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On closures of finite permutation groups

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In this thesis we investigate the properties of k closures of certain finite permutation groups. Given a permutation group of G on a finite set Ω, for $k \geqslant 1$ the k-closure $G^{(k)}$ of G is the largest subgroup of $\mathrm{Sym}(\Omega)$ with the same orbits as G on the set Ω^k of k-tuples from Ω.

The first problem in this thesis is to study the 3-closures of affine permutation groups. In 1992, Praeger and Saxl Showed if G is a finite primitive group and $k \geqslant 2$ then either $G^{(k)}$ and G have the same socle or $(G^{(k)}, G)$ is known. In the case where the socle of G is an elementary Abelian group, so that G is a primitive group of affine transformations of a finite vector space, the fact that $G^{(k)}$ has the same socle as G gives little information about the relative sizes of the two groups G and $G^{(k)}$. In this thesis we use Aschbacher's Theorem for subgroups of finite general linear groups to show that, if $G \leqslant AGL(d,p)$ is an affine permutation group which is not 3-transitive, then for any point $\alpha \in \Omega$, G_α and $\left(G^{(3)} \cap AGL(d,p)\right)_\alpha$ lie in the same Aschbacker class. Our results rely on a detailed analysis of the 2-closures of subgroups of general linear groups acting on non-zero vectors and are independent of the finite simple group classification. In addition, modifying the work of Praeger and Saxl in [1], we are able to give an explicit list of affine primitive permutation groups G for which $G^{(3)}$ is not affine.

The second research problem is to give a partial positive answer to the so-called Polycirculant Conjecture, which states that every transitive 2-closed permutation group contains a semiregular element, that is, a permutation whose cycles all have the same length. This would imply that every vertex-transitive graph has a semiregular automorphism. In this thesis we make substantial progress on the Polycirculant Conjecture by proving that every vertex-transitive, locally-quasiprimitive graph has a semiregualr automorphism. The main ingredient of the proof is the determination of all biquasiprimitive permutation groups with no semiregular elements.

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References

[1] C.E. Praeger and J. Saxl, 'Closures of finite primitive permutation groups', *Bull. London Math. Soc* **24** (1992), 251–258.

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