by Pappus, namely "that this (line) cuts off from (lines) given in position (abscissas) containing a given" (Jones 1986, 102), whose meaning is fixed with the equation $Im.J'm' = \nu$. But then, Chasles stresses, "the things that are sought are multiple and cannot be explicitly indicated, some of them remaining implied [*sous-entendues*]" (Chasles 1860a, 63). Thus, there is a certain amount of implicitness inherent in the genus, for the statement can be understood in different ways. In the case in point, two different porisms arise depending on the way one makes explicit what, in the genus, is only implicit. According to Chasles' rephrasing, one may indeed consider a straight line of variable position forming, on two fixed straight lines, two segments whose rectangle is constant, so that one has to determine the value of this rectangle and the position of both origins. But one may also interpret the configuration in a different way by assuming that both origins are "actually given," and that what is to be deemed as "virtually given," or what one has to find or to determine, consists of both the directions of the fixed straight lines through these given points and the value of the rectangle. In due course, Chasles would then associate two porisms, namely, the porisms LXXVI and LXXVII, to the fifteenth genus (Chasles 1860a, 174–175), the first of which being none other than a porism already "restored" by Simson as his forty-first Proposition (Tweedle 2000, 115–116). While serving to fix the meaning of geometrical configurations which they do not aim to exhaustively recapture, Chasles' equations provide an elaborate scaffolding, structuring a definite textual strategy. Owing to this, Chasles is able to systematically connect and organize the interpretive bits and pieces found Pappus' and Simson's texts, as well as to evaluate them.

9. The pitfalls of "archeolatry"

Shortly after its publication, Chasles' restoration of the porisms was the subject of a lengthy and dithyrambic review in the *Bulletin de bibliographie, d'histoire et de biographie mathématiques* (de Jonquières 1861). This *Bulletin*, sometimes regarded as the first journal entirely devoted to the history of mathematics, was published between 1855 and 1862 as an appendix to the *Nouvelles Annales de Mathématiques.* Both journals were then co-edited by the French mathematician Olry Terquem (1782–1862).[82] The author of this review, the vice-admiral Ernest de Jonquières (1820–1901), had long been a self-identifying disciple of Chasles, despite having scarcely set foot in the latter's lecture halls.[83] He was known to take Chasles' publications with him aboard his vessels, quickly embracing the language and arguments of his remote professor as his own. In his review of the book on the *Porisms*, de Jonquières strongly reiterated Chasles' main arguments regarding the necessity of developing modern Geometry in order to uncover the mystery behind Pappus' statements, especially the sameness of form between Euclid's porisms and the propositions of modern mathematics. Towards the last section of his review, having faithfully summarized Chasles' main claims, de Jonquières offered some more personal remarks. This successful "divination" of the porisms, he commented, was a welcome distraction from the hegemonic flow of publications by analysts. More broadly, Chasles' methods, which at this point de Jonquières had tied to the tradition of the porisms, refuted the widespread claim that analytic methods had any intrinsic superiority over pure geometry.

[82]Both journals were also co-edited by the mathematician Camille-Christophe Gerono (1799–1891). On the status of Terquem's and Gerono's *Annales* in the editorial world of nineteenth-century French mathematics, see Boucard and Verdier 2015, 65–66.

[83]The fast expansion of the French colonial empire under Napoléon III meant that de Jonquières's military and administrative services were constantly required overseas. De Jonquières took part, between 1859 and 1861, in military operations in Venice, the Levant (in modern-day Syria), and Mexico.

By connecting the porisms and this broader polemical context, de Jonquières explicitly mentioned a review penned a few months prior by Terquem himself, in which a recent memoir by Charles Méray written in the style of Chasles' higher geometry was negatively assessed (Méray 1860; Terquem 1860). In this memoir, only one (symbolic) equation was written, only to be immediately set aside and followed by geometrical derivations of the properties of surfaces of the second order. For Terquem, such mathematical writing was insufficiently "equational," a vice he also found, to a lesser extent, in Chasles' 1852 *Traité* (Terquem 1860, 70). Perhaps aware of Chasles' contention that there might be geometrical equations of a non-algebraic nature, Terquem immediately made his complaint more precise: while geometrical treatises such as Méray's may contain several "spoken equations," these are always of lesser value than "written equations" (i.e., algebraic ones). To limit one's expressive resources to verbal and other non-symbolic components, for Terquem, was a typical symptom of a larger epistemic vice he dubbed "archeolatry": the unrestrained idolization of an invented past. This geometrical practice, he hammered, was akin to "rusting a freshly minted medal, just to give it a gloss of antiquity" (ibid.). Not only was this of no scientific utility, Terquem concluded, but it was not even true to the spirit of the ancient geometers unduly imitated.

