THE PROSPECTS FOR HIGH SENSITIVITY GRAVITATIONAL ANTENNAE

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Abstract. The sensitivity of a resonant gravitational wave detector which is necessary for the detection of pulses from asymmetric stellar collapses or from black-hole collisions in nearby galaxies is examined. Limitations on this sensitivity due to the resonating quality of the detector, thermal noise, and reaction of the detection system upon the detector are studied. No severe difficulties for the detection of the above-mentioned pulses are expected. For the detection of pulses from a cluster at the galactic center, a nonresonant system based on Doppler ranging to a drag-free satellite would be more appropriate. At present the sensitivity of Doppler-ranging is still two orders of magnitude below the requirements.

(I) The collision of two collapsed stars ($M \simeq 10\ M_\odot$) or of two neutron stars, and the nonsymmetric collapse of one star ($M \simeq 10\ M_\odot$) give a pulse of gravitational radiation with energy $E_g \simeq 10^{52}$–10^{54} erg, duration $\hat{\tau}_g \simeq 10^{-3}$–$10^{-4}$ s and mean frequency $W_g \simeq 10^4$–10^5 rad s^{-1} (Zel'dovich and Novikov, 1964; Thorne, 1967; Burke and Thorne, 1969; Braginsky *et al.*, 1969). To realize a reasonable experiment to detect such pulses it is necessary that an experimentalist observe at least 10 events (coincidences in two or more antennae) during one year. Taking account of the fact that the density of galaxies is approximately 3 per (Mpc)^3, one can obtain a simple connection between the sensitivity of the gravitational wave detector $\tilde{I}$ [erg cm^{-2}], the number of the galaxies N from which gravitational-wave burts can be observed and the frequency Ω of collisions and collapses in one galaxy collisions and collapses for which $E_g \simeq 10^{52}$–10^{54} erg).

The Table I shows some value of $\tilde{I}$, N, Ω for three distances $R_1 = 3 \times 10^{25}$ cm, $R_2 = 3 \times 10^{26}$ cm, $R_3 = 3 \times 10^{27}$ cm.

TABLE I

10 events observed by experimentalist per year ($\varepsilon_g \simeq 10^{+52}$–$10^{+54}$ erg, $\hat{\tau}_g \simeq 10^{-3}$–$10^{-4}$ s)

R (distance)	N (the number of the galaxies in a sphere $\frac{4}{3}\pi R^3$)	Ω (the frequency of events in one galaxy)	$\tilde{I}$ (the density of grav. wave energy near the antenna)
3×10^{25} cm $= 10$ Mpc	$\simeq 10^{+4}$	$\simeq 10^{-3}$ yr^{-1}	10^{+1} erg cm^{-2}
3×10^{26} cm $= 10^2$ Mpc	$\simeq 10^{+7}$	$\simeq 10^{-6}$ yr^{-1}	10^{-1} erg cm^{-2}
3×10^{27} cm – optical horizon	$\simeq 10^{+10}$	$\simeq 10^{-9}$ yr^{-1}	10^{-3} erg cm^{-2}

Thus if a terrestrial gravitational antenna registers pulses of flux $\tilde{I} \simeq 10^{-3}$ erg cm^{-2}, and these pulses appear $\simeq 10$ time per year, then it is possible to conclude that probably collisions of black holes or neutron stars take place approximately one time in a billion years in one galaxy.

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Conventional astrophysics suggests that a sensitivity of $\tilde{I}=10^{+1}$–10^{-3} erg cm^{-2} for $\hat{\tau}\simeq 10^{-3}$ s, seems to be reasonable for the time scheme of coincidence of Weber (1969, 1970a, b). This required sensitivity is much better than the sensitivity which has been achieved in the recent experiments (Bonazolla, 1973; Tyson, 1972; Braginsky *et al.*, 1970; Kafka, 1973). Let us examine the conditions necessary for obtaining such sensitivity.

