FIELD GALAXIES AND CLUSTER GALAXIES: ABUNDANCES OF MORPHOLOGICAL TYPES AND CORRESPONDING LUMINOSITY FUNCTIONS

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Abstract. With reference to theory published earlier, formulas are given for the estimation of (i) abundances of morphological types among field galaxies, (ii) of selection probabilities, and (iii) of 'space luminosity functions'. Strictly, the theory applies to 'homogeneous classes' of galaxies. This term designates a category of galaxies, say C, so finely defined that the probability, say $\Phi(m \mid C)$, that a galaxy of category C will be included in the catalogue depends on its photographic apparent magnitude m and on nothing else. The practical use of the theory is illustrated on data in the HMS Catalogue. It appears that certain combinations of the Hubble morphological types satisfy the definition of a homogeneous class. Such, for example, is the case for combinations of ellipticals E0–E3 and, separately, of spirals Sc, Scp, SBc. However, the combination of these two categories is not a homogeneous class.

In order to validate the theory empirically, calculations were performed to predict the abundances of eight combinations of morphological types among cluster galaxies listed in the HMS Catalogue, each combination being treated as a distinct homogeneous class. Additional hypotheses underlying these calculations are: (a) abundances of morphological types, (b) luminosity functions of these types, and (c) selection probabilities for cluster galaxies coincide with those for field galaxies. A comparison with the observations, reaching the value of $z=0.07$, is satisfactory. This tends to validate the combination of formulas (i), (ii), (iii) with the additional hypotheses (a), (b) and (c). Incidentally, the result tends to support the steady state cosmology.

1. Introduction

The theoretical statistical discussions that follow center around the astronomical problem of using the apparent magnitude and the redshift data in a given catalogue of galaxies in order to estimate the luminosity function of some specified type of galaxies. The difficulty is that every imaginable catalogue involves some selection of objects, the exact nature of the selection being not known *a priori*. Thus, one of the subproblems of the problem of luminosity functions consists in using the same catalogue data in order to gain information about the process of selecting galaxies for measurements of both the apparent magnitude and redshift.

The theoretical problem was solved by Neyman and Scott (1961) under certain special assumptions. Also, three sets of results of practical applications have been announced by Marcus (1962), by Neyman *et al.* (1962), and by Neyman and Scott (1962), but the methodology of using the theory has never been explained in detail. Because of the relevance of this methodology to certain problems of cosmology now widely discussed, some details are given in this paper.

2. Basic Concepts

To be realistic, a theory concerned with the effects of selection on the contents of a catalogue of galaxies must be based on plausible assumptions regarding the process

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of compiling the catalogue. Having in view the HMS Catalogue (Humason *et al.*, 1956), we visualized the following procedure.

Before determining the exact observational program, the cooperating astronomers examined the available survey plates and marked the objects which they felt could be observed without an excessive outlay of time and effort. One element that they were likely to consider must have been the apparent brightness of the objects, perhaps reflected in the apparent photographic magnitude say, m (not corrected for any effects). However, the apparent magnitude by itself does not determine the relative ease with which a given object could be observed. There are other features, such as the objects being diffuse or concentrated, etc. Thus, if one takes two galaxies G_1 and G_2 visible on a survey plate, both having the same photographic apparent magnitude, the chances of G_1 and G_2 being included in the catalogue may be very different.

In order to be able to use probability theory, we explicitly assume that the inclusion of a given galaxy in the particular catalogue is a chance event with a probability determined by the characteristics of the galaxy and by the process of compiling the catalogue. The following discussion is concerned with these probabilities.

The first basic assumption of our theory is that certain categories of galaxies can be defined so finely that, if G_1 and G_2 belong to the same category, say C, then the probabilities of their being included in the catalogue depend solely on their apparent photographic magnitudes. A category so defined is termed a *homogeneous class*. To each homogeneous class, then, say the tth class, there corresponds a function $\Phi(m \mid t)$ representing the probability that a galaxy of this class will be included in the catalogue. This function is termed the selection probability of class t.