Borrowing a well-known line, which he attributed to a courtesan of Frederick the Great, but which had also been used by Stendhal in his defense of romanticism against classicism, Terquem drove a final wedge between Chasles' modern geometry and the Ancients by arguing that Euclid reborn in the nineteenth century would learn how to use equations just like everyone else, within a few months master them on account of his mathematical genius, and eventually regain his status as an exceptional geometer (Terquem 1860, 70–71; Stendhal 1823, 24).[84] Of course, this was not admissible for de Jonquières, whose argument rested largely on this hidden yet pervasive connection between Euclid's porisms (as reinterpreted by Chasles) and the kind of propositions that had successfully reinvigorated pure geometry. In his view, Chasles' treatises had successfully demonstrated that homographic divisions and other crucial theories were better served by words than symbols, both in terms of concision and clarity (de Jonquières 1861, 10). But, more profoundly, de Jonquières rejected Terquem's normative account of scientific progress and its modalities. In an earlier issue of the *Bulletin*, whilst discussing the history of logarithms, Terquem had compared the introduction of algebra and logarithm in mathematical practice to that of railroads and telegraphs (Terquem 1855, 1). All three inventions, Terquem explained, had enabled the swift transportation of (respectively) mathematical ideas, goods, and messages over immense swathes of conceptual and material land. As a train connects two remote places, so do algebraic computations connect seemingly disjointed hypotheses and conclusions—and in no time at all. Quoting from Benjamin Franklin's *Almanack*, Terquem stressed how essential such celerity had become to modern life, mathematical and otherwise: "Time is the Stuff Life is made of" (Franklin 1746, entry for 4 June). To divest oneself of these modern means of transportation in the name of a hypothetical appreciation of the past, in this regard, was simply unreasonable.

On that score, Terquem had already clashed with Chasles himself, albeit only in the form of short comments scribbled on his eminent colleague's (and friend's) unpublished manuscripts. For reasons yet unknown, Chasles had written down and shared with Terquem a copy of the opening lecture of his course on higher geometry in the year 1847–1848.[85] Unlike Chasles' teaching of the previous year (which he presented in his 1852 *Traité*), this set of lectures bore solely on the theory of conic sections. In the aforementioned opening lecture, Chasles elected to take a stroll through

[84]"If Caesar came back from the dead, he would have all of Louis XIV's generals beat." This line is also sometimes attributed to the Prince of Condé (for instance in Muller 1838, 112). In all of these cases, the then-common trope of Caesar's military genius was employed as a rhetorical device defending the worth of modern art and science against a perceived reactionary (and damaging) veneration of the past. The argument goes that geniuses such as Caesar, were they to live again, would apply their talent first to the learning of modern techniques, and only then exert strategical dominance. On this basis, it would be recommended that young scientists or artists learn modern methods instead of mimicking the ancient past.

[85]We have transcribed and analyzed this text in full in Michel and Smadja 2021b.

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the history of all geometrical methods that had been employed for the study of these curves; at the end of which, he claimed, it would appear clearly that, of all these methods, "only one, namely the rational and progressive Method ~~of the Ancients~~ [sic] is fit for the establishment of the relations and the connection between all parts of the subject, so as to constitute a true theory [of conic sections]" (Michel and Smadja 2021b, 288).[86] Whilst reading this manuscript, Terquem—who was equally erudite and invested in the history of mathematics—had made various annotations in the margins, either to signal faulty grammar, to ask for more precise bibliographical references, or to express his own opinion on Chasles' historical exposition. Next to Chasles' conclusion and argument for the necessity of a return to "the method of the Ancients," Terquem expressed, in no uncertain terms, his disapproval:

> One cannot force the stream of time to flow back to its source; I now see that this advice will not be followed. One loves one's wet nurse, one respects her, but one does not suckle anymore. . . . Which method must be followed? None, and all of them; always that method which fits the subject, which leads us as promptly as possible to our goal. From the Ancients, we have received but geometry to cultivate; yet there is also physics, chemistry etc. A life is still of the same length, but the task at hand is not; we must prove with rigor and promptitude. Such is the current character of science.[87] (Michel and Smadja 2021b, 288)

De Jonquières faced this argumentative line head-on; and, in his review of Chasles' *Porisms*, he turned Terquem's metaphors on their heads. Even if one were to concede that analysis had the upper hand with regard to speed and invention (which he was not quite ready to do), pure geometry would still be necessary because knowledge imparted on its practitioners has other, fundamental qualities. If the celerity of analysis was like that of a locomotive, de Jonquières countered, the slower geometrical method was like a pedestrian stroll, whereby one gains a more complete "lay of the land," a fuller understanding of the figures being surveyed. Chasles' work, in this regard, was a dazzling display of the treasures which can be expected by the geometers who prove to be "patient, observing, and critical" in the face of algebra's promise of immediate and thoughtless results (de Jonquières 1861, 9). In other words, to Terquem's preference for prompt (and modern) methods, de Jonquières opposed other epistemic norms and virtues.[88]

