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ERRATA

VOLUME 13

Page 112, line 22 from the bottom. The subscript after P should be $n + 1$ instead of n .

VOLUME 16

Page 286, line 15. For «xxi», read «xii».

VOLUME 21

Page 89, line 10. Insert right parenthesis before first square bracket.

Page 123, line 11. For the second occurrence of $\mathfrak{A} \rightarrow \mathfrak{B}$, read $\mathfrak{B} \rightarrow \mathfrak{A}$.

VOLUME 22

Page iv, line 12. For «FRIEDMANN», read «FRIEDMAN».

Page 293, line 15. For «1953», read «1955».

Page 304, line 13 from the bottom. For «quantifier free», read «quantifier-free».

Page 351, line 12 from the bottom. The first word in the line should be «von».

Page 351, line 8 from the bottom. For «Axione», read «Axiome».

Page 351, line 7 from the bottom. For «widerspruchsfrie», read «widerspruchsfrei».

Page 351, line 2 from the bottom. Supply a period at the end of the line.

Page 351, last line. Add the first s of «Widerspruchsfreiheitsbeweis».

Page 361, line 14. For the first «to» read «the».

Page 371, lines 2, 3, 6. The inverted circumflexes over R and S are intended as breves.

Page 371, line 10 from the bottom. For «mosti mportant», read «most important».

Page 380. Throughout line 23 from the bottom, wherever the letter s has two subscripts, read the second of these subscripts as a subscript to the first subscript.

Page 386, line 8. For «1925j 26» read «1925/26».

Page 387, line 16. For «ari-stotelischen», read «aristotelischen».

Page 432, column 1, line 15. For «Los», read «Łoś».

Page 435, column 2, line 4. For «Ismann», read «Issmann».

Page 439, column 2, line 46. For «caseu», read «cause».

Page 441, column 1, line 38. For «Fefreman», read «Feferman».

VOLUME 23

Page 27, line 9. For «thaupt», read «thought».

Page 31, line 6 from the top, and line 8 from the bottom. For «petito», read «petitio».

Page 40, line 10. For «notions», read «concept».

Page 59, line 2. For « F », read « ϕ », twice.

Page 73, line 7. For «“almost” every», read «“almost every”».

Page 73, line 7. The last word in the line should be «is».

Page 97, line 14. In *Sectiunea*, there should be a comma below the t as an accent.

ERRATA

- Page 97, line 23. Delete the dash.
- Page 98, line 13. Delete the period after «*élémentom*».
- Page 100, line 12 from the bottom. After «(XV 210)», insert: «*Philosophical Analysis* (cf. XVI 298–301), and Pap's *Elements of Analytic Philosophy* (XIX 74)».
- Page 102, line 3. For XIV, read XVI.
- Page 104. In line 5 of the second column at the top of the page, change p to P , and in line 7 of the same column, change the last p to q .
- Page 224, line 12 from the bottom. For «*množestvo*», read «*množéstvo*».
- Page 225, line 22. For «*Trétégo*», read «*Trét'égo*».
- Page 230, line 13. For «*isčislénie*», read «*isčisléníé*».
- Page 234, line 16 from the bottom. For «*Akadem*», read «*Akadémii*».
- Page 239, line 1. For «*Stážkin*», read «*Stážkín*».
- Page 321, second line of footnote. For $\neg(Ex)$, read $(Ex)\neg$.
- Page 322, Third line of Theorem 4. After «*quantifier*» insert «*,with each prime formula preceded by at least one negation symbol,*».
- Page 357, line 6. For «(1956)», read «(1955)».
- Page 361, line 5. Insert «no. 1/2» after «vol. 16».
- Page 365, line 8. The comma should be after «1956» rather than before it.