RESONANT THREE-WAVE INTERACTIONS AND AN APPLICATION TO SOLAR G MODE OSCILLATIONS

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We present here some initial results of a long-term investigation of resonant three-wave interactions in the solar interior, which indicate that these nonlinear interactions take place in the sun and are, in fact, responsible for the observed g mode spectrum. Resonant three-wave interactions, as the name implies, involve the coupled interaction of three waves which satisfy the resonant condition

$$\omega_o = \omega_1 + \omega_2,$$

where ω_o, ω_1, and ω_2 are the frequencies of the three waves. Two of the waves, mode 0 and 1 for example, couple together to produce a third beat wave which has a frequency equal to the frequency difference of the first two waves. This wave is normally very weak. If its frequency happens to equal the frequency of an oscillation mode of the sun, the wave can be pumped by resonance to very large amplitudes. In the sun, resonant interactions would take place if three natural oscillation modes of the sun satisfy the resonance condition.

The observed p mode spectrum is complex and, therefore, it would be difficult to test for the existence of resonant interactions of these modes. The observed g mode spectrum, on the other hand, consists of only a few modes. We are assuming that the 160-minute oscillation is a g mode and that the recently identified modes by Delache and Scherrer (1983) are also g modes. We believe that the amplitudes of these selected few modes have been enhanced by resonant interaction so that they may be observed at the sun's surface. Several authors (see papers in Dziembowski and Gough 1983) have already made similar suggestions to explain the existence of the 160-minute oscillation, but almost no detailed work has been done in this area.

We have solved the pulsation equations (Guenther 1983) describing resonant interactions for many low degree triads of g modes. The particular interactions investigated were calculated from adiabatic frequency data for low degree g modes of the calculations of Christensen-Dalsgaard, Gough, and Morgan (1979). Modes were selected from this data which satisfy the resonance condition within a frequency tolerance of 2.5 microHz (see Guenther and Demarque 1983 for a

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discussion of this tolerance). After calculating the coupling coefficients for the selected modes the equations were solved. We did not find anything specifically unusual about interactions involving g modes with frequencies near 104 microHz (periods = 160 minutes), and are forced to conclude that no single resonant interaction of three g modes is likely to account for the outstanding amplitude of the 160-minute oscillation. However, we have found some evidence which indicated that the 160-minute oscillation may be due to the collective interactions of many resonant three-wave interactions. This can be seen in the resonant count diagram (figure 1), which we will now describe.

The diagram is a plot of the number of possible three-wave interactions involving two waves and their beat wave versus the beat wave's period. The numbers along the ordinate correspond to the number of possible resonant interactions of the lowest degree g modes which satisfy the resonance condition within a frequency tolerance of 2.5 microHz (similar curves are obtained using finer tolerances). The modes included in the calculation are marked with short vertical lines. All the modes calculated by Christensen-Dalsgaard et al., based upon a standard solar model, are included. Higher degree and order modes are probably not important because their complex pulsation shapes prevent strong coupling (this was found in our detailed calculations, Guenther 1983).

The curve contains several pronounced peaks, one of which is located at 160 minutes. It is apparent that the separation of g modes for the sun favor resonant interactions near 160 minutes. Interestingly, the tentatively identified g modes of Delache and Scherrer, which fall within the range of the diagram, correspond to the other peaks. We interpret this as evidence that many resonant three-wave interactions work together to stimulate the few modes which fall on the peaks of the diagram.

We note that the correspondence is not precise. The frequencies used to generate the curve have not been fine tuned to match the observed g mode spectrum, therefore some shift may be expected when more accurate frequencies are calculated. In light of our interpretation of the observed g modes, a complete spectrum of g modes will not be observable and it will therefore be difficult to compare the complete theoretical spectrum with the sparse observed spectrum. The initial data of Delache and Scherrer indicates that the actual spectrum of solar g modes have lower frequencies than those used to generate the curve and, hence, the curve will be shifted to the left. We will be examining this in more detail in the near future.