Our second basic assumption is motivated by the terms 'field galaxies' and 'cluster galaxies'. The mathematical counterpart of the concept of field galaxies that we used is that the objects so labeled are members of 'clusters', each composed of a single object, and that the clustering of galaxies is of 'first order' in the sense of our earlier paper (1952). In practice, this means that the field galaxies are Poisson distributed in space. If the totality of galaxies studied is divided into a certain number s of homogeneous classes, then the number of galaxies of the tth class in a volume in space is a Poisson variable independent of others, with its expectation proportional to a number λ_t termed the abundance of class t field galaxies in space. These abundances add up to unity, $\sum_1^s \lambda_t = 1$. The discussion in the HMS Catalogue indicates that some of the objects treated as field galaxies were really members of small groups. We believe that a few such small group members would not invalidate the use of our theory.

Our third basic assumption regarding field galaxies was that all of them in the HMS Catalogue are relatively nearby objects, so that the correction of magnitudes for redshift and evolution can be neglected. In consequence, the relationship between the apparent and the absolute magnitude of a field galaxy is written as

$$m = M - 5 + 5 \log_{10} \xi, \tag{1}$$

where ξ stands for the distance, or with a change of the origin of coordinates and the use of natural logarithms, rather than those to the base 10,

$$m = M + a \log \xi \tag{2}$$

with $a = 5/\log 10$.

The last concept we must introduce is the distinction between the distributions of variables in space and in the catalogue. Ordinarily, the term luminosity function of a specified type of galaxies means what we statisticians call the probability density of a random variable $\mathcal{M}$, the absolute magnitude. This probability density could be 'observed' if it were possible to make a census of the particular galaxies. With reference to a specified homogeneous class of galaxies, this density is termed the 'space luminosity function' and denoted by $p_{\mathcal{M}}(M \mid t)$. Roughly speaking, this function, multiplied by the increment $\mathrm{d}M$, represents the probability that a galaxy will have its absolute magnitude between a specified value M and $M + \mathrm{d}M$.

The space luminosity function is contrasted with the 'catalogue luminosity function'. This contrast stems from the basic assumption that the inclusion of a galaxy in the catalogue is a chance event. The catalogue luminosity functions is denoted by $p^*_{\mathcal{M}}(M \mid t)$.

For each galaxy in the catalogue belonging to a particular homogeneous class we consider two random variables, the apparent magnitude denoted by μ (with particular values denoted by m) and the absolute magnitude $\mathcal{M}$ (with particular values denoted by M). The joint catalogue probability density of these two variables is denoted by $p^*_{\mu, \mathcal{M}}(m, M \mid t)$.

3. Fundamental Theorem

For any specified homogeneous class of galaxies the joint catalogue probability density of the apparent and the absolute magnitudes is given by the formula

$$p^*_{\mu, \mathcal{M}}(m, M \mid t) = C[\Phi(m \mid t)\, e^{3m/a}]\,[p_{\mathcal{M}}(M \mid t)\, e^{-3M/a}], \tag{3}$$

where C is a 'norming constant', such that the double integral for m and M from $-\infty$ *to* $+\infty$ *is equal to unity.*

The consequences of the fundamental theorem are rather important. One is that, in the catalogue, the apparent and the absolute magnitudes of galaxies belonging to the same homogeneous class are mutually independent; the probability density of the apparent magnitude being given by

$$p^*_{\mu}(m \mid t) = C_1 \Phi(m \mid t)\, e^{3m/a}, \tag{4}$$

and that of the absolute magnitude, by

$$p^*_{\mathcal{M}}(M \mid t) = C_2 p_{\mathcal{M}}(M \mid t)\, e^{-3M/a}, \tag{5}$$

where C_1 and C_2 are norming constants. Formula (5), then gives the catalogue luminosity function of the given homogeneous class of galaxies. The two Formulas (4) and (5) indicate that, once the catalogue densities of the apparent and of the absolute magnitude are estimated using the data in the catalogue, then Formula (5) will determine unambiguously the space luminosity function and Formula (4) the selection probability Φ, up to a multiplicative constant.

