INVERSE CASCADE IN HYDRODYNAMIC TURBULENCE AND ITS ROLE IN SOLAR GRANULATION

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ABSTRACT. Self-organization i.e. the formation of large ordered structures in a turbulent medium is a consequence of inverse cascade where energy preferentially transfers towards large spatial scales. It is envisaged that this may be one way of explaining solar granulation at various scales.

1. Introduction

Radiation and convection are the two main energy transport processes in the solar interior. The convective transport becomes operative where the temperature and density gradients are such that a fluid element, when displaced from its equilibrium position, keeps moving away from it. This stratification, through unstable convection produces turbulence in the medium. The fluid eddies of varying sizes then carry energy as they propagate and dissipate. The cellular patterns observed on the solar surface are believed to be the manifestations of convective phenomena occuring in the sub-photospheric layers. The cellular velocity fields are seen prominently on two scales: the granulation and the supergranulation, though mesogranulation and giant cells are also suspected to be present. The formation of granules with an average size of 1000 km and a life time of a few minutes can be understood either from the mixing length (Schwarzschild 1975) or from the linear instability (Bogart, Gierasch and MacAuslan 1980) description of the convection in the hydrogen ionization zone of the subphotospheric medium. The supergranules with an average size of 30,000 km and a life time of 20 hours do not have an unambiguous association with a subphotospheric region. The attempts have been to seek an explanation for the energy concentration at the supergranular scale and to identify the region. Simon and Leighton (1964) suggested helium ionization to be responsible for accumulation of energy at supergranular scales. Convective modes with dominant growth rates at the two scales have been favoured by Simon and Weiss (1968), Bogart, Gierasch and MacAuslan (1980) and Antia, Chitre and Narasimha (1984). Here, a new mechanism of making large eddies from small eddies is presented.

J. O. Stenflo (ed.), Solar Photosphere: Structure, Convection, and Magnetic Fields, 329–332.

2. Self-organization in Two-Dimensional Turbulence

Formation of ordered structures in a turbulent medium relates to the concept of self-organization which occurs when a system has two or more invariants which suffer selective decay in the presence of dissipation. One invariant has a higher decay rate than the others. The cascading process is such that the slowly decaying quantity transfers towards large spatial scales and thus appears in the form of large organized pattern. The system can be described using a variational principle where the fast decaying quantity is minimised keeping the slowly decaying quantity constant. Kraichnan (1967) found that in a 2-D hydrodynamic turbulence, the energy cascades towards large spatial scales and enstrophy, which is the total squared vorticity ω , towards small spatial scales. It is this property of selective decay and inverse cascade that facilitates the formation of large structures whose dimensions are determined from the ratio of energy and enstrophy. The total energy W and the enstrophy U are defined as

$$W = \int \frac{1}{2} v^2 \mathrm{d}^3 r, \quad U = \int \frac{1}{2} \omega^2 \mathrm{d}^3 r. \tag{1}$$

Using Kolmogorovic arguments one finds two inertial ranges operating in different spectral regions i.e.

$$W(k) \propto k^{-5/3} \text{ for } k < k_s \tag{2}$$

and

$$W(k) \propto k^{-3} \text{ for } k > k_s \tag{3}$$

where K_s may be the source wave vector. The validity of equations (2) and (3) has been shown by considering inverse cascade through mode-mode coupling (Hasegawa 1985) and by numerically solving Navier-Stokes equations for a 2-D situation.

