

# Towards physics-based helioseismic inversions of subsurface sunspot structure

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**Abstract.** Numerical computations of wave propagation through sunspot-like magnetic field structures are critical to developing and testing methods to deduce the subsurface structure of sunspots and active regions. We show that helioseismic analysis applied to the MHD sunspot simulations of Rempel and collaborators, as well as to translation-invariant models of umbral-like fields, yield wave travel-time measurements in qualitative agreement with those obtained in real sunspots. However, standard inversion methods applied to these data fail to reproduce the true wave-speed structure beneath the surface of the model. Inversion methods which incorporate direct effects of the magnetic field, including mode conversion, may be required.

**Keywords.** Sun: helioseismology, Sun: magnetic fields

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## 1. Introduction

Current controversy exists in the interpretation and modeling of helioseismic measurements of sunspots (see the review by Gizon, Birch & Spruit 2010). A major issue is the discrepancy between the relatively deep two-layer wave-speed models derived from standard time-distance helioseismic inversions (Kosovichev *et al.* 2000; Couvidat, *et al.* 2005) and shallow, positive wave-speed perturbations inferred from forward models which explicitly include magnetic fields (e.g. Crouch *et al.* 2005; Cameron *et al.* 2010).

Structural (i.e. wave-speed) models of sunspots are inferred from *p*-mode travel-time perturbations relative to travel times through quiet Sun. These travel-time perturbations typically show a variation with phase-speed  $w$  (the temporal frequency divided by the horizontal wavenumber), ranging from positive (longer times) at small phase-speeds, to negative (shorter times) at larger phase-speeds. Deeper penetrating modes have increasing phase-speed, so the travel-time variation with  $w$  provides the basis for the two-layer wave-speed models (e.g. Kosovichev *et al.* 2000) which extend downward to approximately 10 Mm below the surface. However, recently observed variations of travel-time perturbations with frequency (at fixed phase-speed) have been suggested as evidence of strong near-surface perturbations (Couvidat & Rajaguru 2007; Braun & Birch 2006; 2008). In addition, the nature and interpretation of the positive (slower) travel-time perturbations and the sensitivity of the travel-time perturbations to the analysis methodology have been questioned (Braun & Birch 2008; Birch *et al.* 2009; Gizon *et al.* 2009; Moradi *et al.* 2010). Positive travel-time shifts (slower waves) have also been measured

in artificial datasets where the wave-speed perturbations are positive (Birch *et al.* 2009; Moradi *et al.* 2009).

## 2. Simulated sunspot models for helioseismology

We discuss here the use of two types of models for developing and testing helioseismic methods. One of these consists of a realistic magnetoconvection simulation using the MURaM (Max Planck Institute for Solar System Research/University of Chicago Radiative MHD) code described by Vögler *et al.* (2005) and modified to simulate realistic sunspot structures (Rempel *et al.* 2009). The simulation spans a 48 by 48 Mm box extending 8 Mm into a solar-like stratification. The data saved for helioseismic use consists of slices of vertical velocity sampled every minute at a constant optical depth near the photosphere.

