

# ON THE ELECTRICAL EFFECTS OF THE PRESENCE OF FLUFF ON THE SURFACE OF COSMIC DUST GRAINS

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ABSTRACT. We give an analytical model describing the effects of "fluff" on the potential and the electric field on and close to a charged spherical body embedded in an astrophysical plasma. The consequences are investigated for dust grains biased at positive or negative potentials but large enough for electron or ion field emission to be active.

## 1. INTRODUCTION

It is usual to determine the electric charge carried by cosmic dust grains embedded in astrophysical plasmas using the assumption that the grain is spherical. However, departure from spherical symmetry can produce some perturbations of the equipotential surfaces that have been underestimated. Thus, we consider a quasi-spherical rough grain with protrusions and recesses; it is assumed that the typical length scales of these irregularities are smaller than the average radius of the quasi-spherical grain and we discuss the topology of the equipotential surfaces and the value of the electric field in the environment of the grain. Since the scale length characterizing the roughness is smaller than the average grain radius, we analyse the effect of each protrusion (or recess) separately, considering that the total effect is the result of the superposition of all individual effects.

## 2. TWO POINT CHARGES MODEL FOR A SPHERE WITH A PROTRUSION

It has been shown elsewhere (Lafon and Millet, 1984a,b) that a simple and fairly good model for a spherical metallic grain with one protrusion is that delimited by one of the

equipotential surfaces of a system of two point charges, one small  $\alpha Q$  at some point  $D$  and the other larger  $(1-\alpha)Q$  ( $0 < \alpha \ll 1$ ) at some point  $O$  separated by the distance  $R(1+d)$  ( $d \ll 1$ ). The surface  $S$  of the grain is that at that at the potential  $V = 4\pi\epsilon_0 RQ$  of the sphere of radius  $R$  carrying the charge  $Q$ ; it is practically identical to the sphere centered at the point  $O$  (greater charge) with radius  $R$ , except close to the small charge, where it is strongly perturbed. The perturbation can be characterized by the maximum difference between the distance from  $O$  of any point of  $S$  and the radius  $R$  of the "equivalent" spherical grain, in units of  $R$ :  $x$ ;  $x$  is the scale length characterizing the protrusion.  $(1+x)R$  is the distance from  $O$  of the end of the protrusion, which is on the axis of symmetry  $OD$ . The degree of attachment of the protrusion can be characterized by a parameter  $g = x^2/\alpha$ , that can be of the order of a few units.

Of course, if  $R$  and  $Q$  are used as units of length and charge,  $\alpha$  and  $d$  are sufficient to fully characterize the model; conversely, for given  $x$  and  $g$

$$\alpha = x^2/g \quad d = x(g-1)/(g+x)$$

Thus, a couple  $x, g$  is also sufficient to fully characterize the model.

However,  $g$  must be smaller than some  $g_m$ , which is a function of  $x$  practically constant and close to 5.8 if  $x \ll 1$ ). Otherwise, the equipotential surface is no longer a deformed sphere, but it is made of two separated lobes, which has no longer any physical meaning for the problem of the protrusion.

Fig. 1 illustrates the meridian sections of the equipotential surfaces of a system of two point charges, in the neighbourhood of the small charge (the straight line  $OD$  defined by the two point charges is an axis of symmetry for the system). In this case  $x = 0.05$  and all lengths are measured in units of  $R$  in the case where  $g = 7$ . The lower index of the curves is the potential  $V_S$  of the equipotential surface, in units of  $V = Q/4\pi\epsilon_0 R$ . The upper index is the corresponding  $g$ . There is a scaling law that says that the equipotential surfaces indexed by values of  $g \neq 7$  and corresponding  $V_S \neq 1$  can be changed into equipotential surfaces at potential 1 by scaling lengths by a factor  $1/V_S$  (Lafon and Millet, 1984a). Small  $g$  correspond to weakly protruding bumps, whereas high  $g$  correspond to "loosely attached" protrusions.

Fig. 2 shows the geometry of the "two point charges" model.  $\lambda$  denotes the distance of any point  $M$  from the small point charge  $\alpha Q$  at  $D$ .

At any point on the surface of the grain, the electric field can be expressed as a function of  $\lambda$  (note that though the derivations are correct the final expression of  $E^2$  is not correct in Lafon and Millet, 1984a)

$$E^2 = \frac{1}{(1-\alpha)^2 \lambda^4} \left\{ (\lambda-\alpha)^4 + \alpha^2 (1-\alpha)^2 - \frac{\alpha(1+d)^2 (\lambda-\alpha)^3}{\lambda^2} \right. \\ \left. + \alpha(\lambda-\alpha) \left[ (\lambda-\alpha)^2 + (1-\alpha)^2 \right] \right\}$$

where E is measured in units of  $Q/4\pi\epsilon_0 R^2$ .

At the end of the protrusion,  $P$ , on the axis of symmetry, this yields

$$E_P = \frac{1-\alpha}{(1+x)^2} \left( 1 + \frac{(x+\alpha)^2}{\alpha(1-\alpha)} \right) \\ E_P \approx (1-2x)(1+g) \quad \text{for } x \ll 1 \text{ and } g = O(1)$$

so that  $g$  characterizes also the enhancement of the electric field on the protrusion.