The equation of motion for a quadrupole gravitational wave detector

$$m\ddot{X}^{\mu}+H\dot{X}^{\mu}+KX^{\mu}=-mc^2 l^{\alpha}R^{\mu}_{0\alpha 0}+F_N, \tag{1}$$

where K is the rigidity which connects the two test masses, separated by a distince l, H is the coefficient of friction, $mc^2 l^{\alpha} R^{\mu}_{0\alpha 0}$ is the force produced by the gravitation wave, $R^{\mu}_{0\alpha 0}$ is the ac component of the Rieman tensor, c is the speed of light, F_N is the fluctuational force. In the case of pure thermal fluctuation the spectral density $(F_N)^2_f$ is equal to

$$(F_N)^2_f=4\varkappa T_M H, \tag{2}$$

where $\varkappa$ is the Boltzman constant, T_M is the temperature of the heat bath in which the mechanical part of the detector is situated. The ratio of the two forces on the right side of the Equation (1) is the ratio of signal to noise. Using the spectral density $\tilde{I}$ instead of $R^{\mu}_{0\alpha 0}$, assuming that the orientation of the detector relative to the wave's polarization and the propagation directions is optimal, and assuming that the signal to noise ratio is equal to one, we obtain an equation for the minimum detectable flux $[\tilde{I}]_{\min}$ in one pulse:

$$[\tilde{I}]_{\min}\simeq\frac{c^3\varkappa T_M}{2\pi G m\omega_g Q_M l^2}. \tag{3}$$

Here $\omega_g\simeq 1/\hat{\tau}_g$ if the mean frequency of gravitational radiation in the pulse, $\omega_g\simeq\omega_M= =\sqrt{K/m}$, $Q_M=m\omega_M/H$ is the mechanical quality factor.

Equation (3) is also valid if $\hat{\tau}\ll\tau^*_M=2Q_M/\omega_M$ (see details in Braginsky, 1970). It is important that the Equation (3) is obtained under the assumption that the registering system is ideal (about the backward fluctuational influence of the registering system see below). Equation (3) gives the *first important condition* for the detection of small gravitational wave pulses.

The simple conclusion that one may obtain from Equation (3) is the following: in order to increase the sensitivity it is necessary to minimize T_M and (or) to increase m and Q_M. Substituting into Equation (3) $m=10^{+6}$ g, $m_g=10^{+4}$ *rad* s^{-1}, $l=10^2$ cm, $Q_M=2\times 10^5$, $T_M=3\times 10^{-3}$K (the parameters of the installations for the experiments of Fairbank and Hamilton (1970)) we obtain $[\tilde{I}]_{\min}\simeq 1$ erg cm^{-2}. This is apparently the limit of sensitivity which may be obtained with heavy aluminium bars, since the Q_M of aluminium even at low temperature, is not very high.

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If we use a dielectrical monocrystal instead of aluminium, the Q_M may be much greater. It is possible to show that in a dislocation-free monocrystal without surface losses (the surface losses can be eliminated (by careful polish) the product $Q_M\omega_M$ is

$$Q_M\omega_M \simeq \frac{4\varrho C_p^2}{\beta T_M \alpha^2}, \tag{4}$$

where ϱ is the density; C_p, the heat capacity; α, the linear expansion coefficient; β, the heat conductivity.

The Table II includes the estimates for the factor $Q_M\omega_M$ of sapphire (Al_2O_3) at different temperatures based on the Equation (4) and experimental data for β, C_p and α (Yates *et al.*, 1972; Goer and Dreyfus, 1967; Wolfmeyer and Dillinger, 1971). Table II also include estimates of $[\tilde{I}]_{\min}$ for a relatively small sapphire mono-crystal ($m = 4 \times 10^4$ g, $l = 40$ cm), The estimates $[\tilde{I}]_{\min}$ in Table II for an ideal monocrystal bar look very optimistic. Below we shall see that another type of difficulty will appear if an experimentalist has an ideal monocrystal at low temperature and wants to obtain sensitivity better than $[\tilde{I}]_{\min} \simeq 10^{-2}$ erg cm^{-2}.

