THEORETICAL SPECTRA OF CATACLYSMIC VARIABLES

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ABSTRACT. We present two methods of modelling accretion disks in dwarf novae. The first one determines self-consistently the structure of a disk together with the radiation field. The computed theoretical spectra agree well with the observations of the dwarf nova WX Hyi in quiescence as well as in superoutburst. The second method is a modification of Tylenda's (1981) approach. We have used it to calculate models of disks around main-sequence accretors. The most interesting result is that no unique solution exists for a moderate mass flux around 10^{-6} $M_{\odot}yr^{-1}$. Such disks are thus probably unstable.

1. GENERAL METHOD

In most studies of accretion disks, the radiative transfer phenomena are treated only approximately. Even recent sophisticated studies using classical model atmosphere fluxes (see, e.g., Wade, 1984) assume that the disk is optically thick. On the other hand, a simple but elegant method of Tylenda (1981) works properly only if the disk is optically thin. Therefore, we have proposed a new self-consistent method which avoids some simplifications used in the previous studies, namely

(i) the disk is not considered as a semi-infinite atmosphere but rather as a finite slab of gas;

(ii) no a priori assumption about the total optical thickness of the disk is made; instead, its value is implied by the model;

(iii) the total radiative flux is not constant as in classical atmospheres, but increases upwards (i.e. from the central plane of the disk to its surface), its value being determined through the energy balance equation;

(iv) analogously, the gravity acceleration is not constant and depends on the distance from the central plane.

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Figure 1. Emergent flux from the disk around a white dwarf of 1 $M_\odot$. The three systems of lines correspond (from top to bottom) to $\dot{M}$ = 10^{-8}, 10^{-10} (dashed lines), and 10^{-11} $M_\odot\,yr^{-1}$, respectively. In each system, the upper curve corresponds to inclination i = 0°, the middle curve to i = 30°, and the lower curve to i = 60°. The observed energy distribution of the dwarf nova WX Hyi after Hassal et al. (1983) is also plotted. The observed values are shifted along the ordinate to yield the best fit between the observed quiescent spectrum and the model with $\dot{M}$ = 10^{-11} $M_\odot\,yr^{-1}$, i = 30°. The solid thin line, which represents a theoretical flux for $\dot{M}$ = 3.6 x 10^{-10} $M_\odot\,yr^{-1}$, i = 30°, obtained by interpolating the calculated models, fits the observed superoutburst energy distribution relatively well.

