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ASTIN Bulletin

43(1), 2013

BERMÚDEZ, LLUÍS; FERRI, ANTONI; GUILLÉN, MONTSERRAT. *A correlation sensitivity analysis of non-life underwriting risk in solvency capital requirement estimation*. 21-37. This paper analyses the impact of using different correlation assumptions between lines of business when estimating the risk-based capital reserve, the solvency capital requirement (SCR), under Solvency II regulations. A case study is presented and the SCR is calculated according to the standard model approach. Alternatively, the requirement is then calculated using an internal model based on a Monte Carlo simulation of the net underwriting result at a one-year horizon, with copulas being used to model the dependence between lines of business. To address the impact of these model assumptions on the SCR, we conduct a sensitivity analysis. We examine changes in the correlation matrix between lines of business and address the choice of copulas. Drawing on aggregate historical data from the Spanish non-life insurance market between 2000 and 2009, we conclude that modifications of the correlation and dependence assumptions have a significant impact on SCR estimation.

HAPP, SEBASTIAN; WÜTHRICH, MARIO V. *Paid-incurred chain reserving method with dependence modeling*. 1-20. The paid-incurred chain (PIC) reserving method is a claims reserving method that allows to combine claims payments and incurred losses information in a mathematical consistent way. The main criticism on the original Bayesian log-normal PIC model presented in Merz-Wüthrich [5] is that it does not respect dependence properties within the observed data. In the present paper, we extend the original Bayesian log-normal PIC model so that dependence is modeled in an appropriate way.

KALUSZKA, MAREK; KRZESZOWIEC, MICHAL. *An iterativity condition for the mean-value principle under cumulative prospect theory*. 61-71. In this paper, we present the full characterization of the iterativity condition for the mean-value principle under the cumulative prospect theory. It turns out that the premium principle is iterative for exactly six pairs of probability distortion functions. Some of the corresponding premium principles are the classical mean-value principle, essential infimum or essential supremum of the random loss. Moreover, from the proof of the main theorem of this paper, it follows that the iterativity of the mean-value principle is equivalent to the iterativity of the generalized Choquet integral.

SHI, TIANXIANG; LANDRIAULT, DAVID. *Distribution of the time to ruin in some Sparre Andersen risk models*. 39-59. The finite-time ruin problem, which implicitly involves the inversion of the Laplace transform of the time to ruin, has been a long-standing research problem in

risk theory. Existing results in the Sparre Andersen risk models are mainly based on an exponential assumption either on the interclaim times or on the claim sizes. In this paper, we utilize the multivariate version of Lagrange expansion theorem to obtain a series expansion for the density of the time to ruin under a more general distribution assumption, namely the combination of n exponentials. A remark is further made to emphasize that this technique can also be applied to other areas of applied probability. For instance, the proposed methodology can be used to obtain the distribution of some first passage times for particular stochastic processes. As an illustration, the duration of a busy period in a queueing risk model will be examined.

ASTIN Bulletin

43(2), 2013

ALBRECHER, HANSJÖRG; LAUTSCHAM, VOLKMAR. *From ruin to bankruptcy for compound poisson surplus processes*. 213-243. In classical risk theory, the infinite-time ruin probability of a surplus process Ct is calculated as the probability of the process becoming negative at some point in time. In this paper, we consider a relaxation of the ruin concept to the concept of bankruptcy, according to which one has a positive surplus-dependent probability to continue despite temporary negative surplus. We study the resulting bankruptcy probability for the compound Poisson risk model with exponential claim sizes for different bankruptcy rate functions, deriving analytical results, upper and lower bounds as well as an efficient simulation method. Numerical examples are given and the results are compared with the classical ruin probabilities. Finally, it is illustrated how the analysis can be extended to study the discounted penalty function under this relaxed ruin criterion.