TABLE II

$T=$	300 K	80 K	4.2 K	2 K	0.4 K	0.01 K
$Q_M\omega_M$ rad s^{-1}	2.2×10^{16}	8.4×10^{15}	2.8×10^{17}	2×10^{18}	10^{21}	$\sim 3.5 \times 10^{22}$
$[\tilde{I}]_{\min}$ erg cm^{-2}	5	3.5	5×10^{-3}	3.5×10^{-4}	1.5×10^{-7}	$\sim 10^{-10}$

Table III shows the preliminary results of measurement of for sapphire obtained by H. Bagdasarov, V. Mitrofanov and by the author. The sapphire monocrystal cylinder has $L = 15$ cm, $D = 4.5$ cm, $M = 1.1 \times 10^{+3}$ g. The results in Table III show that our monocrystal cylinder was not ideal (the factor $Q_M\omega_M$ at 80 K is two order smaller than the theoretical limit). But if we should decide to make a gravitational

TABLE III

($\omega_M = 2 \times 10^5$/rad s^{-1})($f_M = 34$ kHz)

T	300 K	80 K
Q_M	4×10^7	1.3×10^8
$Q_M\omega_M$	8×10^{12}	2.6×10^{13}
$\tau_M^* = \dfrac{2Q_M}{\omega_M}$	400 s	1200 s

antenna with this small cylinder we may reach approximately the sensitivity of a heavy aluminium bar which has $Q_M\omega_M \simeq 2 \times 10^9$ (Weber 1969, 1970a, b; Bonazolla, 1973; Braginsky *et al.*, 1972; Tyson, 1972; Kafka, 1973).

The *second condition* necessary for the detection gravitationalwave pulses is to construct a very sensitive system for registering small vibration of the detector. The ferroelectrical transducers seem not to be suitable for $\tilde{I}$ less than 10 erg cm^{-2}. Active

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transducers (modulator-demodulator type) are more promising. Study of different types of active transducers is now in progress in many groups (Braginsky *et al.*, 1972; Dick and Yen, 1972; Paik, 1972).

Figure 1 shows the scheme of an active capacity choice of system connected with a quadrupole mechanical oscillator.

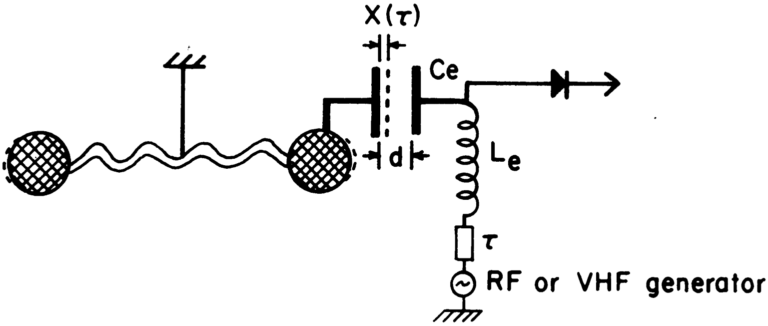

Fig. 1.

If it is possible to eliminate (using a compensation scheme) the noise of the RF generator, then the smallest displacement $[X(\tau)]_{\min}$ between the plates of the capacity depends on the thermal fluctuation in the electrical resonator and on the amplitude $U_{\sim}$ of RF voltage on the plates (see details in Braginsky, 1970):

$$[X(\tau)]_{\min} \simeq \frac{2d}{U_{\sim}} \sqrt{\frac{4\varkappa T_e \Delta f}{\omega_e Q_e C_e}}. \tag{5}$$

Here d is the mean distance between the plates; $\varkappa$, the Boltzman constant; T_e, the temperature of the electrical resonator; $\omega_e = 1/\sqrt{L_e C_e}$, the eigen frequency of the resonator; $Q_e = \omega_e L_e/\tau$, the quality factor of the resonator; C_e, the capacity; $\Delta f \simeq 1/\tau$. Substituting into Equation (5) the values $T_e = 2\,\mathrm{K}$, $W_e = 6 \times 10^{10}$ rad s^{-1}, $C_e = 10^{-12}\,F$, $Q_e = 10^{+11}$ (this Q_e has been achieved by Weissman and Turneaure (1968) we obtain $[X(\tau)]_{\min} \simeq 10^{-21}\sqrt{\Delta f}$ cm.