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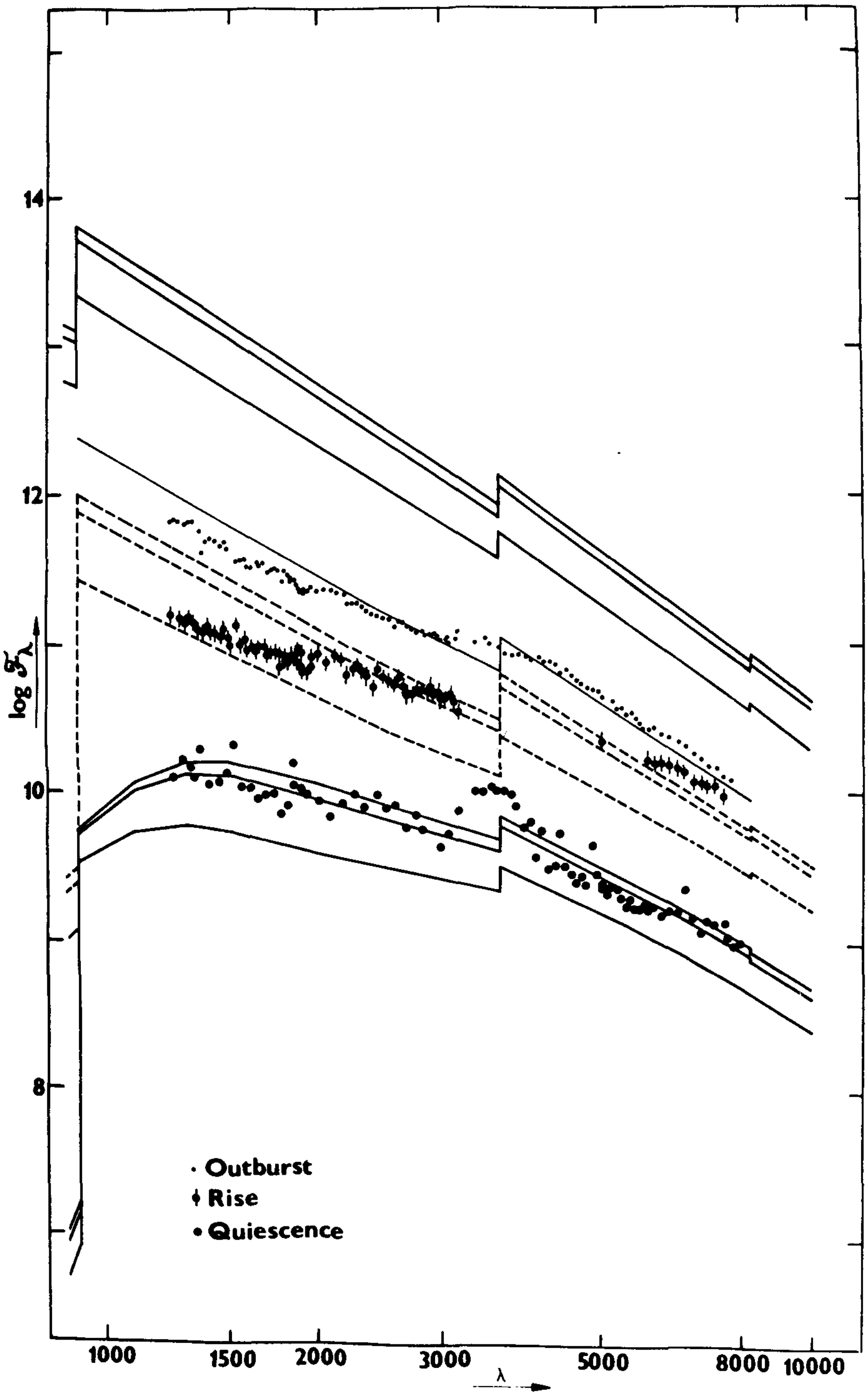
14
12
10
8
log $\mathcal{F}_\lambda$
1000
1500
2000
3000
5000
8000
10000
λ
• Outburst
ϕ Rise
• Quiescence

We divide the disk into a set of concentric rings, each of them behaving like an independent plane-parallel slab. The vertical structure of each ring is determined by solving simultaneously the hydrostatic equilibrium, the energy balance, and the radiative transfer equations. The appropriate boundary conditions, which depend on the radial distance from the central star, are given by the canonical model by Lynden-Bell and Pringle (1974). To calculate the viscosity we use Lynden-Bell and Pringle's (1974) assumption that the kinematic viscosity is determined by the critical Reynolds number Re, assuming Re = 5000. The resulting set of non-linear, highly coupled equations is solved by means of a suitable modification of the complete linearization technique.

The numerical calculations were carried out for a stationary disk model with a white-dwarf central star of mass 1 $M_\odot$ and radius 5 x 10^8 cm, the mass flux through the disk being set to 10^{-8}, 10^{-10} and 10^{-11} $M_\odot$ yr^{-1}, respectively. A comparison of the theoretical flux distribution with the observed distribution for WX Hyi is displayed in Figure 1. It indicates that the quiescent state of WX Hyi corresponds to a mass flux of about 10^{-11} $M_\odot$ yr^{-1}, while its superoutburst is probably caused by an increase of the mass flux by 1.5 dex.

More details can be found elsewhere (Kříž and Hubený, 1986).

2. A SIMPLIFIED METHOD FOR OPTICALLY THIN DISKS

The method described in the previous section has two disadvantages:

(i) The corresponding calculations are very time-consuming. Consequently, this method can hardly be used for calculating an extensive grid of models.

(ii) The present version of the method uses the surface density as a boundary condition fixed in advance. Thus we had to use a very simple, temperature-independent viscosity. Consequently, the adopted model a priori excludes any thermal instability, found by Meyer and Meyer-Hofmeister (1981) and many others, as this instability is in fact a consequence of the temperature dependence of viscosity.