BIAGINI, FRANCESCA; RHEINLÄNDER, THORSTEN; WIDENMANN, JAN. *Hedging mortality claims with longevity bonds*. 123-157. We study mean-variance hedging of a pure endowment, a term insurance and general annuities by trading in a longevity bond with continuous rate payments proportional to the survival probability. In particular, we discuss the introduction of a gratification annuity as an interesting insurance product for the life insurance market. The optimal hedging strategies are determined via their Galtchouk-Kunita-Watanabe decompositions under specific, yet sufficiently general model assumptions. The results are then further illustrated by assuming a general affine structure of the mortality intensity process. The optimal hedging strategies as well as the residual hedging error of a gratification annuity and a simple life annuity are finally investigated with numerical simulations, which illustrate the nice features of the gratification annuity for the insurance industry.

BOLTHAUSEN, ERWIN; WÜTHRICH, MARIO V. *Bernoulli's law of large numbers*. 73-79. This year we celebrate the 300th anniversary of Jakob Bernoulli's path-breaking work Ars conjectandi, which appeared in 1713, eight years after his death. In Part IV of his masterpiece, Bernoulli proves the law of large numbers which is one of the fundamental theorems in probability theory, statistics and actuarial science. We review and comment on his original proof.

EMBRECHTS, PAUL; HOFERT, MARIUS. *Statistical inference for copulas in high dimensions: a simulation study*. 81-95. Statistical inference for copulas has been addressed in various research papers. Due to the complicated theoretical results, studies have been carried out mainly in the bivariate case, be it properties of estimators or goodness-of-fit tests. However, from a practical point of view, higher dimensions are of interest. This work presents the results of large-scale

simulation studies with particular focus on the question to what extent dimensionality influences point and interval estimators.

MAURER, RAIMOND; ROGALLA, RALPH; SIEGELIN, IVONNE. *Participating payout life annuities: lessons from Germany*. 159-187. This paper analyzes the framework of German participating payout life annuities (PLAs), which offer guaranteed minimum benefits as well as participation in insurers' surpluses. We show that the process of sharing surpluses between shareholders and policyholders follows transparent and consistent rules. Subsequently, we develop an asset-liability model for a stylized German life insurer that offers PLAs to evaluate benefit variability and insurer stability given stochastic mortality and capital market developments. Our results suggest that guaranteed benefits can be provided with high credibility via PLAs, while, at the same time, annuitants receive attractive money's worth ratios. Moreover, we show that it might be difficult to offer a fixed benefit annuity providing the same lifetime utility as a PLA for the same premium and a comparably low insolvency risk. Overall, PLA schemes may be an efficient way to deal with risk factors that are highly unpredictable and difficult to hedge over the long run, such as systematic longevity and investment risks.

NGUYEN, TILO; SAMORODNITSKY, GENNADY. *Multivariate tail estimation with application to analysis of CoVaR*. 245-270. The quality of estimation of multivariate tails depends significantly on the portion of the sample included in the estimation. A simple approach involving sequential statistical testing is proposed in order to select which observations should be used for estimation of the tail and spectral measures. We prove that the estimator is consistent. We test the proposed method on simulated data, and subsequently apply it to analyze CoVaR for stock and index returns.

TSAI, JEFFREY T; TZENG, LARRY Y. *The pricing of mortality-linked contingent claims: an equilibrium approach*. 97-121. This study introduces an equilibrium approach to price mortality-linked securities in a discrete time economy, assuming that the mortality rate has a transformed normal distribution. This pricing method complements current studies on the valuation of mortality-linked securities, which only have discrete trading opportunities and insufficient market trading data. Like the Wang transform, the valuation relationship is still risk-neutral (preference-free) and the mortality-linked security is priced as the expected value of its terminal payoff, discounted by the risk-free rate. This study provides an example of pricing the Swiss Re mortality bond issued in 2003 and obtains an approximated closed-form solution.