4. Practical Steps to Estimate the Selection Probability and the Space Luminosity Function

All the above results refer to galaxies of some particular homogeneous class. The first practical question to consider is whether in the real world any category of galaxies exists that, with a degree of interpretation and approximation, corresponds to the mathematical concept of a homogeneous class. The criterion is Formula (3) and the question is whether, for any specified category of field galaxies, the values of variables $\mathscr{M}$ and μ found in the catalogue show independence. Figures 1 and 2 give scatter diagrams of the apparent and absolute magnitudes (measured from an arbitrary origin) for two combinations of Hubble morphological types of field galaxies as found in the HMS Catalogue, not too elongated ellipticals and spirals Sc, Scp and SBc. After several tests indicated lack of dependence, we decided to treat these two categories as sufficiently approximating the concept of homogeneous classes. On the other hand, the examination of the two figures indicates that an attempt to treat the combination of all six morphological types as a single homogeneous class would be risky. As indicated

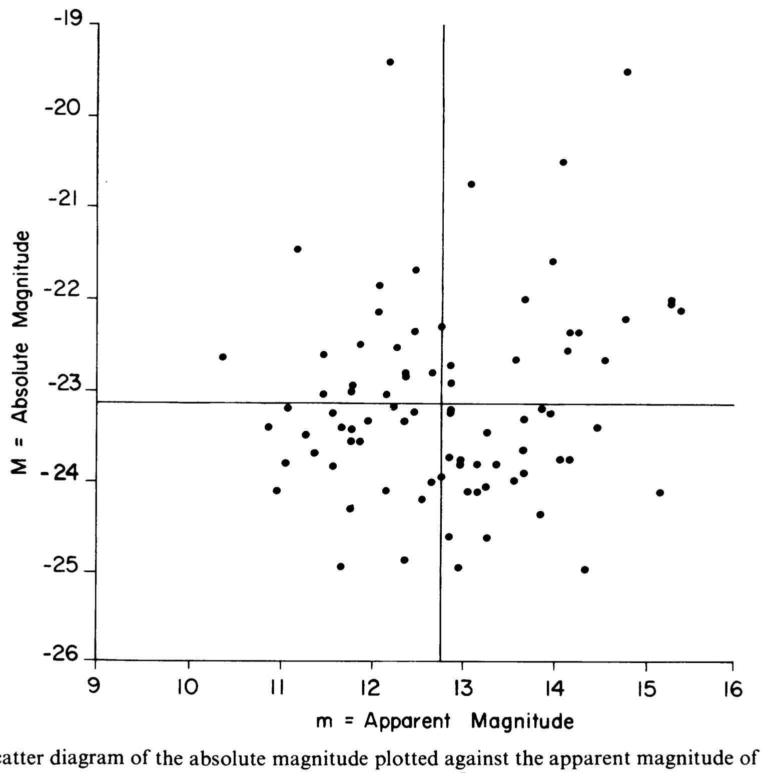

Fig. 1. Scatter diagram of the absolute magnitude plotted against the apparent magnitude of HMS field galaxies E0–E3. The means are $\bar{m} = 12.83$ and $\bar{M} = -23.13$ (arbitrary origin).

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by mean values, ($\bar{m}=12.83$, $\bar{M}=-23.13$) for ellipticals and ($\bar{m}=11.49$, $\bar{M}=-22.64$) for spirals, the superposition of the two scatter diagrams would have shown a negative correlation. Similar analysis led us to adopt as homogeneous classes the eight combinations of morphological types listed in the tables to be discussed below.