Finally, note that, as explained in Lafon and Millet 1984a, this model, which is based on the theory of electric images, is strictly valid for metals but it is also practically valid with high accuracy for materials with high enough relative permittivity  $\epsilon$ , "high enough" meaning  $\epsilon > 3$ , i.e. for most of the insulators usually expectable in astrophysical grains.

### 3. DISCUSSION OF THE MODEL

Assuming that the small charge is a point charge we have described the fundamental features due to a protrusion with only two parameters,  $x$  for the radial departure from the spherical symmetry and  $g$  for the shape of the bump. Now we investigate the effects of this simplification using more complicated models of the same type.

First assume (Fig. 3) that the small charge  $\alpha Q$  is replaced by a small charge  $\alpha_N Q$  evenly distributed along a needle with length  $2L$  aligned on the axis of symmetry  $OD$ , and centered at the same distance  $OD = R(1+d)$  than the point charge. Assume also that the resulting projection is salient in a similar way, i.e. the distance  $(1+x)R$  between  $O$  and the of the protrusion on  $OD$  is not changed. Then a tedious but straight forward analytical discussion of the equations leads to the following results (the method is sketched in Lafon and Millet, 1984b)

$\alpha_N < \alpha$   
The maximum admissible  $g$ ,  $g_N$  is slightly reduced  
For small needles ( $L \ll x-d$ ),  $\alpha_N = \alpha - \delta\alpha$  with  $\delta\alpha = O(L^2/(x-d))$ . Moreover, the electric field at the end of the protrusion is increased by a small amount of the second order in  $L$

$$dE = \frac{2\alpha}{3(x-d)^2} \frac{L^2}{(x-d)^2}$$

In a similar way, assume (Fig. 4) that the small

charge  $\alpha Q$  is replaced by a small charge  $\alpha_D Q$  evenly distributed over a disc with radius  $L$ , perpendicular to the axis of symmetry  $OD$ , centered at the same distance  $OD = (1+d)R$  as the point charge. Assume also that the resulting protrusion is still salient in a similar way:  $OP = (1+d)R$  unchanged. A similar analysis leads to the following results:

$\alpha_D > \alpha$   
 The maximum admissible  $g$ ,  $g_m$  is slightly reduced  
 For small discs ( $L \ll x-d$ )  $\alpha_D = \alpha - \delta\alpha$  with  $\delta\alpha =$   
 $O(L^2/(x-d)^2)$ . Moreover, the electric field at the end  $P$  of  
 the protrusion is reduced by a small amount of the second  
 order in  $L$ :

$$dE = - \frac{1}{2} \frac{\alpha}{(x-d)^2} \frac{L^2}{(x-d)^2}$$

To summarize, more sophisticated charge distributions will describe protrusions with different shapes but the description will be qualitatively unchanged and quantitatively weakly perturbed. The parameters  $x$  and  $g$  are sufficient to account for the fundamental physical phenomena.

#### 4. THE ELECTRICAL EFFECTS OF FLUFF

The main results of the previous sections is that even small protrusions ( $x \ll 1$ ) can produce large enhancements of the electric field, (multiplication by a factor of the order of  $1+g$ , where  $g$  is independent of  $x$  and can be of the order of a few units). On the other hand, an asymptotic analysis (Lafon and Millet, 1984a) shows that the equipotential surfaces surrounding a grain with a protrusion characterized by  $x, g$  becomes spherically symmetric at distances from  $O$  of the order of  $R(1 + x(1+g))$  i.e. close to the grain surface, independently of  $x$ .

Now, note that a recess can be described with a similar model but with  $\alpha < 0$ . However, the electric field is reduced close to a recess, which produces less noticeable effects.

Finally, the important parameters for a fluffy grain are the greatest  $x$  and  $g$  characterizing the sizes and the degrees of attachment of its protrusions.

The protrusions will be important for charging/discharging processes sensitive to the local electric field, like the field emission (for which a small enhancement of the local field increases the emitted current by orders of magnitude), but not for those sensitive to the potential structure (like the collection of plasma particles). In particular, the presence of fluff can enlarge significantly the range of particle sizes for which the field emission fixes an upper bound for the magnitude of the potential. The field emission limited potential of a grain with protrusions characterized by  $x, g$  will be lower by a factor  $(1-2x)(1+g) \approx 1+g$

than for a smooth grain. Fig. 5 (from Lafon and Millet, 1984 a) illustrates this effect under magnetospheric conditions.

Of course, the violent discharge of fluffy charged grains by field emission (Lafon and Millet, 1983) can be triggered for larger grains (with sizes larger by a factor  $\approx 1+g$ ).

For positively charged grains, the emission of ions produces mass loss from the emitting site. Strong field emission reduces the lifetime of a grain. In other words, the the grain can survive so far as ion field emission is not active. Fluffy grains will tend to loose their fluff and have smooth surfaces; they can even be destroyed if their potential becomes excessive for their size. Smaller grains will be sensitive to such effects when fluffy.