Therefore, we have adopted a simpler method, developed originally by Tylenda (1981) and Williams and Ferguson (1982) for modelling optically thin disks. Their approach is based on a drastic simplification of the vertical structure of the disk. They assume that the temperature and density are independent of the distance Z from the central plane of the disk. In reality, these assumptions are not so bad for optically thin disks. Indeed, more exact calculations mentioned in Section 1 reveal that the temperature is nearly z-independent for optically thin disks. The corresponding distribution of density is Gaussian. With some caution, this distribution can be approximated by a rectangular distribution with appropriately chosen semithickness of the disk z_o. The choice $z_o = v_s \omega^{-1} \sqrt{2}$ matches the Gaussian distribution much better than the original Tylenda's choice, $z_o=v_s\omega^{-1}$. Here v_s is the velocity of sound and ω the angular velocity of disk rotation. Then the density is given by $\rho = \Sigma/2z_o$; Σ being the usual viscosity-dependent surface density following from the canonical theory of stationary accretion. The viscosity can be written as $\nu = \alpha\, v_s 2z_o$, using the α - approach by Shakura and Sunyaev (1973).

Assuming that the source function is given simply by the Planck function and using the above assumptions we can integrate the equation of radiative transfer analytically. The emergent flux of radiation is then given by

$$F_\nu = B_\nu(T)\,[\,1 - 2E_3(\tau_\nu)\,].$$

where τ_ν is the monochromatic optical thickness $\tau_\nu = 2z_o\chi_\nu$; χ_ν being the volume absorption coefficient. E_3 is the third integral exponential function. In the stationary case, the energy dissipated in the disk is compensated by the radiative losses, i.e.

$$D = 2\pi \int_0^\infty F_\nu \, d\nu.$$

D is the dissipation rate per unit area given by the standard theory.

The structure of this simplified model is determined by the prescribed values of the mass M and radius R_* of the central star, the mass flux through the disk $\dot{M}$, and the viscosity parameter α. The

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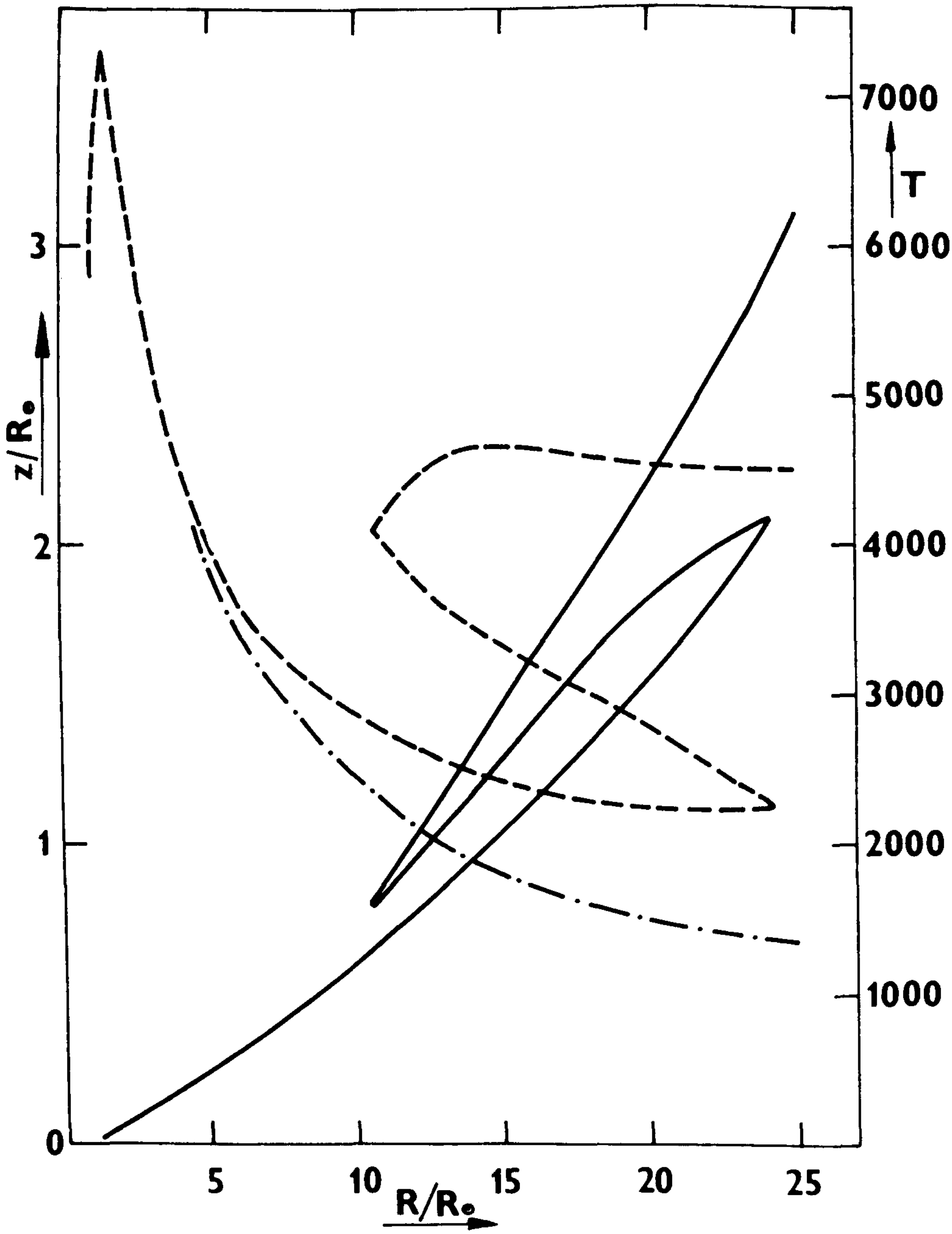
Figure 2. Structure of the disk with $\dot{M} = 10^{-6}\ M_{\odot}\ yr^{-1}$ around a main-sequence star with $M = 1\ M_{\odot}$, $R = 1\ R_{\odot}$. The semithickness of the disk z_o is shown by full lines, the temperature T by dashed lines. For illustration, the effective temperature calculated using the black-body assumption is also drawn (dot-dashed line). R is the distance from the center of the star.