Having thus satisfied oneself that a given combination of morphological types of

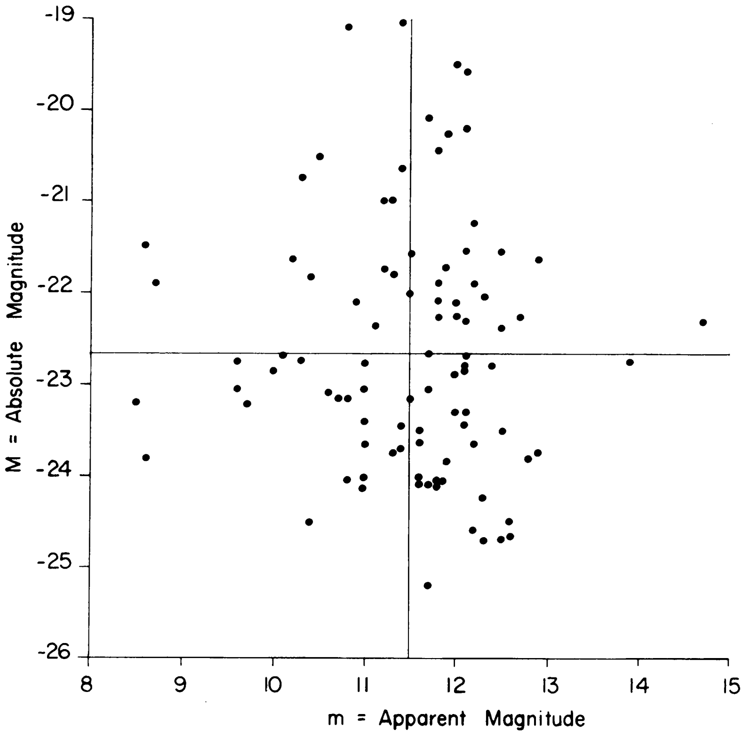

Fig. 2. Scatter diagram of the absolute magnitude plotted against the apparent magnitude of HMS field galaxies Sc, Scp, SBc. The means are $\bar{m}=11.49$ and $\bar{M}=-22.64$.

galaxies as listed in the given catalogue may be treated as a homogeneous class, what does one do to estimate its selection probability and its space luminosity function?

In both cases, one selects an interpolatory formula to fit the catalogue distribution separately of the apparent and separately of the absolute magnitudes of the given galaxies and then one uses Formulas (4) and (5) to obtain the desired estimates.

The process of estimating the space luminosity function is unambiguous. Having fitted $p^*_{\mathcal{M}}(M \mid t)$ directly from the data, all one has to do is to multiply it by $\exp(3M/a)$ and to norm so that the integral of the product taken from $-\infty$ to $+\infty$ is equal to unity. The estimation of $\Phi(m \mid t)$ is a little more complicated. Here again we have from (4)

$$\Phi(m \mid t)=p^*_{\mu}(m \mid t)\, C_1^{-1}\, e^{-3m/a}. \tag{6}$$

We emphasize that the constant C_1 is not uniquely determined and, for the same category of galaxies, may well vary from one catalogue to the next. All depends on the effort made in compiling the catalogue.

The simplest case is when it can be taken for granted that the astronomers compiling the catalogue make a special effort to include in it *all* the galaxies of the specified type bright enough for the observations to be possible without excessive expenditure of time and work. If this is so, then the constant C_1 in Formula (6) can be adjusted so that, as the value of m is decreased, the product on the right-hand side of (6) tends to unity. This can be conveniently done by selecting *a priori* a formula to represent, or to approximate Φ. The required properties of such a fomula are: that for small m it be close to unity, that it be strictly decreasing and tend to zero as m grows, that it be reasonably flexible and that it depend only on a few adjustable parameters, say on two of them. After adopting such a formula, all that is needed is to substitute it for Φ in (4) and to adjust the parameters involved so as to obtain the best fit to the empirical distribution of apparent magnitudes of galaxies in the given catalogue.

Our own choice of the function to represent the selection function Φ is

$$\Phi(m) = \int_{(m-\alpha)/\beta}^{\infty} \exp\{-x^2/2\}\, dx/(2\pi)^{1/2}, \tag{7}$$

where α and $\beta > 0$ are two adjustable parameters. After substituting (7) in (4), the best fitting values of α and β are found by the method of maximum likelihood. The appropriate equations are easy to write. However, they are somewhat messy and their solution requires the use of a digital computer.

Table I gives the values of parameters α and β, for eight combinations of morphological types which we treated as homogeneous classes. Also given in the table are constants characterizing the space luminosity function of the same categories of field galaxies, which we tentatively tried to approximate by the 'normal' distributions. Then M_0 designates the 'space mean' and σ the 'space standard deviation' of absolute magnitude.