3. Solar Granulation and 3-D Hydrodynamic Turbulence

But how good is the assumption of 2-D for the solar atmosphere? Levich (1985 and references therein) has shown that the inverse cascade occurs even in 3-D hydrodynamic turbulence. A qualitative description of this phenomenon and a possible answer to the very important question of how a 3-D situation develops into a 2-D or a quasi 2-D are attempted here briefly. The energy injection into the solar atmosphere occurs by the convective upward motions and the energy associated with the latent heat of ionization is released. This begs the question whether the excitation of random small scale motions can lead to large organized structures that are observed in the form of granules, supergranules and giant cells. Here, a picture that emphasizes the role of large helicity fluctuations in the cascading process, as developed by Levich and coworkers is presented. The helicity density, a measure of the knottedness of the vorticity field is given by $\gamma = \mathbf{v} \cdot \boldsymbol{\omega}$ and the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{v}$. A turbulent medium exhibits large fluctuations in helicity even though the mean helicity $\langle \mathbf{v} \cdot \boldsymbol{\omega} \rangle = 0$. The fluctuating topology of the vorticity field in such a medium is characterized by statistical helicity invariant 'I', represented by conserved mean square helicity per unit volume:

$$I = \lim_{v' \to \infty} \left\langle \left(\int \mathbf{v} \cdot \boldsymbol{\omega} \, d^3 r \right)^2 \right\rangle \frac{1}{v'}, \tag{4}$$

and 'I' is a constant for a nondissipative system. Again using Kolmogorov arguments, the inertial range of 'I' and energy W is determined as

$$I(k) \propto k^{-1} \tag{5}$$

$$W(k) \propto k^{-5/3}. \tag{6}$$

Thus, analogous to the 2-D case, 'I' is expected to cascade towards large spatial scales and W(k) to small scales. It is more appropriate to say that the correlation length of helicity fluctuations increases without carrying much energy with it. When this correlation length becomes equal to a fixed vertical scale, as for example restricted by superadiabaticity, the correlation can grow only in the horizontal plane. This gives rise to anisotropy. In the case of highly anisotropic flow characterized by $L_z \ll L_{x,y}$ and $v_z \ll (v_x, v_y)$ it can be shown that the invariant 'I' attains the form $I(K) \propto K^{-5/3}$ which is indistinguishable from that of the energy spectrum in 2-D. Thus, as anisotropy increases, the fraction of energy transferred to larger scales also increases. The growth of large structures in an anisotropic turbulence can again be interrrupted as a result of symmetry breaking, for example cuased by the coriolis force. At the length scale L_c where the nonlinear term of Navier-Stokes equation becomes comparable to the coriolis force, the inverse cascade is inhibited. In the quasi 2-D situation that obtains, coriolis force together with a lack of reflexional symmetry with respect to the horizontal plane favours helical structures with a definite sign of helicity. It is found that in quasi 2-D, the coriolis force favors cyclonic circulation, the sign of helicity corresponding to the updraft cyclonic motion can be fixed. If downward motion is present, it must be anticyclonic to retain the same sign of helicity. There are several related questions of the energetics, the life time, the spatial and temporal structure which need to be investigated keeping in view the available observations of solar granulation and this may give clues as to what more needs to be measured about solar granulation.

References

Antia,H.M., Chitre,S.M. and Narasimha,D. 1984 Convection in the envelope of Red Giants Ap. J. 282: 574-583.

Bogart,R.S., Gierasch,P.J., and Mac Auslan,J.M. 1980 Linear modes of convection in the solar envelope Ap. J. 236: 285-293.

Hasegawa,A. 1985 Self-organization processes in continuous media Advance in Physics 34; 1-42.

Kraichnan,R.H. 1967, Inertial ranges in two-dimensional turbulence Phys. Fluids 10, 1417-1423.

Levich,E. 1985 Certain problems in the theory of developed hydrodynamic turbulence Phys. Rep. 151, 129-238.

Schwarzschild,M. 1975, On the scale of photospheric convection in red giants and supergiants Ap. J. 195: 137-144.

Simon,G.W. and Leighton,R.B. 1964 Velocity fields in the solar atmosphere III. Large-scale motions, the chromospheric network and magnetic fields Ap. J. 140:1120-1147.

Simon,G.W. and Weiss,N.O. 1968 Supergranules and the Hydrogen convection zone, Zeitschrift fur Astrophysik 69: 435-450.