This implicit rebuttal did not escape Terquem, who used his editorial discretion to add a short answer at the end of de Jonquières's review. In his response, Terquem re-asserted that "in the two *analytic* centuries that have gone by since Descartes, the science of space has made more considerable progress than in the fifty-six *geometrical* centuries that preceded" (Terquem 1861, 11). Even more crucial to Terquem was the fact that the advances made by pure geometry had only been made possible by earlier discoveries made by analysts. Using the example of Poinsot's mechanics (which Chasles and De Jonquières had, on many occasions, lauded as one of the leading achievements of pure geometry), Terquem noted that:

> The composition of moments, of rotational motions, the helicoidal character of all motions of solid bodies, spheroidal attractions etc., have been *formulated* by *analysts*, and since then admirably *materialized*, presented to the external eye in a picturesque manner . . . . This is an immense service to science. It shows infinite talent, an extremely lucid imagination, but no creation; and it is *creation* that is the hallmark of *genius.* (Terquem 1861, 12)

[86]Note: This sentence is taken from a handwritten, first draft of a lecture, in which many words are struck through. See the introduction to Michel and Smadja 2021b for more details on the genesis of this text.

[87]See Michel and Smadja 2021b, 239–248 for a justification of the attribution of these handwritten notes to Terquem, and for a broader discussion of Chasles's and Terquem's differing attitudes towards the history of science.

[88]Recall Chasles' emphasis on the value of geometrical thought which lays bare the entire "chain of truths" joining hypotheses and conclusions, and the value of mathematical work which, in lieu of new properties, establishes the centrality of one particular proposition recognized as fundamental (see section 5).

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Terquem, who had trained in the same schools as Chasles and shared his erudite interest in the history of mathematics as well as in this science itself, mobilized the same categories when discussing Ancient Greek mathematics: the dichotomy between Ancients and Moderns, the concept of equation, the trope of genius etc. However, his position was the exact opposite of Chasles' (and de Jonquières's). Where Chasles called for a geometrical practice in which genius was no longer necessary, it remained, for Terquem, the ideal to which mathematicians ought to aspire. Where Chasles defended the slow and patient construction of geometrical understanding, Terquem opposed the value of prompt methods at a time of ubiquitous recourses to science in technological and commercial applications. And where Chasles wished to pursue the methods of the Ancients by reconnecting to them those of the Moderns, Terquem saw an unbridgeable gap, leading to the ultimate necessity of staying firmly rooted in modern methods, whilst relegating the study of the Ancients to the status of an (admittedly important) cultural and erudite endeavor.

10. Conclusion

Chasles' mode of engagement with the past thus appears to be a complex and reflexive process of ampliative adjustment and creative connection, in which both present and past were jointly and reciprocally modelled. This modelling required the shaping of a new notational technology originating in Chasles' innovative emphasis on the anharmonic ratio, a deliberately hybrid concept, partly derived, and partly crafted, as an interpretive tool from his reading of Pappus' lemmas. At the juncture of past and present, Chasles' move would both command a new gaze on ancient sources, and spawn a new approach to geometry—the germ of the latter supposedly being discernable from the former. Far from being a mere passive rewriting of ancient texts into modern notations, this self-controlled and highly constrained process amounted to a back-and-forth adaptation of modern abstract equations to the multiplicity of ancient cases, so as to settle on the provisionally most stable and illuminating adjustments. Neither pouring new wine into old wineskins, nor old wine into new ones, it was a different process altogether, conditioned upon a full acknowledgment of the breach between the present and the past. The task of connecting, in a meaningful way, the multiple lemmas of the Ancients to the all-encompassing theorems of the M oderns, the lengthy expressions of older to newer and swifter formulations, indeed presupposed (on Chasles' part) an identification of differing levels of abstraction and generality between ancient and modern science. This identification, we have shown, was the result of a historical-epistemological practice initiated, encouraged, and disseminated within the French scientific institutions born of the Revolution, of which Chasles (like Comte) was heir. The historical merits of Chasles' productive approach to ancient sources, however, would come under fire from a new generation of interpreters for whom establishing and comparing texts constituted the be all and end all of classical studies.

With Chasles, such rebukes fell on deaf ears. This essay opened on Chasles' somewhat surprising assertion, made in the 1870 *Rapport*, that modern geometry had unknowingly incorporated the doctrine of porisms. On his mind, at the time, was the theory of the organic generation of curves, which he and de Jonquières had developed in close contact with these studies of the porisms. Indeed, parallel to his "practical experiment[ing]," Chasles had mobilized his analysis of the role of the givens, and of the indeterminates, to center not only the theory of conics, but also that of cubics, on "porism-like" propositions. It is in this context, that he and de Jonquières decisively shaped what they termed "geometrical equations" of curves: generations via homographic pencils of lower-degree curves. Thus, to the reciprocal adaptation of past and present, Chasles had adjoined an experiment in connecting them, and so the porisms appeared to be the form that modern geometrical methods were destined to take.