The pulse of gravitational radiation which lasts for a time $\hat{\tau}_g$ and has an energy flux of energy $\tilde{I}$ produces a change in the oscillation amplitude of the detector $X_g(\tau)$ equal to

$$X_g(\tau) \simeq l \sqrt{\frac{8\pi G}{C^3} \tilde{I} \hat{\tau}_g}, \tag{6}$$

if $\omega_M \simeq \omega_g$ and $\hat{\tau}_g \ll \tau_M^* = 2Q_M/\omega_M$.

Three estimates of $X_g(\tau)$ for three different $\tilde{I}$ are given in Table IV.

TABLE IV

($l = 40$ cm, $\hat{\tau}_g = 10^{-3}$ s)

$\tilde{I}$[erg cm^{-2}]	10^{-3}	10^{-1}	10^{+1}
$X_g(\tau)$ [cm]	$\simeq 1 \times 10^{-20}$	$\simeq 1 \times 10^{-19}$	$\simeq 1 \times 10^{-18}$ cm

Comparing the data in Table IV with the estimate $X(\tau)\simeq 10^{-21}\sqrt{\Delta f}$ cm based on the present state of art for Q_e, one can reach a preliminary conclusion that it is possible to reach the sensitivity $I=10^{-3}$ erg cm^{-2} for millisecond gravitational wave pulses. But a *third condition* for a detector exists. In Equations (3), (5) and (6) the reaction of the registering system on the masses of the detector is not taken into account (see details in Braginsky, 1970; Braginsky, 1973). It is possible to show that if $Q_M=\infty$, definite classical limits exist for the minimum detectable force $[F(\tau)]_{\min}$ acting on a mechanical oscillator. These limits depend on the relations between $\hat{\tau}$, $\tau_e^*=2Q_e/\omega_e$ and ω_M.

If $\hat{\tau}\gg\tau_e^*$, and $1/\omega_M\gg\tau_e^*$, then

$$[F(\tau)]_{\min}\simeq\frac{4}{\hat{\tau}}\sqrt{m\varkappa T_e\frac{\omega_M}{\omega_e}}, \tag{7}$$

and if $\hat{\tau}\ll\tau_e^*$, and $\tau_e^*\gg 1/\omega_M$, then

$$[F(\tau)]_{\min}\simeq\frac{4}{\hat{\tau}}\sqrt{m\varkappa T_e\frac{\omega_M}{\omega_e}\frac{2\hat{\tau}}{\tau_e^*}}=4\sqrt{\frac{m\varkappa T_e\omega_M}{\hat{\tau}Q_e}}. \tag{8}$$

The Equations (7) and (8) are classical, they are valid if Q_M is high enough (when the estimates from (7) or (8) are greater than estimates from Equation (2)). Equations (7) and (8) show that for to minimize $[F(\tau)]_{\min}$, it is necessary to decrease the ratios ω_M/ω_e and $\hat{\tau}/\tau_e^*$, and to increase the Q-factor $Q_e=\tau_e^*\omega_e/2$ of the electrical resonator.

The estimates for $[F(\tau)]_{\min}$ from Equation (8) are less than those from Equation (7), if $\hat{\tau}\ll\tau_e^*$. Equation (8) will not be valid if $[F(\tau)]_{\min}$ from (8) is equal or less than $[F(\tau)]$ quantum:

$$[F(\tau)]_{\text{quantum}}\simeq\frac{4}{\hat{\tau}}\sqrt{\frac{m\hbar\omega_M}{n}}, \tag{9}$$

where $\hbar$ is the Planck constant, n is the quantum level of the mechanical oscillator before $F(\tau)$ acted on it. Equation (9) is based on the uncertainty principle (see details in Braginsky (1970, 1973)). Assuming $[F(\hat{\tau}_g)]_{\min}$ from Equations (7) and (8) to be equal to $mc^2l^\alpha R^\mu_{0\alpha0}$, it is easy to obtain two new limits for $\tilde{I}$ which are valid for high Q_M:

$$[\tilde{I}]_{\min}\simeq\frac{2c^3\varkappa T_e}{\pi Gm\omega_M\omega_e l^2\hat{\tau}_g}, \tag{10}$$

if $\hat{\tau}_g\gg\tau_e^*$ and $1/\omega_M\gg\tau_e^*$;

$$[\tilde{I}]_{\min}\simeq\frac{c^3\varkappa T_e}{\pi Gm\omega_M Q_e l^2}, \tag{11}$$

if $\hat{\tau}_g\ll\tau_e^*$ and $1/\omega_M\ll\tau_e^*$.