The resonant count diagram, along with Delache and Scherrer's data is suggestive that resonant interactions are responsible for the selective enhancement of g modes. The enhanced amplitudes allow these few modes to be observed at the surface of the sun. The diagram and our detailed calculations indicate that the enhancements are the result of several resonant three-wave interactions, i.e. no single resonant three-wave interaction can account for the 160-minute oscillation, for example. The use of this diagram for testing the mass distribution in the inner parts of models of the sun is being investigated.

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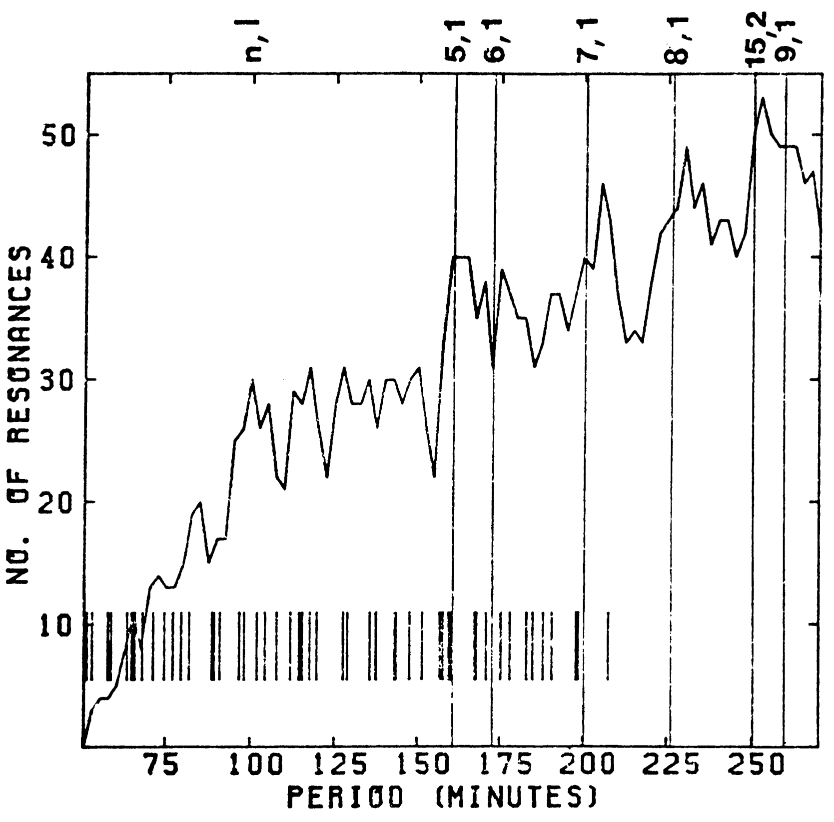

Figure 1. The resonant count diagram. The short vertical lines mark the positions of the modes calculated for a standard solar model, and the long vertical lines mark the position of the g modes identified by Delache and Scherrer. These tentative identifications are shown along the top of the diagram.

DISCUSSION

Hesser: Is it intuitively obvious how the interior mass distribution would have to be modified to bring about agreement between theory and observation?

Demarque: No. We have, of course, tried to determine, at least in a rough way, how to modify the mass distribution of the standard model to achieve agreement with the observed modes, but the available information is insufficient.

Zahn: Have you taken the selection rules between modes into account, in combining the frequencies?

Demarque: We are aware of these selection rules, although they have not been taken into account in the diagram I have shown. On the average, the selection rules will reduce the total number of resonances by a factor of two.

Cox: What is the range of ℓ in the g-mode list of Gough and Christensen-Daalsgaard? Would the coupling peak be even better fitted if ℓ were limited to 1, 2 or 3?

Demarque: The Christensen-Dalsgaard, Gough and Morgan list contains modes with $\ell < 5$. I don't know the answer to your second question, but it would be easy to check.