Here, too, Chasles was not without critics: many saw but fool's gold in this injection of Euclid's lost texts into new mathematical treatises. In this sense, what the case of the porisms reveals is an orchestrated reverberation of the quarrel of the Ancients and the Moderns, distantly echoing in the midst of the nineteenth century: debates about the purposes of the past, about its usefulness and authority in the present, and about the ways to measure this usefulness, had come to take on a new flavor when transposed in mid-nineteenth-century French academia. What perhaps best distinguishes these resurgent discussions from their older counterparts is how self-conscious and critical the very act of restoring past texts had become. Both components of Chasles' "divination" of the *Porisms*—namely the adaptation to the plurality of cases and the connection with modern geometry—involved (potentially) revisable work. The adaptation itself was a "practical experiment," a tentative unfolding of the many potentialities latent in Chasles' unifying concept of the homographic division, so as to best fit the incomplete mold given by Pappus' cryptic genera. The connection itself was at least partly the fruit of "artifice," to borrow Comte's term. It was revealed only after the centrality of the anharmonic ratio, and the role (and plurality of meanings) of the givens and the indeterminates, had been recognized and highlighted by modern geometrical research. Only after this process of (for the most part, invisible) maturation had taken place, could one see with clear eyes the lost meaning of Euclid's edifice, the historical distance accommodated but not erased.

References

Acerbi, Fabio. 2011. "The Language of the 'Givens': Its Forms and Its Use as a Deductive Tool in Greek Mathematics." *Archive for History of Exact Sciences* **65**: 119–153.

Alder, Ken. [1997] 2010. *Engineering the Revolution: Arms and Enlightenment in France, 1763–1815*, 2nd edition. Chicago: University of Chicago Press.

Apollonius. 1566. *Conicorum libri quattuor, una cum Pappi Alexandrini lemmatibus et commentariis Eutocii Ascalonitae*. Latin translation by Federico Commandino. Bologna: Officina Alexandri Benatil.

Apollonius. 1706. *Apollonii Pergæi De Sectione Rationis Libri duo, . . . Pappi Alexandrini Præfatio ad VIImum Collectionis Mathematicæ, nunc primum Græce edita, cum lemmatibus ejusdem Pappi ad hos Apollonii Libros*. Edition and Latin translation by Edmond Halley. Oxford: Sheldonian Theater.

Apollonius. 1710. *Apollonii Pergæi Conicorum Libri Octo, et Sereni Antissensis De Sectione Cylindri & Coni Libri Duo*. Oxford: Sheldonian Theater.

Belhoste, Bruno. 2001. "La préparation aux grandes écoles scientifiques au XIXe siècle: établissements publics et institutions privées." *Histoire de l'éducation* **90**: 1–23.

Belhoste, Bruno. 2003. *La formation d'une technocratie: l'École polytechnique et ses élèves de la Révolution au Second Empire*. Paris: Belin.

Blåsjö, Viktor. 2014. "A Critique of the Modern Consensus in the Historiography of Mathematics." *Journal of Humanistic Mathematics* **4** (2): 113–123.

Blåsjö, Viktor, and Jan P Hogendijk. 2018. "On Translating Mathematics." *Isis* **109** (4):774–781.

Boucard, Jenny, and Norbert Verdier. 2015. "Circulations mathématiques et congruences dans les périodiques de la première moitié du XIXe siècle." *Philosophia Scientiæ* **19** (2): 57–78.

Boudin, Amédée. 1869. *Panthéon de la Légion d'honneur. Biographie de M. Chasles, membre de l'Institut*. Paris.

Bos, Henk. 2001. *Redefining Geometrical Exactness. Descartes' Transformation of the Early Modern Concept of Construction*. New York: Springer-Verlag.

Breton de Champ, Paul Émile. 1849. "Mémoire sur les Porismes d'Euclide." *Comptes rendus de l'Académie des sciences* **29**: 479–482.

Breton de Champ, Paul Émile. 1853. "Note sur un point important de la question des Porismes." *Comptes rendus de l'Académie des sciences* **36**: 1008–1012.

Breton de Champ, Paul Émile. 1855. "Recherches nouvelles sur les Porismes d'Euclide." *Journal de mathématiques pures et appliquées* **20**: 209–304.

Breton de Champ, Paul Émile. 1857. "Observation sur le mémoire de M. Housel, intitulé: Les Porismes d'Euclide." *Journal de mathématiques pures et appliquées* **2**:185–205.

Breton de Champ, Paul Émile. 1858. "Deuxième Supplément aux Recherches nouvelles sur les Porismes d'Euclide." *Journal de mathématiques pures et appliquées* **3**:89–142.

Breton de Champ, Paul Émile. 1860a. "Réclamation de priorité au sujet de l'interprétation des énoncés de Porismes que Pappus nous a transmis." *Comptes rendus de l'Académie des sciences* **50**: 938–940.

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Breton de Champ, Paul Émile. 1860b. "Addition à la réclamation de priorité présentée dans la précédente séance au sujet de l'interprétation des énoncés de Porismes que Pappus nous a transmis." *Comptes rendus de l'Académie des sciences* **50**: 995–996.