TABLE I

Estimated parameters in the selection probabilities Φ and in the space luminosity functions of 8 categories of field galaxies

Category	α	β	M_0	σ
E0–E3	12.0	1.13	−18.0	1.02
E4–E7	10.9	1.30	−17.0	1.33
SBO	11.4	1.10	−18.5	0.58
SBb	11.2	1.10	−18.7	0.86
SO, SOp	11.7	1.10	−17.0	1.25
Sa, Sap, Sab	11.9	0.90	−18.4	0.82
Sb, Sbc	10.8	1.10	−17.9	1.05
Sc, Scp, SBc	11.6	0.75	−16.7	1.28

Note: M_0 is recorded using Hubble constant 100 km s^{-1} Mpc^{-1}.

The data in Table I are taken from Marcus (1962). They refer to the HMS Catalogue.

Of the two parameters in the formula for Φ, the first, α has the following interpretation. It represents that value of the apparent magnitude for which the probability of a galaxy being included in the given catalogue is exactly equal to $\frac{1}{2}$. The value of β determines the steepness of the curve representing Φ: the smaller the value of β is, the steeper the curve. Figure 3 was constructed to illustrate the implications of different values of α and β.

One circumstance that may appear surprising is that the selection probability for elongated ellipticals E4–E7 is so much lower and so much flatter than that for roundish ellipticals. Some time ago this circumstance was discussed with Rudolph Minkowski. To begin with he was skeptical. Later on, however, he inspected a list of some of his own observations of elliptical galaxies in a rather concentrated cluster (may have been the cluster 'around NGC 6166') and told us, with a degree of surprise, that really he could have observed one of the elongated galaxies just as easily as a round one, yet, without thinking about any particular reason, he observed the round elliptical. We mention this detail particularly in order to emphasize the fact that selection probabilities characterize not only the instruments used but also the unforseeable preferences of the observers. The two curves in Figure 3 relating to ellipticals illustrate the fact that the observers who compiled the HMS Catalogue somehow 'preferred' the 'round' to the 'flat' elliptical galaxies even if they are of exactly the same apparent photographic magnitude.

The difference between the Φ curves in Figure 3 corresponding to E0-E3 and to the spirals is most instructive. The two curves show that the very bright spirals, with

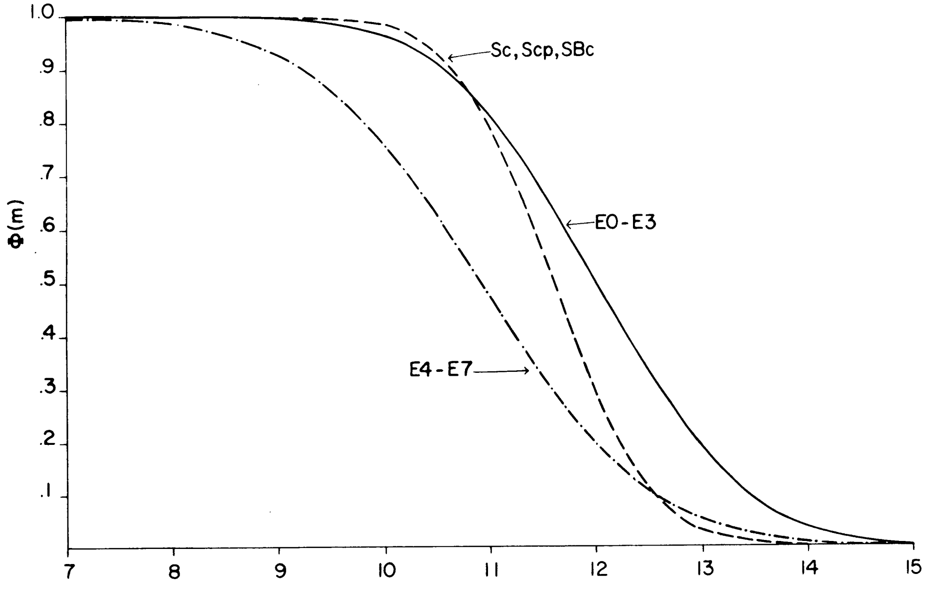

Fig. 3. Selection probabilities for 3 assumed homogeneous classes of field galaxies in the HMS catalogue.