WILLMOT, GORDON E; WOO, JAE-KYUNG. *Some distributional properties of a class of counting distributions with claims analysis applications*. 189-212. We discuss a class of counting distributions motivated by a problem in discrete surplus analysis, and special cases of which have applications in stop-loss, discrete Tail value at risk (TVaR) and claim count modelling. Explicit formulas are developed, and the mixed Poisson case is considered in some detail. Simplifications occur for some underlying negative binomial and related models, where in some cases compound geometric distributions arise naturally. Applications to claim count and aggregate claims models are then given.

ASTIN Bulletin

43(3), 2013

BAYRAKTAR, ERHAN; KYPRIANOU, ANDREAS E; YAMAZAKI, KAZUTOSHI. *On optimal dividends in the dual model*. 359-372. We revisit the dividend payment problem in the dual model

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of Avanzi *et al.* Using the fluctuation theory of spectrally positive Lévy processes, we give a short exposition in which we show the optimality of barrier strategies for all such Lévy processes. Moreover, we characterize the optimal barrier using the functional inverse of a scale function. We also consider the capital injection problem and show that its value function has a very similar form to the one in which the horizon is the time of ruin.

CHRISTIANSEN, MARCUS C; STEFFENSEN, MOGENS. *Safe-side scenarios for financial and biometrical risk*. 323-357. Premium settlement and calculation of reserves and capital requirements are typically based on worst- or just bad-case assumptions on interest rates, mortality rates, and other transition rates between states defined according to the insurance benefits. If interest and transition rates are chosen independently from each other, the worst choice, i.e. the combination of interest rates and transition rates that maximizes the reserve, can be found by dynamic programming. Here, we generalize this idea by choosing the interest and transition rates from a set that allows for mutual dependence. In general, finding the worst case is much more complicated in this situation, but we characterize a set of relatively tractable problems and present a series of examples from this set. Our approach with mutual dependence is relevant e.g. for internal models in Solvency II.

CONG, JIANFA; TAN, KEN SENG; WENG, CHENGGUO. *VaR-based optimal partial hedging*. 271-299. Hedging is one of the most important topics in finance. When a financial market is complete, every contingent claim can be hedged perfectly to eliminate any potential future obligations. When the financial market is incomplete, the investor may eliminate his risk exposure by superhedging. In practice, both hedging strategies are not satisfactory due to their high implementation costs, which erode the chance of making any profit. A more practical and desirable strategy is to resort to the partial hedging, which hedges the future obligation only partially. The quantile hedging of Föllmer and Leukert (Finance and Stochastics, vol. 3, 1999, pp. 251-273), which maximizes the probability of a successful hedge for a given budget constraint, is an example of the partial hedging. Inspired by the principle underlying the partial hedging, this paper proposes a general partial hedging model by minimizing any desirable risk measure of the total risk exposure of an investor. By confining to the value-at-risk (VaR) measure, analytic optimal partial hedging strategies are derived. The optimal partial hedging strategy is either a knock-out call strategy or a bull call spread strategy, depending on the admissible classes of hedging strategies. Our proposed VaR-based partial hedging model has the advantage of its simplicity and robustness. The optimal hedging strategy is easy to determine. Furthermore, the structure of the optimal hedging strategy is independent of the assumed market model. This is in contrast to the quantile hedging, which is sensitive to the assumed model as well as the parameter values. Extensive numerical examples are provided to compare and contrast our proposed partial hedging to the quantile hedging.

MILHAUD, XAVIER. *Exogenous and endogenous risk factors management to predict surrender behaviours*. 373-398. Insurers have been concerned about surrenders for a long time especially in saving business, where huge sums are at stake. The emergence of the European directive Solvency II, which promotes the development of internal risk models (among which a complete unit is dedicated to surrender risk management), strengthens the necessity to deeply study and understand this risk. In this paper, we investigate the topics of segmenting and modelling surrenders in order to better take into account the main risk factors impacting policyholders' decisions. We find that several complex aspects must be specifically dealt with to predict surrenders, in particular the heterogeneity of behaviour as well as the context faced by the insured. Combining them, we

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develop a new methodology that seems to provide good results on given business lines, and that moreover can be adapted for other products with little effort.