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temperature, density and semithickness of the disk at each point can be found by interactive solution of the above quoted equations.

Using this method, we recomputed the set of models of the main-sequence accretors by Kenyon and Webbink (1984). We adopted the same central star, i.e. $M = 1\ M_\odot$, $R = 1\ R_\odot$, and chose $\alpha = 1$. Our results show that the disks with $\dot{M} \leqq 3\times10^{-6}\ M_\odot\ yr^{-1}$ are at least partially optically thin and, therefore, the basic assumption adopted by Kenyon and Webbink that the disks are optically thick is not fulfilled.

Moreover, we found that our equations do not have a unique solution for moderate values of the mass flux. Figure 2 shows some properties of the theoretical disk, corresponding to the model of the recurrent nova T CrB by Kenyon and Garcia (1986), consisting of a lobe-filling gM3 star and a main-sequence companion accreting mass at a rate of $10^{-6}\ M_\odot\ yr^{-1}$. The disk is optically thick in the central region and thin in the extended outer parts. Accordingly, the calculated temperature is much higher than the effective temperature in the outer parts of a disk. However, the most striking result was that three different solutions were obtained for $10.7 < R/R_\odot < 24.5$. Consequently, it seems that the disk is unstable, similarly as in the case of some disks around white dwarfs (Meyer and Meyer-Hofmeister, 1981, and others). This effect could also explain the variations of the ultraviolet continuum observed for T CrB by Cassatella et al. (1982). On the other hand, the models with mass fluxes $10^{-7}\ M_\odot\ yr^{-1}$ and $10^{-5}\ M_\odot\ yr^{-1}$ have unique solutions and are thus probably stable.

We have also calculated the emergent spectrum from the whole system using the same procedure for calculating the light of the boundary layer as Kenyon and Webbink (1984). The computed spectrum matches the observed flat ultraviolet continuum of T CrB (Cassatella et al., 1982) satisfactorily. However, we stress that this theoretical spectrum represents only a rough approximation due to the simplified method used for calculating the radiation of the boundary layer.

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