Equation (3) is similar to Equation (11), but in (11) the ratio T_M/Q_M is replaced by the ratio T_e/Q_e.

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Substituting into Equation (11) the values $m = 4 \times 10^4$ g, $l = 40$ cm, $\omega_M = 10^{+4}$ rad s^{-1}, $T_e = 0.01$ K and $Q_e = 10^{+12}$, we obtain $[\tilde{I}]_{\min} \simeq 10^{-2}$ erg cm^{-2}. This estimate is very promising, but at the same time it is much worse than the estimate 10^{-10} erg cm^{-2} for the same m, W_M and T_M (see Table II) which is based on Equation (3). In other words the third condition which gives Equation (10) and (11) is more severe, and for to reach a sensitivity better than $\tilde{I} \simeq 10^{-2}$ erg cm^{-2}, it is necessary to have $T_e < 10^{-2}$ K, and (or) $Q_e > 10^{+12}$.

Equations (3), (7), (8), (10) and (11) were obtained under the assumption that the time of measurement (the resolution time) τ_{res} is equal to the expected duration of the gravitation wave pulse τ_g. If one chooses τ_{res} greater than τ_g it is easy to obtain new equations for $[\tilde{I}]_{\min}$ which are similar to Equations (3), (10) and (11). These estimates for the sensitivity from these new equations are better than the estimates from Equations (3), (10) and (11). But in the case $\hat{\tau}_g \ll \hat{\tau}_{\text{res}}$ the experiment is not informative enough.

Concluding this review of various necessary conditions for achieving high-sensitivity detectors for the frequency $\omega_g \simeq 10^{+4}$–10^{+5} rad s^{-1} it is possible to say that the present state of experimental art and the present level of detector theory do not predict severe obstacles to observing the gravitational waves from relatively rare collisions of black holes and nonsymmetric collapsing stars.

(II) The programme and conditions described above are suitable for a definite level of Ω – the frequency of black holes collisions or nonsymmetric collapse. If the real frequency is less, and therefore the experimentalists cannot observe enough events per year to convince one that they are seeing anything we shall never obtain information about gravitational radiation from nonsymmetric collapsing systems.

Another probable source of gravitational radiation exists. Zel'dovich and Polnarev (1973) showed that, if clusters exist in the center of our Galaxy, they may produce pulses of gravitational radiation with intensity $\tilde{I} \simeq 10^{-3}$–1 erg cm^{-2}, mean frequency $\omega_g \simeq 10^{-1}$–10 rad s^{-1}, and duration $\tau_g \simeq 1$ s. This type of pulses may occur approximately 25 times in a year.

It is evident that the type of detector described above is not suitable for the frequency range $\omega_g \simeq 10^{-1}$–10 rad s^{-1}. More fruitful perhaps is the idea (Braginsky and Hertzenstein, 1967) to use two free masses: the Earth and a drag-free satellite or two drag-free satellites with a Doppler ranging system between them.

The variation of the speed Δv_g of these two masses due to a gravitational wave pulse is equal to

$$\Delta v_g \simeq l \sqrt{\frac{8\pi G}{c^3} \frac{\tilde{I}}{\hat{\tau}_g}}, \tag{12}$$

where l is the distance between the masses.

Equation (12) is valid if $l \lesssim \pi c/\omega_g$. It is easy to show that the thermal noise condition similar to Equation (3) is not important for this type of antenna, and it is necessary only to construct a Doppler ranging system. Substituting in Equation (12) the values $\tilde{I}=1$ erg cm^{-2}, $\hat{\tau}_g=1$ s, $l=10^{12}$ cm, we obtain $\Delta v_g \simeq 2.5\times10^{-7}$ cm s^{-1}. The present level of sensitivity (Anderson, 1972) is $\Delta v \simeq 10^{-5}$ cm s^{-1}. Thus, if experimentalists can improve the sensitivity of Doppler ranging systems about two orders, then it will be possible to realize this type of gravitational wave antennae.

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