PIGEON, MATHIEU; ANTONIO, KATRIEN; DENUIT, MICHEL. *Individual loss reserving with the multivariate skew normal framework*. 399-428. The evaluation of future cash flows and solvency capital recently gained importance in general insurance. To assist in this process, our paper proposes a novel loss reserving model, designed for individual claims developing in discrete time. We model the occurrence of claims, as well as their reporting delay, the time to the first payment, and the cash flows in the development process. Our approach uses development factors similar to those of the well-known chain-ladder method. We suggest the Multivariate Skew Normal distribution as a multivariate distribution suitable for modeling these development factors. Empirical analysis using a real portfolio and out-of-sample prediction tests demonstrate the relevance of the model proposed.

TSANAKAS, ANDREAS; WÜTHRICH, MARIO V; CERNÝ, ALEŠ. *Market value margin via mean-variance hedging*. 301-322. We use mean-variance hedging in discrete time in order to value an insurance liability. The prediction of the insurance liability is decomposed into claims development results, that is, yearly deteriorations in its conditional expected values until the liability is finally settled. We assume the existence of a tradeable derivative with binary pay-off written on the claims development result and available in each development period. General valuation formulas are stated and, under additional assumptions, these valuation formulas simplify to resemble familiar regulatory cost-of-capital-based formulas. However, adoption of the mean-variance framework improves upon the regulatory approach by allowing for potential calibration to observed market prices, inclusion of other tradeable assets, and consistent extension to multiple periods. Furthermore, it is shown that the hedging strategy can also lead to increased capital efficiency.

ASTIN Bulletin

44(1), 2014

BÖRGER, MATTHIAS; FLEISCHER, DANIEL; KUKSIN, NIKITA. *Modeling the mortality trend under modern solvency regimes*. 1-38. Stochastic modeling of mortality/longevity risks is necessary for internal models of (re)insurers under the new solvency regimes, such as Solvency II and the Swiss Solvency Test. In this paper, we propose a mortality model which fulfills all requirements imposed by these regimes. We show how the model can be calibrated and applied to the simultaneous modeling of both mortality and longevity risk for several populations. The main contribution of this paper is a stochastic trend component which explicitly models changes in the long-term mortality trend assumption over time. This allows to quantify mortality and longevity risk over the one-year time horizon prescribed by the solvency regimes without relying on nested simulations. We illustrate the practical ability of our model by calculating solvency capital requirements for some example portfolios, and we compare these capital requirements with those from the Solvency II standard formula.

CHI, YICHUN; LIN, X SHELDON. *Optimal reinsurance with limited ceded risk: A stochastic dominance approach*. 103-126. An optimal reinsurance problem from the perspective of an insurer is studied in this paper, where an upper limit is imposed on a reinsurer's expected loss over a prescribed level. In order to reduce the moral hazard, we assume that both the insurer and the

reinsurer are obligated to pay more as the amount of loss increases in a typical reinsurance treaty. We further assume that the optimization criterion preserves the convex order. Such a criterion is very general as most of the criteria for optimal reinsurance problems in the literature preserve the convex order. When the reinsurance premium is calculated as a function of the actuarial value of coverage, we show via a stochastic dominance approach that any admissible reinsurance policy is dominated by a stop-loss reinsurance or a two-layer reinsurance, depending upon the amount of the reinsurance premium. Moreover, we obtain a similar result to Mossin's Theorem and find that it is optimal for the insurer to cede a loss as much as possible under the net premium principle. To further examine the reinsurance premium for the optimal piecewise linear reinsurance policy, we assume the expected value premium principle and derive the optimal reinsurance explicitly under (1) the criterion of minimizing the variance of the insurer's risk exposure, and (2) the criterion of minimizing the risk-adjusted value of the insurer's liability where the liability valuation is carried out using the cost-of-capital approach based on the conditional value at risk.