$m < 10$ mag., had a better chance of being included in the catalogue than the equally bright ellipticals. However, with fainter magnitudes, say $m > 13$, the situation is changed radically. This detail of Figure 3 indicates that at substantial distances the proportion of spirals Sc, Scp, SBc included in the catalogue would be much smaller than that corresponding to normal ellipticals. This point will be referred to below.

5. Space Abundances of Morphological Types of Field Galaxies

As may be anticipated on intuitive grounds, the selection probabilities and the space luminosity function are of particular importance for estimating the space abundances λ_t. We visualize that the totality of field galaxies in a catalogue has been divided into s homogeneous classes, or types, and we denote by N_t the number of those objects belonging to the tth class. Then the formula yielding the estimate of the corresponding space abundance is

$$\lambda_t = \frac{N_t/x_t}{\sum_1^s N_i/x_i}, \tag{8}$$

where the symbol x_t designates the product of two integrals, each from $-\infty$ to $+\infty$,

$$x_t = I_t J_t, \tag{9}$$

one depending only on the selection probability and the other only on the space luminosity function,

$$I_t = \int \Phi(m \mid t) \exp\{3m/a\} \, \mathrm{d}m \tag{10}$$

$$J_t = \int p_{\mathcal{M}}(M \mid t) \exp\{-3M/a\} \, \mathrm{d}M. \tag{11}$$

Formulas characterizing the precision of these estimates are given in our original publication (Neyman and Scott, 1961). Combined with (4), (10) and (11), the Formula (8) has an easy intuitive interpretation. The more 'favorable' the selection probability Φ (that is, the larger I_t), and the brighter generally the particular type of galaxies (that is, the larger the integral J_t), the greater the overrepresentation of the given type of field galaxies in the catalogue, and vice versa.

Using the above formulas and the earlier estimates of selection probabilities, we computed the estimates of space abundances of the eight combined morphological types given in the third column of Table II, other details of which will be discussed in the next section. Here we notice that, while the number 83 of field galaxies of type E0–E3 in the HMS Catalogue is quite large, the space abundance of this particular type is very small, only 3.7%. On the other hand, while in the catalogue the number 94 of Sc, Scp, SBc type field galaxies is less than one quarter of the total, the estimated space abundance is practically 50%. These differences between abundances in the catalogue and in space are due to the combined effect of the relative brightness of the given type and of how 'favorable' its selection probability is.

6. Indirect Validation of the Theory

Given a degree of skill it is easy to write formulas of one kind or another and to claim that they correspond to some physical phenomena. Thus, after having obtained the results of our initial paper (Neyman and Scott, 1961), we had to face the problem of at least partial validation of the theory. One, and a quite convincing way of doing so would be to apply the theory to two different catalogues and see whether the estimates of space abundances of the various morphological types of field galaxies and also the corresponding space luminosity functions estimated through the use of the two catalogues would be similar, within the unavoidable chance variations. (Of course, the selection probabilities corresponding to two different catalogues would be naturally expected to be different.) We meant to perform such a test but, for a variety of resons, thus far we did not. For one thing, in the early 1960's there was no catalogue of galaxies comparable to the HMS in size, but having little overlap with it. In consequence, in company with W. Zonn, we attempted an indirect verification. This involved not only the theory leading to estimates of space luminosity functions, of selection probabilities and of space abundances as described above, but also some extraneous hypotheses. As we realized later, certain of these additional hypotheses are implicit in the steady state theory.