Breton de Champ, Paul Émile. 1860c. "Question des Porismes -Lettre de M. Breton." *Comptes rendus de l'Académie des sciences* **50**: 1034–1037.

Breton de Champ, Paul Émile. 1865. *Notice sur les débats de priorité auxquels a donné lieu l'ouvrage de M. Chasles sur les Porismes d'Euclide*. Paris: Bouchard-Huzard.

Breton de Champ, Paul Émile. 1867. "Découverte d'un résumé des Porismes d'Euclide." *Nouvelles annales de mathématiques* **6**: 522–526.

Brianchon, Charles-Julien. 1817. *Mémoire sur les lignes du second ordre*. Paris: Bachelier.

Carnot, Lazare. 1803. *Géométrie de Position*. Paris: Duprat.

Chasles, Michel. 1837. *Aperçu historique sur l'origine et le développement des méthodes en géométrie*. Bruxelles: Hayez.

Chasles, Michel. 1847. "Sur l'enseignement de la Géométrie supérieure. Discours d'introduction au Cours de Géométrie supérieure fondé à la Faculté des Sciences de l'Académie de Paris. - Séance d'ouverture, le 22 décembre 1846." *Journal de Mathématiques Pures et Appliquées (1ère série)*, **12**:1–40.

Chasles, Michel. 1852. *Traité de Géométrie Supérieure*. Paris: Bachelier.

Chasles, Michel. 1859. "Les trois livres de Porismes d'Euclide (Introduction §§ I & II)." *Comptes rendus de l'Académie des sciences* **48**: 1033–1041.

Chasles, Michel. 1860a. *Les trois livres de Porismes d'Euclide*. Paris: Mallet-Bachelier.

Chasles, Michel. 1860b. "Sur les Porismes. Réponse de M. Chasles aux Réclamations adressées par M. Breton." *Comptes rendus de l'Académie des sciences* **51**: 1043–1061.

Chasles, Michel. 1870. *Rapport sur les progrès de la géométrie*. Paris: Imprimerie nationale.

Chasles, Michel. 1880. *Traité de Géométrie Supérieure*, 2nd edition. Paris: Gauthier-Villars.

Chemla, Karine. 1998. "Lazare Carnot et la généralité. Variations sur le théorème dit de Menelaus." *Revue d'histoire des mathématiques* **4**: 163–190.

Chemla, Karine. 2012. *The History of Mathematical Proof in Ancient Traditions*. Cambridge: Cambridge University Press.

Chemla, Karine. 2016. "The Value of Generality in Michel Chasles's Historiography of Geometry." In *The Oxford Handbook of Generality in Mathematics and the Sciences*, edited by Karine Chemla, Renaud Chorlay, and David Rabouin, 47–89. Oxford: Oxford University Press.

Claudin, Anatole. 1881. *Catalogue de la bibliothèque scientifique, historique et littéraire de feu M. Michel Chasles*. Paris, Claudin.

Comte, Auguste. 1824. *Système de politique positive, tome 1*. Paris.

Comte, Auguste. 1830. *Cours de philosophie positive, tome 1*. Paris: Bachelier.

Cook, Alan. 1998. *Edmond Halley: Charting the Heavens and the Seas*. Oxford: Clarendon Press.

Dahan-Dalmedico, Amy. 1992. "Lagrange. La méthode critique du mathématicien-philosophe." In *Les Leçons à l'École Normale de l'an III, t. 1, Les leçons de mathématiques : Lagrange, Laplace et Monge*, edited by Jean Dhombres, 171–192. Paris: Dunod.

de Jonquières, Ernest. 1861. "Les Trois Livres de Porismes d'Euclide, rétablis pour la première fois, d'après la Notice et les Lemmes de Pappus, etc., par M. Chasles." *Nouvelles Annales de Mathématiques (1ère série)* **20**: 1–11.

Fontenelle, Bernard Le Bovier de. [1688] 2001. "Digression sur les Anciens et les Modernes." In *La Querelle des Anciens et des Modernes*, edited by Anne-Marie Lecoq, 294–313. Paris: Gallimard.

Fontenelle, Bernard Le Bovier de. [1704] 1707. "Sur les spirales à l'infini." *Histoire de l'Académie Royale des Sciences (année 1704)* 57–70.

Franklin, Benjamin. [1746] 1961. "Poor Richard, 1746. An Almanack For The Year of Christ 1746." In *The Papers of Benjamin Franklin*, vol. **3**. January 1, 1745, through June 30, 1750, edited by Leonard W. Labaree, 60–67. New Haven: Yale University Press.

Freudenthal, Hans. 1977. "What Is Algebra and What Has It Been in History?" *Archive for History of Exact Sciences* **16** (3): 189–200.

Fried, Michael N. 2011. *Edmond Halley's Reconstruction of the Lost Book of Apollonius's Conics: Translation and Commentary*. New York: Springer.

Fried, Michael N. 2014. "The Discipline of History and the 'Modern Consensus in the Historiography of Mathematics'." *Journal of Humanistic Mathematics* **4** (2): 124–136.