GUERREIRO, GRACINDA RITA; MEXIA, JOÃO TIAGO; MIGUENS, MARIA DE FÁTIMA. *Statistical approach for open bonus malus*. 63-83. In this paper, following an open portfolio approach, we show how to estimate a Bonus-malus system evolution. Considering a model for the number of new annual policies, we obtain ML estimators, asymptotic distributions and confidence regions for the expected number of new policies entering the portfolio in each year, as well as for the expected number and proportion of insureds in each bonus class, by year of enrollment. Confidence regions for the distribution of policyholders result in confidence regions for optimal bonus scales. Our treatment is illustrated by an example with numerical results.

NIRMALENDRAN, MAATHUMAI; SHERRIS, MICHAEL; HANEWALD, KATJA. *Pricing and solvency of value-maximizing life annuity providers*. 39-61. This paper provides a detailed quantitative assessment of the impact of capital and default probability on product pricing and shareholder value for a life insurer providing life annuities. A multi-period cash flow model, allowing for stochastic mortality and asset returns, imperfectly elastic product demand, as well as frictional costs, is used to derive value-maximizing capital and pricing strategies for a range of one-year default probability levels reflecting differences in regulatory regimes including Solvency II. The model is calibrated using realistic assumptions. The sensitivity of results is assessed. The results show that value-maximizing life insurers should target higher solvency levels than the Solvency II regulatory one-year 99.5% probability under assumptions of reasonable levels of policyholder's aversion to insolvency risk. Even in the case of less restrictive solvency probabilities, policyholder price elasticity and solvency preferences are shown to be important factors for a life insurer's value-maximizing strategy.

RIEGEL, ULRICH. *A bifurcation approach for attritional and large losses in chain ladder calculations*. 127-172. We introduce a stochastic model for the development of attritional and large claims in long-tail lines of business and present a corresponding "chain ladder-like" IBNR method which allows the use of claims payment data for attritional and claims incurred data for large losses. We derive formulas for the mean squared error of prediction and apply the method to a German motor third party liability portfolio.

SHI, PENG. *A copula regression for modeling multivariate loss triangles and quantifying reserving variability*. 85-102. This article proposes a claims reserving model for dependent lines of business with the accommodation of association among triangles by a copula function. We show that the family of elliptical copulas is a pretty convenient choice to capture the dependencies introduced by

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various sources, including the common calendar year effects. To quantify the associated reserving variability, we resort to parametric bootstrapping techniques for simulating the predictive distribution of outstanding liabilities and for calculating the three components of predictive uncertainty: the model error, the process error and the estimation error. Numerical analysis is performed for a portfolio of casualty insurance from a major U.S. insurer.

ASTIN Bulletin

44(2), 2014

CHANG, CHIA-CHIEN. *Valuation of mortgage insurance contracts with counterparty default risk: reduced-form approach*. 303-334. In the recent subprime mortgage crisis, which has caused banks and insurance companies to go bankrupt or into acquisition, the lender and insurer have exhibited not only correlated defaults when exposed to common risk factors but also counterparty default risk, which is triggered by mortgage defaults. Given the correlated defaults and the counterparty default risk, we use the reduced-form approach to derive the closed-form formulas of mortgage insurance contracts with premium refunds, annual premiums and upfront premiums. Regardless of the nature of the premium structures, the numerical analysis with parameter calibration demonstrates that both the correlated defaults and the counterparty default risk significantly impact mortgage insurance premiums, particularly in long-term mortgage loans.

DEELSTRA, GRISELDA; RAYÉE, GRÉGORY; VANDUFFEL, STEVEN; YAO, JING. *Using model-independent lower bounds to improve pricing of asian style options in lévy markets*. 237-276. Albrecher *et al.* [H. Albrecher, P. Mayer, W. Schoutens. General lower bounds for arithmetic Asian option prices. Applied Mathematical Finance (2008), *15: 123-149]* have proposed model-independent lower bounds for arithmetic Asian options. In this paper we provide an alternative and more elementary derivation of their results. We use the bounds as control variates to develop a simple Monte Carlo method for pricing contracts with Asian-style features. The conditioning idea that is inherent in our approach also inspires us to propose a new semi-analytic pricing approach. We compare both approaches and conclude that these both have their merits and are useful in practice. In particular, we point out that our newly proposed Monte Carlo method allows to deal with Asian-style products that appear in insurance (e.g., unit-linked contracts) in a very efficient way, and outperforms other known Monte Carlo methods that are based on control variates.