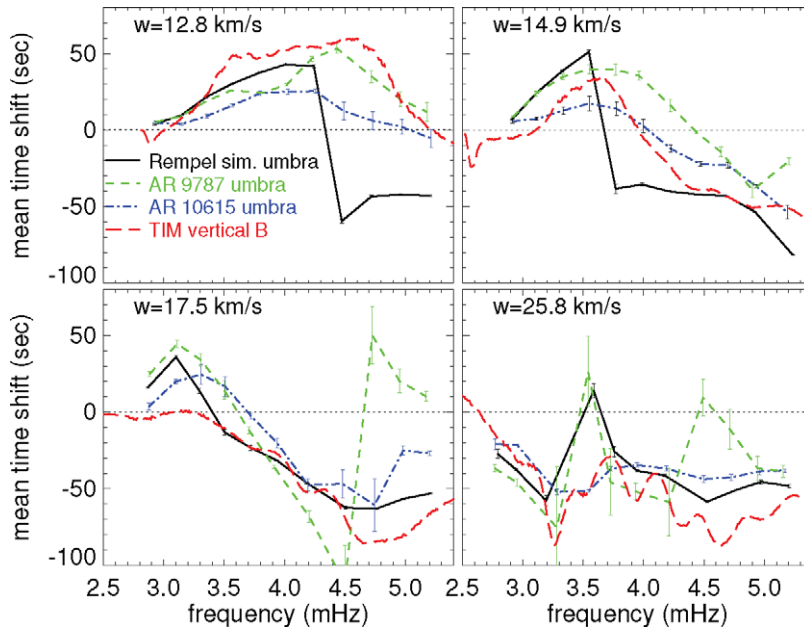
A second type of model propagates waves through (horizontally) translation-invariant background models (Crouch *et al.* 2010). The power spectra from the translation-invariant models (TIMs) are then converted into a time series of synthetic helioseismic data using the algorithm outlined by Gizon & Birch (2004).

Using helioseismic holography, we produce sets of maps of travel-time perturbations, using an analysis analogous to those involving center-annulus time-distance correlations (Braun & Birch 2008). Our analysis is applied to MDI observations of two sunspots (AR 9787 & AR 10615), 27 hrs of the Rempel simulation, and a TIM consisting of a vertical 3 kG field embedded into a model S background stratification. Our analysis is similar to previous time-distance methods in that we employ standard phase-speed filters and their corresponding annuli (see Table 1 of Couvidat *et al.* 2006), but differs in the additional use of box-car filters to isolate narrow ranges in temporal frequency.

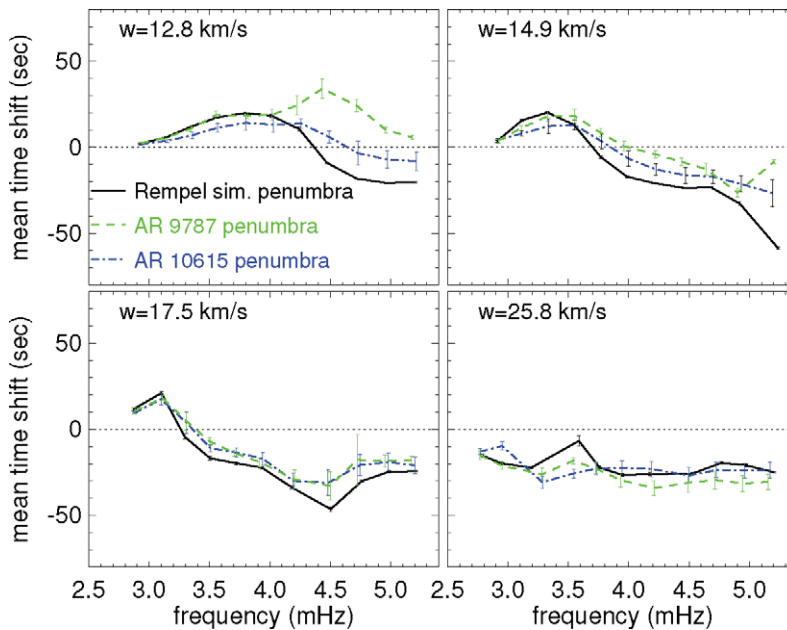
Comparing the spatial average of the travel-time perturbations over the umbrae (Figure 1) and penumbrae (Figure 2), we see at least a qualitative similarity between results for the real and artificial sunspots. In many cases (especially in the penumbral measurements) the agreement is remarkably quantitative as well. Of particular note is the presence, in both real and artificial spots, of positive travel-time perturbations at smaller phase-speeds, which tend to decrease to negative values with increasing frequency, and the predominantly negative values at higher phase-speeds.