Fumaroli, Marc. 2001. "Les abeilles et les araignées." In *La Querelle des Anciens et des Modernes*, edited by Anne-Marie Lecoq, 7–218. Paris: Gallimard.

Gergonne, Joseph-Diez. 1816–1817a. "Recherche du cercle qui en touche trois autres sur un plan." *Annales de mathématiques pures et appliquées* **VII** (10): 289–303.

Gergonne, Joseph-Diez. 1816–1817b. "Solution et construction, par la géométrie analytique, de deux problèmes dépendant de la géométrie de la règle." *Annales de mathématiques pures et appliquées* **VII** (11): 325–334.

Gergonne, Joseph-Diez. 1817–1818. "Réflexions sur l'article précédent." *Annales de mathématiques pures et appliquées* **VIII** (5): 156–161.

Goldstein, Leon J. 1976. *Historical Knowing*. Austin/London: University of Texas Press.

Goldstein, Catherine 2010. "Des passés utiles: mathématiques, mathématiciens et histoires des mathématiques." *Noesis* **17**: 135–152.

Gordin, Michael D. 2014. "The Tory Interpretation of History." *Historical Studies in the Natural Sciences* **44** (4): 413–423.

Grattan-Guinness, Ivor. 2004. "The Mathematics of the Past: Distinguishing Its History from Our Heritage." *Historia Mathematica* **31**: 163–185.

Hachette, Jean Nicolas Pierre. 1818. *Second Supplément de la Géométrie descriptive*. Paris: Firmin-Didot.

Heath, Thomas. 1921. *A History of Greek Mathematics*. Oxford: Clarendon Press.

Herz-Fischler, Roger. 1984. "What are propositions 84 and 85 of Euclid's Data all about?" *Historia Mathematica* **11**: 86–91.

Hogendijk, Jan P. 1987. "On Euclid's Lost *Porisms* and Its Arabic Traces." *Bollettino di Storia delle Scienze Matematiche* **VII** (1): 93–115.

Hogendijk, Jan P. 1991. "Desargues' *Brouillon Project* and the *Conics* of Apollonius." *Centaurus* **34**: 1–43.

Housel, Charles Pierre. 1856. "Les Porismes d'Euclide." *Journal de mathématiques pures et appliquées* **1**:193–209.

Housel, Charles Pierre. 1861. "Les Porismes d'Euclide." *Revue archéologique* **3**: 221–229.

Hultsch, Friedrich Otto. 1876–1878. *Pappi Alexandrini Collectionis quae supersunt*. Berlin: Weidman.

Jones, Alexander. 1986. *Pappus of Alexandria: Book 7 of the Collection*, **2 vols**. New York: Springer.

Knorr, Wilbur Richard. 1975. *The Evolution of the Euclidean Elements: A Study of the Theory of Incommensurable Magnitudes and Its Significance for Early Greek Geometry*. Dordrecht: Reidel.

Koppelman, Elaine. 1971."Chasles, Michel." In *Dictionary of Scientific Biography*, edited by Charles Coulston Gillispie, 212–215. New-York: Charles Scribner's Sons.

Langins, Janis. 2004. *Conserving the Enlightenment: French Military Engineering from Vauban to the Revolution*. Cambridge: MIT Press.

Leslie, John. [1809] 1811. *Elements of Geometry, Geometrical Analysis, and Plane Geometry*, 2nd ed. Edinburgh: Ballantyne.

Levine, Joseph M. 1981. "Ancients and Moderns Reconsidered." *Eighteenth-Century Studies* **15** (1): 72–89.

Levine, Joseph M. 1991. *The Battle of Books: History and Literature in the Augustan Age*. Ithaca and London: Cornell University Press.

Levine, Joseph M. 1999. *Between the Ancients and the Moderns: Baroque Culture in Restoration England*. New Haven and London: Yale University Press.

Lorenat, Jemma. 2015. "Figures Real, Imagined, and Missing in Poncelet, Plücker, and Gergonne." *Historia Mathematica* **42**: 155–192.

Macherey, Pierre. 1989. *Comte, la philosophie et les sciences*. Paris: Presses universitaires de France.

Méray, Charles. 1860. "Mémoire sur la théorie géométrique des surfaces du second ordre." *Annali di Matematica* **3**(1): 30–51.

Michel, Nicolas. 2020. "The Values of Simplicity and Generality in Chasles's Geometrical Theory of Attraction." *Journal for General Philosophy of Science* **51**: 115–146.

Michel, Nicolas, and Ivahn Smadja. 2021a. "Mathematics in the Archives: Deconstructive Historiography and the Shaping of Modern Geometry (1837–1852)." *British Journal for History of Science* **54**: 423–441.

Michel, Nicolas, and Ivahn Smadja. 2021b. "Un cours inédit de Chasles en Sorbonne: 'Considérations sur la théorie des section coniques', Discours d'ouverture du cours de Géométrie Supérieure (1847-1848)." *Revue d'Histoire des Mathématiques* **27**: 229–296.