## REFERENCES

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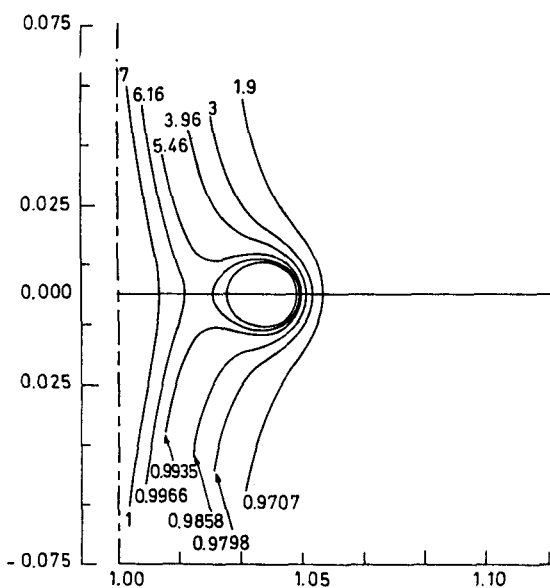


Fig. 1 Meridian sections of the equipotential surfaces around a sphere with a protrusion for fixed  $x$ ,  $g$  (or  $x, z$ ) and various  $d$  indexed by the value of the potential  $V_s$  (lower part of the figure). They are also the meridian section of the surface of spheres with protrusions for various  $g$  and same  $x$  after scaling by the factor  $1/V_s$ . The indexes of the upper part of the figure are the corresponding values of  $g$ . Here  $x=0.05$ ,  $g=7$

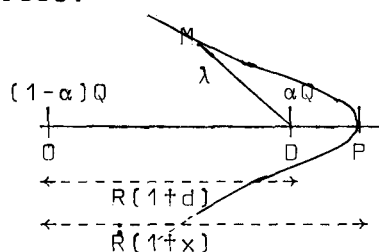


Fig. 2



Fig. 3

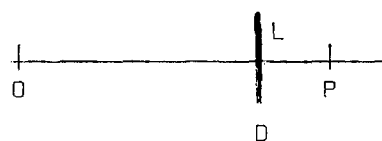


Fig. 4

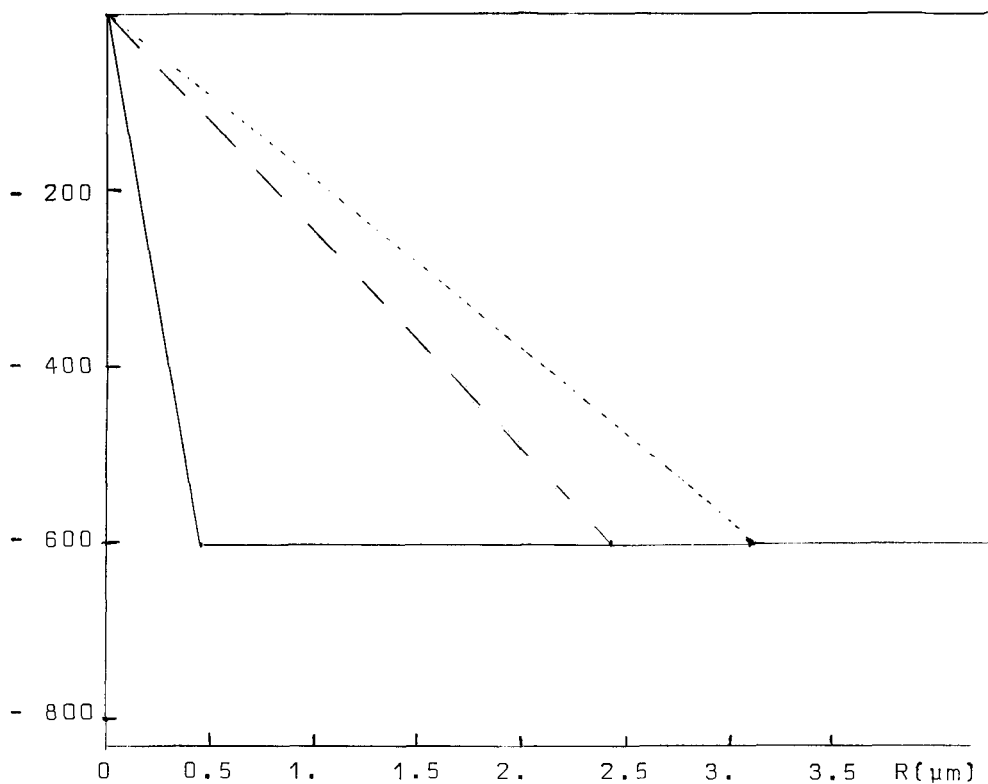


Fig. 5 Potential of fluffy (dashed or dotted curves) and smooth (solid curve) spherical dust grains as functions of their radius  $R$  under the conditions of the terrestrial magnetosphere.  $g$  is taken equal to 4 (dashed curve) and 5.5 (dotted curve).  $x = 0.05$   
From Lafon and Millet, 1984 a,b