DONNELLY, CATHERINE; ENGLUND, MARTIN; NIELSEN, JENS PERCH. *The importance of the choice of test for finding evidence of asymmetric information*. 173-195. We put one of the predictions of adverse-selection models to the test, using data from the Danish automobile insurance market: that there is a positive correlation between claims risk and insurance coverage. We can find a statistically significant insurance coverage–risk correlation when coverage is expressed relative to the insurance premium, but not when it is expressed in monetary terms.

GRAF, STEFAN; HAERTEL, LENA; KLING, ALEXANDER; RUß, JOCHEN. *The impact of inflation risk on financial planning and risk-return profiles*. 335-365. The importance of funded private or occupational old-age provision is expected to increase due to demographic changes and the resulting problems for government-run pay-as-you-go systems. Clients and advisors therefore need reliable methodologies to match offered products with clients' needs and risk appetite. In Graf *et al.* (2012), the authors have introduced a methodology based on stochastic modeling to properly assess the risk-return profiles – i.e. the probability distribution of future benefits – of

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various old-age provision products. In this paper, we additionally consider the impact of inflation on the risk-return profile of old-age provision products. In a model with stochastic interest rates, stochastic inflation and equity returns including stochastic equity volatility, we derive risk-return-profiles for various types of existing unit-linked products with and without embedded guarantees and especially focus on the difference between nominal and real returns. We find that typical "rule of thumb" approximations for considering inflation risk are inappropriate and further show that products that are considered particularly safe by practitioners because of nominal guarantees may bear significant inflation risk. Finally, we propose product designs suitable to reduce inflation risk and investigate their risk-return profile in real terms.

KLING, ALEXANDER; RUß, JOCHEN; SCHILLING, KATJA. *Risk analysis of annuity conversion options in a stochastic mortality environment.* 197-236. While extensive literature exists on the valuation and risk management of financial guarantees embedded in insurance contracts, both the corresponding longevity guarantees and interactions between financial and longevity guarantees are often ignored. The present paper provides a framework for a joint analysis of financial and longevity guarantees, and applies this framework to different annuity conversion options in deferred unit-linked annuities. In particular, we analyze and compare different versions of so-called guaranteed annuity options and guaranteed minimum income benefits with respect to the value of the option and the resulting risk for the insurer. Furthermore, we examine whether and to what extent an insurance company is able to reduce the risk by typical risk management strategies. The analysis is based on a combined stochastic model for both financial market and future survival probabilities. We show that different annuity conversion options have significantly different option values, and that different risk management strategies lead to a significantly different risk for the insurance company.

PÉREZ-SÁNCHEZ, J M; NEGRÍN-HERNÁNDEZ, M A; GARCÍA-GARCÍA, C; GÓMEZ-DÉNIZ, E. *Bayesian asymmetric logit model for detecting risk factors in motor ratemaking.* 445-457. Modelling automobile insurance claims is a crucial component in the ratemaking procedure. This paper focuses on the probability that a policyholder reports a claim, where the classical logit link does not provide a right model. This is so because databases related with automobile claims are often unbalanced, containing more non-claims than the presence of claims. In this work an asymmetric logit model, which takes into account the large number of non-claims in the portfolio, is considered. Both, logit and asymmetric logit models from a Bayesian point of view, are used to a sample that was collected from a major automobile insurance company in Spain in 2009, resulting in a dataset of 2,000 passenger vehicle. We establish the validity of the asymmetric model in front of the conventional logit link. The use of a garage, the age of the vehicle and the duration of the client's relation with the company are all shown to be significant explanatory variables by the logit model. The asymmetric model includes, in addition, the length of time the policyholder has held a driving licence and the type of use made of the vehicle. The asymmetric model provides a better fit to the data examined.