Muller, Jean. 1838. *Histoire de la Confédération Suisse*, vol. **4**, translated from the German by Charles Monnard. Paris: Ballimore.

Netz, Reviel. 1999. *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*. Cambridge: Cambridge University Press.

Pappus. 1588. *Pappi Alexandrini mathematicae collectiones a Federico Commandino Urbinate in latinum conversæ at commentariis illustratæ*, Latin edition and translation by Federico Commandino. Pesaro.

Perrault, Claude. 1684. *Les dix livres d'architecture de Vitruve*, 2nd ed. Paris: Jean-Baptiste Coignard.

Perrault, Claude. 1688. *Paralelle des Anciens et des Modernes*. Paris: Jean-Baptiste Coignard.

Pickering, Mary. 2009. *Auguste Comte: An Intellectual Biography, Volume II*. Cambridge: Cambridge University Press.

Poncelet, Jean-Victor. 1817–1818. "Réflexions sur l'usage de l'analyse algébrique dans la géométrie; suivies de la solution de quelques problèmes dépendant de la géométrie de la règle." *Annales de mathématiques pures et appliquées* **VIII** (5): 141–155.

Poncelet, Jean-Victor. 1822. *Traité des propriétés projectives des figures*. Paris: Bachelier.

Poncelet, Jean-Victor. 1862. *Applications d'analyse et de géométrie*. Paris: Mallet-Bachelier.

Poncelet, Jean-Victor. 1866. *Traité des propriétés projectives des figures*, **2 vols**. Paris: Gauthier-Villars.

Rashed, Roshdi. 2002. *Les Mathématiques Infinitésimales du IXe au XIe siècle, vol. IV: Ibn Al-Haytham. Méthodes Géométriques Transformations Ponctuelles et Philosophie des Mathématiques*. London: Al-Furqan.

Remmert, Volker R., Martina R. Schneider, and Henrik Kragh Sørensen, eds. 2016. *Historiography of Mathematics in the 19th and 20th Centuries*. Basel: Birkhäuser.

Robison. 1817. "Simson, Dr Robert." In *Encyclopædia Britannica*, 5th ed.,. **XIX**:369–374. Edinburgh: Encyclopædia Press.

Rollin, Charles. 1817. *Œuvres complètes VI*. Paris: Didot, Ledoux & Tenré.

Saito, Ken. 1998. "Mathematical Reconstructions Out, Textual Studies In: 30 Years in the Historiography of Greek Mathematics." *Revue d'Histoire des Mathématiques* **4**:131–142.

Schneider, Martina R. 2016. "Contextualizing Unguru's 1975 Attack on the Historiography of Ancient Greek Mathematics." In *Historiography of Mathematics in the 19th and 20th Centuries*, edited by Volker R. Remmert, Martina R. Schneider, and Henrik Kragh Sørensen, 245–267. Basel: Birkhäuser.

Sédillot, Louis-Amélie. 1834. "Notice du Traité des connues géométriques de Hassan ben Haithem", *Journal asiatique* **13**: 435–458.

Schubring, Gert. 2005. *Conflicts between Generalization, Rigor, and Intuition. Number Concepts Underlying the Development of Analysis in 17th–19th Century France and Germany*. New-York: Springer Science.

Serret, Joseph-Alfred. 1861. "Rapport sur diverses Notes de M. Breton (de Champ) relatives à la question des Porismes." *Comptes rendus de l'Académie des sciences* **53**: 699–714.

Simson, Robert. 1723. "Pappi Alexandrini Propositiones duæ generales, quibus plura ex Euclidis Porismatis complexus est, Restituæ a Viro Doctissimo Rob. Simson, Math. Prof. Glasc. Vid. Pappi praefationem ad Lib. 7. Coll. Math. Apollonii de Sectione rationis libris duobus a Clariss. Hallejo praemissam pag. VIII. & XXXIV." *Philosophical Transactions* **377**: 330–340.

Simson, Robert. 1776. *De Porismatibus Tractatus: Quo Doctrinam Porismatum Satis Explicatam, Et in Posterum ab Oblivione Tutam Fore sperat Auctor. In Opera quaedam reliqua nunc primum post auctoris mortem in lucem edita, cura vero Jacobi Clow*, section II: 315–594. Glasgow: Robert and Andreas Foulis.

Smadja, Ivahn. 2015. "Sanskrit versus Greek 'Proofs': History of Mathematics at the Crossroads of Philology and Mathematics in Nineteenth-century Germany." *Revue d'Histoire des Mathématiques* **21**: 217–349.

Smadja, Ivahn. 2016. "On Two Conjectures that Shaped the Historiography of Indeterminate Analysis: Strachey and Chasles on Sanskrit Sources." *Historia Mathematica* **43**: 241–287.

Stendhal. 1823. *Racine et Shakspeare*. Paris: Bossange, Delaunay et Mongie.

