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# Multiplier problems for spaces of continuous functions with p—summable transforms

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The space $A^p$ , $p \in [1, \infty)$ , is defined as the set of those functions on the circle group $\mathbb{T}$ that are continuous and have $p$-summable Fourier transforms. Each $A^p$ space will be normed by $N_p : f \to \|f\|_\infty + \|\hat{f}\|_p$ , under which it is a Banach space. In this thesis we are concerned with these spaces and both their Fourier (or convolution) and pointwise multiplier spaces.

In Chapter 2 we consider the spaces $A^p$ in detail. In particular, we prove that $A^p$ , $p \in (1, 2)$ , is not an algebra and establish constructively the strict inclusion results

$$\bigcup_{p\in[1,q)} A^p \subsetneq A^q \text{ , if } q \in (1, 2] \text{ ,}$$

and

$$A^q \subsetneq \bigcap_{p\in(q,2]} A^p \text{ , if } q \in [1, 2) \text{ .}$$

In Chapter 3 we consider the spaces $(A^p, A^q)$ of Fourier multipliers from $A^p$ to $A^q$ . We identify $M + Fl^{p'}$ as the strong dual of $A^p$ , and prove that $(A^p, A^q) = M + Fl^{p'}$ if $p \in [1, 2]$ , $q \in [p, 2]$ . In the

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case $p \in (1, 2]$ , $q \in [1, p)$ , we give a sufficient and several necessary conditions for membership of $(A^p, A^q)$ . In addition, we establish constructively the strict inclusion results

$$(A^q, A^q) \subsetneqq \bigcap_{p\in[1,q)} (A^p, A^p) , \text{ if } q \in (1, 2] ,$$

and

$$\bigcup_{p\in(q,2]} (A^p, A^q) \subsetneqq (A^q, A^q) , \text{ if } q \in [1, 2) .$$

In Appendix B we identify the idempotent elements of $(A^p, A^q)$ .

In Chapter 4 we consider the pointwise multiplier spaces $M(A^p, A^p)$ . It is proved there that $M(A^p, A^q) = \{0\}$ when $2 \geq p > q \geq 1$ . Here, and in Appendix B, we establish various inclusion results for these multiplier spaces and prove several results involving the translation-continuity of functions in $M(A^p, A^p)$ . With the aid of results in Appendix A we obtain sufficient conditions for membership of $M(A^p, A^p)$ in such a way as to show that the functions so obtained are translation-continuous. In Appendix B we prove that $M(A^p, A^p) = M_p \cap C$ , $p \in [1, 2]$ , where $M_p$ is the set of $f \in L^\infty$ for which $\hat{f} * l^p \subseteq l^p$ . We then discuss some consequences of this, including the result that the maximal ideal space $\Delta_p$ of $M(A^p)$ cannot be identified with $T$ .

An analogue, for non-discrete locally compact abelian groups, of the strict inclusion results for the $A^p$ spaces has been given by Hewitt and Ritter in [4]. The material from Chapter 2 and Chapter 3 has appeared in [3] and [1]. The results concerning idempotent elements of $(A^p, A^q)$ have since been extended to arbitrary compact abelian groups and will appear in [2].

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