## 3. Inversions of the Rempel simulation

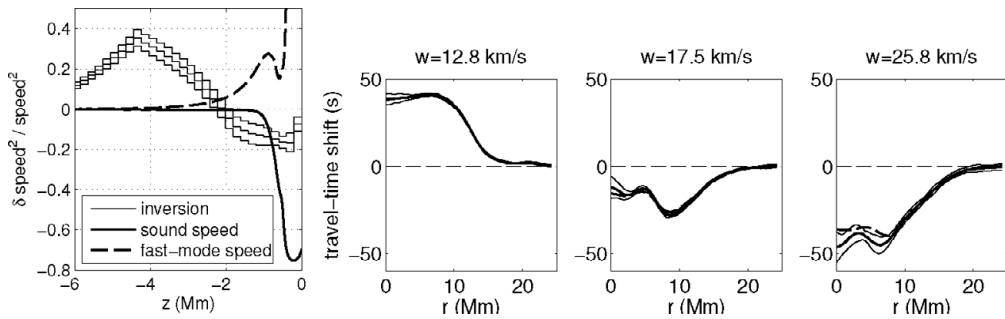
We carried out three-dimensional ray-approximation based wave-speed inversions for a 12 hour time-series of the MURaM simulation. We used the ray approximation for the travel-time shifts caused by wave-speed perturbations as described by Kosovichev & Duvall (1997) using a RLS MCD approach. Figure 3 shows the results of applying the inversion procedure to travel-time maps measured from the simulation. The maps were made using a wide frequency bandpass (i.e. 2.5–5.5 mHz) for each phase-speed filter. The wave-speed structure that is recovered from the inversion (left panel) is reminiscent of the two-layer wave-speed structure seen from inversions of solar data (e.g. Kosovichev *et al.* 2000; Couvidat, *et al.* 2005). While the results are consistent with the mean-travel time measurements (right panels of Figure 3) it is clear that the inversion procedure does not recover either the sound-speed or fast-mode speed perturbations present in the model. This shows that one or more of the assumptions of the inversion is not met.



**Figure 1.** Comparison of mean travel-time shifts, averaged over the umbrae, determined from both MDI observations of two sunspots as well as the umbra of the MURaM simulated spot of Rempel and collaborators and a translation-invariant model (TIM) with a vertical magnetic field embedded within a solar model. The phase-speed filters used are indicated in each panel. Box-car frequency filters of width 0.5 mHz were used in all cases except for the TIM results which use a considerably narrower frequency range.



**Figure 2.** Comparison of mean travel-time shifts, averaged over the penumbrae, determined from both MDI observations of two sunspots as well as the MURaM simulated sunspot.



**Figure 3.** Inversion results for the MURaM simulated sunspot. The left panel shows the relative wave-speed perturbation inferred from the inversion (with errors) as well as the true perturbations to the fast-mode speed (dashed line) and sound speed (solid line) averaged over the region  $r < 6$  Mm. The right panels show a comparison of the azimuthal average of the travel-time shifts predicted from the inverted results (dashed lines) with the azimuthal average of the measured (input) travel-time shifts (solid lines with errors).

#### 4. Discussion

It is clear that standard helioseismic inversion methods fail to recover the subsurface wave-speed structure within the Rempel sunspot simulation. Consequently, the similarities between the helioseismic travel-time measurements made using the artificial sunspot and actual observations of sunspots adds to the uncertainty in our inferences of subsurface structure below strong magnetic fields. Numerical and semi-analytic MHD models provide considerable insight into the physics of sunspots and the propagation of waves in magnetic regions. Making full use of the year-round, almost limb-to-limb, coverage provided by HMI onboard the Solar Dynamics Observatory will likely require an efficient and reliable inversion method incorporating magnetic effects. A first step towards this end is the development of inversion kernels which include the physical effects of magnetic fields (e.g. Crouch *et al.* 2010).

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