Swift, Jonathan. [1704] 1824. *A Full and True Account of the Battle fought last Friday between the Ancient and the Modern Books in Saint James's Library*. In *The Works of Jonathan Swift*, 2nd ed., **X**:219–264. Edinburgh & London: Archibald Constable and Co, and Hurst, Robinson and Co.

Taisbak, Christian Marinus. 1996. "Zeuthen and Euclid's Data 86 Algebra – or a Lemma about intersecting Hyperbolas?" *Centaurus* **38**: 122–139.

Taisbak, Christian Marinus. 2003. ΔΕΔΟΜΕΝΑ: *Euclid's Data or the Importance of Being Given*. Copenhagen: Museum Tusculanum Press.

Tannery, Paul. [1882] 1912. "De la solution géométrique des problèmes du second degré avant Euclide." In *Mémoires scientifiques. I. Sciences exactes dans l'Antiquité, 1876–1884*, edited by Johann Ludwig Heiberg & Hieronymus Georg Zeuthen, 254–280. Toulouse & Paris: Privat, Gauthier-Villars.

ten Hagen, Sjang, 2021. *History and Physics Entangled: Disciplinary Intersections in the Long Nineteenth Century*. PhD dissertation, University of Amsterdam.

Terquem, Olry. 1855. "Notice sur la découverte des logarithmes." *Nouvelles annales de mathématiques* 1ère série (supplément) **14**: 1–7; 40–49.

Terquem, Olry. 1860. "Bulletin de bibliographie, d'histoire et de biographie mathématiques." *Nouvelles annales de mathématiques* 1ère série **19**: 70–71.

Terquem, Olry. 1861. "Note du Rédacteur." *Nouvelles annales de mathématiques* 1ère série **20**: 11–13.

Trail, William. 1812. *Account of the Life and Writings of Robert Simson*, M.D. Bath: Richard Cruttwell.

Tweddle, Ian. 2000. *Simson on Porisms: An Annotated Translation of Robert Simson's Posthumous Treatise on Porisms and Other Items on this Subject*. London-Berlin-Heidelberg: Springer.

Turner, James. 2014. *Philology: The Forgotten Origins of the Modern Humanities*. Princeton and Oxford: Princeton University Press.

Unguru, Sabetai. 1975. "On the Need to Rewrite the History of Greek Mathematics." *Archive for History of Exact Sciences* **15** (1): 67–114.

Unguru, Sabetai. 1979. "History of Ancient Mathematics – Some Reflections on the State of the Art." *Isis* **70** (4): 555–565.

Valat, Pierre. 1870. *Lettres d'Auguste Comte à M. Valat, 1815–1844*. Paris: Dunod.

van der Waerden, Bartel Leendert. 1954. *Science Awakening*. Groningen: Noordhoff.

van der Waerden, Bartel Leendert. 1976. "Defence of a 'Shocking' Point of View." *Archive for History of Exact Sciences* **15** (3): 199–210.

Varignon, Pierre. [1704] 1707. "Nouvelle formation de spirales." *Mémoires de l'Académie Royale des Sciences (année 1704)*, 69–131.

Vérin, Hélène. 1984. "Le mot : ingénieur." *Culture technique* **12**: 69–131.

Vincent, Alexandre Joseph Hydulphe. 1857a. "Notice sur les porismes." *La Science* **10**. Reprint Paris: Dubuisson et Cie, 1–15.
Vincent, Alexandre Joseph Hydulphe. 1857b. "Seconde notice sur la théorie des porismes." *La Science* **40–41**: 1–16.
Vincent, Alexandre Joseph Hydulphe. 1859. "Considérations sur les Porismes en général et sur ceux d'Euclide en particulier. Examen et réfutation de l'interprétation donnée par M. Breton (de Champ) aux textes de Pappus et de Proclus relatifs aux Porismes." *Journal de mathématiques pures et appliquées* **4**: 9–46.
Wang, Xiaofei. 2022. "How Jean-Baptiste Delambre Read Ancient Greek Arithmetic on the Basis of the Arithmetic of 'Complex Numbers' at the Turn of the 19th century." *Historia Mathematica* **59**: 146–163.
Weil, André. 1978. "Who Betrayed Euclid? (Extract from a letter to the Editor)." *Archive for History of Exact Sciences* **19**: 91–93.
Wilson, Adrian. 2017. "Science's Imagined Pasts." *Isis* **108** (4): 814–826.
Zeuthen, Hieronymus Georg. 1886. *Die Lehre von den Kegelschnitten im Altertum*. Copenhagen: Höst and Sohn.
Zeuthen, Hieronymus Georg. 1919. *Sur l'origine de l'algèbre*. Copenhagen: Bianco Lunos Bogtrykkeri.
Zilsel, Edgar. [1926] 1972. *Die Entstehung des Geniesbegriffes. Ein Beitrag zur Ideengeschichte der Antike und des Frühkapitalismus*. Tübingen: Mohr. 2nd ed. Georg Olms.

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