REFERENCES

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McKean, H. and Moll, V. *Elliptic curves: function theory, geometry, arithmetic* (Cambridge University Press, Cambridge, 1997), xiii+280pp., 0 521 58228 8 (hardback), £40 (US$59.95).

One of the main trends in mathematics during the last few decades has been the development of links between what were originally regarded as separate disciplines. A major catalyst in this process has been the theory of elliptic curves, that is, Riemann surfaces of genus 1. These objects have so many different facets that they have served, since the early 19th century, as a common meeting-ground for mathematicians from a wide range of backgrounds. In recent years general interest in elliptic curves has been greatly enhanced by Andrew Wiles's work on the Taniyama-Shimura Conjecture, which relates elliptic curves to modular forms, with its spectacular corollary of Fermat's Last Theorem. This book's subtitle, *Function theory, geometry, arithmetic*, gives an indication of the authors' broad approach, tracing the development of elliptic curves through the 19th and early 20th centuries. In addition they briefly outline links with several other topics such as Galois theory, partitions and even applied mathematics (solitons and the KdV equation for instance).

Chapter 1 is a brisk introduction to Riemann surfaces and projective curves, while Chapter 2 covers the classical theory of elliptic integrals and elliptic functions, due to Abel, Gauss, Jacobi and Legendre, and shows how they lead to complex tori $\mathbb{C}/L$, where L is a lattice, or equivalently to elliptic curves such as $y^2 = (1-x^2)(1-k^2x^2)$. Chapter 3 explains the development of theta functions, from Jacobi to Ramanujan, with interesting digressions into quadratic reciprocity, sums of squares, and partitions. The modular group $\Gamma_1 = PSL(2,\mathbb{Z})$, with its associated modular forms and congruence subgroups, is the main theme of Chapter 4; if this can be seen as a modern introduction to the work of Fricke and Klein, then Chapter 5 plays the same role for Klein's book on the icosahedron, with an algebraic and geometric discussion of the quintic equation, and Hermite's solution using $\sqrt[4]{k}$. Chapter 6 is about algebraic number theory, with special attention to imaginary quadratic number fields, where the connection is that ideal classes in the ring of integers correspond to equivalence classes of elliptic curves. The book closes with a rather brief chapter on the arithmetic of elliptic curves, leading up to a proof of the Mordell-Weil Theorem that the group of rational points on a nonsingular cubic curve has finite rank.

Throughout this book the style of writing is brisk, clear and informal with routine verifications generally left to the reader. There are plenty of references to alternative sources, such as the excellent book by Silverman and Tate **[1]**, for readers wanting a more detailed treatment. This approach allows an impressive range of topics to be covered and some fascinating connections to be explored. Although this is not a history book, there are numerous historical references and the authors often take care to explain not just what the 19th-century masters did but also how they did it; this approach can be of great benefit to those lacking the time, the library facilities or the linguistic skills to read the original papers. The bibliography alone, with more than 300 references, makes this book a valuable resource.

A good research student with a solid grounding in basic complex function theory should cope well with this book, though some familiarity with surface topology, number theory and group theory would help. For the more experienced reader the book's great attraction is the insight it offers into the links between so many different topics.

There are a few places, mainly in group theory, where the authors' generally impressive command of detail is a little less sure. For instance, the group $PSL(2, \mathbb{F}_p)$ contains subgroups of *index* (not order) p for the primes $p \leq 11$ (not just $p = 5, 7, 11$ as stated on page 212). On page 167 the kernel of the epimorphism from the free group $\langle X, Y \mid - \rangle$ onto the modular group is the *normal* subgroup generated by X^3 and Y^2. On page 143 the coefficients of the elliptic modular function are not degrees of irreducible representations of the monster simple group but sums of such degrees, such as $196884 = 1 + 196883$; the Galois group of the general quintic is S_5, not A_5 (page 206), and the polyhedron described on page 219 is the *great* (not stellated) dodecahedron. These are small quibbles, however, and the general standard of mathematical accuracy is high.

To sum up, this is a very enjoyable and instructive book, which I strongly recommend to anyone wanting to learn how the theory of elliptic curves developed up to the first part of the 20th century and how these fascinating objects fit into the general scheme of modern mathematics. My only real regret is that the authors did not continue the story beyond the Mordell-Weil Theorem.

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