As is well known, in addition to data relating to field galaxies, the HMS Catalogue

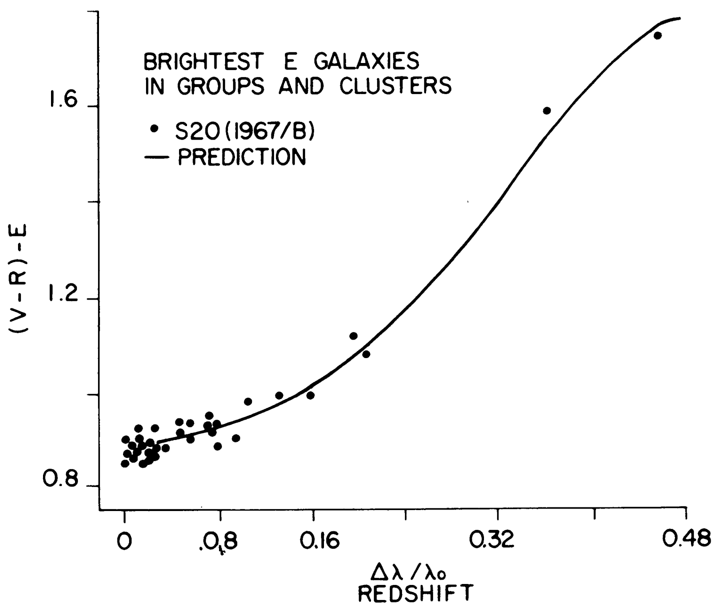

Fig. 4. Extent of one of the sets of data available to Sandage (1973).

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contains information about a number of small groups and about clusters. For Virgo and Coma the information is quite extensive; but for clusters beyond Coma the information is relatively scarce. We took into consideration all those systems for which the Catalogue contained data for at least 3 objects and classified them according to distance as indicated in Table II: 'near' groups, Virgo by itself, 'intermediate groups', Coma and 'far clusters'. The latter category included 5 systems: Perseus, 'around NGC 6166', Hercules, Pegasus II and Corona Borealis, with redshift values varying from 0.018 to 0.07. Figure 4, redrawn from Sandage (1973), provides a comparison between the great volume of data available to him and our 5 clusters.

The particular question we asked was: what would the catalogue percentages of the eight different categories of galaxies be in each of the 5 kinds of systems if (a) the space abundances of those categories in all the systems were the same as in the field, (b) if the luminosity functions in the systems (groups and clusters) were the same as in the field, and (c) if the selection probabilities were also the same as in the field?

The answer to this question is given in Table II reproduced from Neyman *et al.* (1962). In order to clarify the meaning of the table, a discussion of just one double column must suffice. The last double column refers jointly to the 5 'far' clusters enumerated above. The total number of objects belonging to the 5 clusters listed in the HMS catalogue is $n=48$. The last column in the table indicates that 47.9% of these 48 objects are of the type E0–E3. Our calculations performed as described above predicted a somewhat smaller percentage, 44.1 %, etc.

TABLE II

Percentage of galaxies of different morphological types

Morphological type	Field galaxies		Near groups $n=32$		Virgo $n=80$		Intermediate groups $n=72$		Coma $n=46$		'Far' clusters $n=48$	
	N_t in HMS	Space λ%	Exp	Obs	Exp	Obs	Exp	Obs	Exp	Obs	Exp	Obs
E0–E3	83	3.7	7.0	6.2	10.6	12.5	18.1	22.2	34.8	34.8	44.1	47.9
E4–E7.Ep	28	7.6	5.3	9.4	5.0	15.0	6.2	9.7	8.8	19.6	12.3	8.3
SBO, SBa	21	2.2	4.1	9.4	5.6	10.0	4.6	13.9	2.3	0.0	1.1	2.1
SBb	26	1.9	3.4	3.1	5.0	2.5	6.3	4.2	7.8	2.2	5.8	0.0
SO, SOp	66	11.5	12.5	6.2	13.7	15.0	16.1	23.6	20.5	28.3	21.0	14.6
Sa, Sap, Sab	51	3.3	7.1	12.5	11.2	8.8	11.6	11.1	8.8	8.7	4.3	18.7
Sb, Sbc	77	20.8	21.8	12.5	20.6	12.5	17.5	8.3	11.0	2.2	8.0	6.2
Sc, Scp, SBc	94	49.0	38.8	40.6	28.5	23.8	19.5	6.9	5.9	4.3	3.3	2.1
Correlation coefficient			0.90		0.73		0.32		0.88		0.90	

