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Left and right zero divisors in group algebras

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We prove that most group algebras of free products have left zero divisors that are not right zero divisors.

In [4, p. 35] the question whether every left zero divisor in a group algebra is also a right zero divisor is asked. The question was originally asked by S. Montgomery. The following theorem shows that there is a large class of group algebras having left zero divisors that are not right zero divisors.

THEOREM 1. *Let* FG *be a group algebra with zero divisors and let* H *be any group of order greater than two. Then the free product group algebra* $F[G * H]$ *has left zero divisors that are not right zero divisors.*

DEFINITIONS. A ring R is (right) strongly prime if for every nonzero $r \in R$ there exists a finite set S (called a right insulator) such that the right annihilator of rS is zero. R is $\mathrm{SP}(n)$ if for each nonzero element, we can choose an insulator with n elements. R is bounded strongly prime if it is $\mathrm{SP}(n)$ for some natural number n.

THEOREM 2. *Let* $G * H$ *be a nontrivial free product of groups (both* G *and* H *have order greater than one), where the order of* H *is greater than two. Then the group algebra* $F[G * H]$ *is bounded strongly prime and is not a Goldie ring.*

Proof. We prove that the group algebra is bounded strongly prime in [2, Proposition 3.3]. In order to show that the group algebra is not a right Goldie ring, it is sufficient to show the existence of a regular

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element x (a right and left nonzero divisor) such that the right ideal generated by x is not right essential [3, Lemma 7.2.3]. Choose $a \in G - \{1\}$ and $b, c \in H - \{1\}$. Then $ab + ba$ is regular and $(ab+ba)F[G * H] \cap (ac+ca)F[G * H] = (0)$. Thus $ab + ba$ is the desired regular element.

THEOREM 3 [1, Theorem 2.3]. *A bounded strongly prime ring is either a Goldie ring or is* SP(1).

Proof of Theorem 1. As $F[G * H]$ is bounded strongly prime and not Goldie, it must be SP(1). Let a be any nonzero element with nonzero left annihilator and let $\{b\}$ be a right insulator for a. Then ab is a left but not right zero divisor.

As a corollary we note that if Montgomery's question has an affirmative answer for all torsion free group algebras, then all group algebras of torsion free groups must be domains.

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