like the hand of a clock, or a boat which is rowed by impulses. Would Petty, who was a competent mathematician, who lived among the astronomers, and who was himself an inventor of machines, have conceived this exquisite bit of knowledge; or, conceiving it, would he have published it at the very time when, owing to the meetings of the Royal Society (of which he was one of the first members of council) he was in almost daily communication with those who would have set him right? Either supposition is hardly possible.

Yours faithfully,

January 17*th*, 1859. A. DE MORGAN.

ON THE INCONGRUITY EXISTING BETWEEN THE RATES OF PREMIUM CHARGED AT CERTAIN AGES AND THE BENEFITS ACCRUING THEREUNDER.

To the Editor of the Assurance Magazine.

SIR,—I should like to bring under your review a matter that I think hardly meets with the consideration it deserves among actuaries—viz., the incongruity that exists between the premiums charged at different ages on "bonus" policies, and the benefits to which they entitle the holder, where, as in the great majority of cases, these premiums are calculated with reference only to the principal sum assured, and the reversionary "bonus" is declared by annual additions which are periodically "vested" or added to the principal amount, forming the capital which determines the amount of bonus for the next succeeding period.

Where the bonus is at the rate of P per £1 per annum, computed at each period of t years, its progress may be stated thus:—

First period.

Sum assured	£1.
Annual bonus	$P=B_1$.

Second period.

Sum assured	$1+tP$.
Annual bonus	$P(1+tP)=B_2$.

Third period.

Sum assured	$(1+tP)+tP(1+tP)=(1+tP)^2$.
Annual bonus	$P(1+tP)^2=B_3$, &c.

Allowing for the altered circumstance of the addition being made to the sum assured after the first term has elapsed, the identity of the above formula for it with that for the amount of £1, is obvious—the sum assured for the nth period being $(1+tP)^{n-1}$,
and the annual bonus $P(1+tP)^{n-1}=B_n$.

By the ordinary commutation tables, the annual premium for such a benefit, at age x, is

$$\frac{M_x+B_1R_x+(B_2-B_1)R_{x+t}+(B_3-B_2)R_{x+2t}+\ \ldots\ \text{&c.}}{N_{x-1}}.$$

I send you the following results of this formula, deduced from the Carlisle 3 per Cent. Table, assuming the sum under the policy to be

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£1,000, and that a bonus of £1. 10*s.* per cent. per annum is added thereto in the usual manner each five years. To avoid any invidious distinction, I have purposely chosen data that, as far as I know, do not accurately represent the operations of any Office.

The first column shows the (net) annual premium required to assure £1,000 at the ages stated; the second, the annual premium required to meet the increasing benefit above described; and the third, the ratio of the first to the second.

Age.	Premium for £1,000.	Premium for £1,000, with Bonus.	Ratio.
20	14·9358	24·3651	1·63
30	19·5192	29·6399	1·52
40	25·9932	36·8245	1·42
50	36·2236	47·7889	1·32
60	57·8955	70·1644	1·21

Assuming the Office to charge the premium in the first column with the ordinary "loading" of 30 per cent., it will be seen that, while the entrants at age 50 get a reasonable equivalent for their payments, those at the more advanced ages have to pay some 30 per cent. of the premiums of their more fortunate younger brethren of 20.

I remain, Sir,

Your most obedient servant,

Aberdeen, 1st February, 1859. H. A. S.

ON THE FACILITIES AFFORDED IN THE COMMUTATION SYSTEM BY THE INTRODUCTION OF COLUMNS OF DIFFERENCES.

To the Editor of the Assurance Magazine.

SIR,—Although your correspondent, "Joshua Milne," will not admit the superiority of the columnar method in life contingency calculations, he has failed, I think, to prove that, even with the help of a complete set of temporary and deferred annuities, the "ancient" method gives facilities equal to those which are claimed for its rival. I regret that the examples which have been adduced, in illustration of the two methods, were not presented by the writers on both sides, with the operations in full, for the merits of each method could then have been better appreciated.

As the old method of computation is, doubtless, greatly facilitated by the tabulation of the annuities, temporary and deferred, so may the power of the columnar method be increased by the tabulation of the differences of the N and M columns, and the summation of those differences, to be used supplementally to the D and N table in its ordinary form. I have long been sensible of the importance of such tables (and doubtless it has been equally apparent to others), for, as far back as the early part of 1854, I tabulated the differences of the N and M columns for terms from 1 to 70 years, and from those differences formed new R and S columns.

As a discussion has recently taken place in your valuable *Journal* with regard to the relative merits of the old and the columnar methods, it may not be uninteresting to give a short account of the difference tables above