In order to judge the degree of correspondence between the theory just explained, on the one hand, and the observations, on the other, it is convenient to follow particular lines in the table. The first line, corresponding to E0–E3 ellipticals, beginning with 'near groups' and ending with 'far clusters', shows a clear cut tendency for an increase in the precentages, both expected and observed; there appears to be no striking systematic differences between prediction and observation. In the next to the last line of the table, there is again full agreement in the general tendency; in this case the percentages of spirals decrease rapidly from about 40% in near groups to 2 to 3% in the far clusters. Here, however, beginning with Virgo and beyond, the predicted percentages are somewhat higher than those observed. In one particular line, that corresponding to the type SBb, there is indicated systematic overestimation of this type. Undoubtedly, these systematic deviations (and also others noticeable in the table) are due to the fact that all the predictions in one line depend on what was found for the given category of objects in the field. In particular, if the space abundance in the field is overestimated, or underestimated, then this error would propagate itself all along the particular line in Table II. Our own degree of optimism is based on the similarity of tendencies in predicted and observed percentages. The correlation coefficients, shown on the last line, tend to be high except in the intermediate groups where the percentage of advanced spirals is overestimated.

In addition to the calculations reported in Table II, we performed others, also intended to provide a partial empirical verification of the theory. The results are shown in Table III, reproduced from an earlier announcement (Neyman and Scott, 1962). They refer to the same 5 categories of galaxy systems as in Table II but are concerned with the average photographic magnitudes of the objects listed in the HMS catalogue. The predictions are based on the several assumptions explained above, including the assumption that the selection probabilities for cluster objects are the same as for those in the field, for each type of galaxy.

TABLE III

Predicted and observed average photographic magnitudes in 5 systems of galaxies

System	n = No. galaxies in HMS	Mean radial velocity	Mean apparent magnitude	
			Predicted	Observed
Near groups	32	629	10.7	10.7
Virgo	80	1 197	11.6	11.4
Intermediate groups	72	2961	12.5	12.6
Coma	46	6866	13.9	14.8
Far clusters	48	11 696	14.5	16.0

It will be seen that for the first 3 systems, the agreement between prediction and observations is excellent. On the other hand, for Coma and for the far clusters the observed average apparent magnitude is increasingly fainter than the predicted. The intuitive explanation is that when clusters become objects of special interest, observers expended much more effort to observe cluster galaxies than those in the field. The natural consequence of this fact is that, for a given relatively faint value of m, a cluster galaxy had a better chance of being included in the catalogue than an equally faint field galaxy.

Our last remark refers to the relevance of the results reported to the steady state cosmology. As explained above, the omnipresent assumption underlying the predictions in Tables II and III is the identity of space abundances of morphological types and of space luminosity functions in systems of galaxies and in the field. If one grants that the predictions compare favorably with the observations, one is led to the conclusion that over the interval of look-back time studied neither the space abundances nor the luminosity functions of particular (combined) morphological types have changed very much, which is consistent with the steady state view. However, it may well be that the look-back time period covered is too short to expect substantial changes in these two characteristics of the population of galaxies.

It would be most satisfactory if the theory explained above could be applied unchanged to modern observations, say, of quasars. However, the chance mechanism involved in cataloguing such objects is likely to be quite different from that underlying the HMS data and a special study is indicated.

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DISCUSSION

Rudnicki: The catalogue of Humason *et al.* was completed when the concept of clusters of galaxies was not quite clear. According to the picture developed by Prof. Neyman and yourself, 100% of galaxies belong to clusters. What do you consider the field galaxies to be? I think they are just the galaxies which are not members of rich clusters. Am I right?

Scott: Humason *et al.* classified galaxies in their catalogue as field galaxies, group members or cluster members. We used their classification. In another indirect verification of our theory, which I did not have time to discuss, we find that *more* effort was indeed made to observe cluster galaxies (as is known).

We say that one can assume that all galaxies are members of clusters since we allow clusters of only one member. Field galaxies are then 'cluster members' where the cluster has only one member. This is perhaps a matter of semantics but it is convenient to consider clusters of 1, 2, 3, ... members, taking all possibilities together, when working out the theory. In any case, in this paper we used the HMS classification.