ROBERT, CHRISTIAN Y; THEROND, PIERRE-E. *Distortion risk measures, ambiguity aversion and optimal effort.* 277-302. We consider the class of concave distortion risk measures to study how choice is influenced by the decision-maker's attitude to risk and provide comparative statics results. We also assume ambiguity about the probability distribution of the risk and consider a framework à la Klibanoff, Marinacci and Mukerji (2005; A smooth model of decision making under ambiguity. Econometrica, 73, 1849-1892) to study the value of information that resolves ambiguity. We show that this value increases with greater ambiguity, with greater ambiguity

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aversion, and in some cases with greater risk aversion. Finally, we examine whether a more risk-averse and a more ambiguity-averse individual will invest in more effort to shift his initial risk distribution to a better target distribution.

TZOUGAS, GEORGE; VRONTOS, SPYRIDON; FRANGOS, NICHOLAS. *Optimal bonus-malus systems using finite mixture models*. 417-444. This paper presents the design of optimal Bonus-Malus Systems using finite mixture models, extending the work of Lemaire (1995; Lemaire, J. (1995) Bonus-Malus Systems in Automobile Insurance. Norwell, MA: Kluwer) and Frangos and Vrontos (2001; Frangos, N. and Vrontos, S. (2001) Design of optimal bonus-malus systems with a frequency and a severity component on an individual basis in automobile insurance. ASTIN Bulletin, 31(1), 1-22). Specifically, for the frequency component we employ finite Poisson, Delaporte and Negative Binomial mixtures, while for the severity component we employ finite Exponential, Gamma, Weibull and Generalized Beta Type II mixtures, updating the posterior probability. We also consider the case of a finite Negative Binomial mixture and a finite Pareto mixture updating the posterior mean. The generalized Bonus-Malus Systems we propose, integrate risk classification and experience rating by taking into account both the a priori and a posteriori characteristics of each policyholder.

VENTURA-MARCO, MANUEL; VIDAL-MELIÁ, CARLOS. *An actuarial balance sheet model for defined benefit pay-as-you-go pension systems with disability and retirement contingencies*. 367-415. In this paper, we develop a theoretical basis for drawing up a "Swedish" type actuarial balance sheet for a defined benefit pay-as-you-go (DB PAYG) scheme with retirement and disability benefits. Our model enables us to obtain the system's expected average turnover duration, measure the scheme's solvency and explore the phenomenon identified as "pension reclassification", a widespread practice that masks the system's real status unless further pension information becomes available. The model is clearly linked to actuarial practice in social security and gives partial support to the practical adaptation of Swedish methodology carried out by OSFI (2012) in applying the concept of the contribution asset to the Canadian Pension Plan (CPP) balance sheet, which includes disability and survivor benefits.

ZHU, JINXIA. *Dividend optimization for a regime-switching diffusion model with restricted dividend rates*. 459-494. We consider the optimal dividend control problem to find an optimal strategy under the constraint that dividend rates is restricted such that the expected total discounted dividends are maximized for an insurance company. The evolution of the reserve is modeled by a diffusion process with drift and volatility coefficients modulated by an observable Markov chain. We consider the regime-switching threshold strategy which pays out dividends at the maximal possible rate when the current reserve is above some critical level dependent on the regime of the Markov chain at the time, and pays nothing when the reserve is below that level. We give sufficient conditions under which such type of strategy is optimal for the regime-switching model.

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1(1), 2014

BUTT, ADAM; EVANS, JOHN; FARMER, JIM; PITT, DAVID. *A pilot survey of actuarial graduates' views on their education*. 63-75. This paper presents the results of a pilot survey of recent graduates of some Australian university actuarial programs. The survey aimed to shed light on graduates' views relating to their education since leaving university. The survey that we report on here has been designed as a pilot to inform the development of a more substantial survey to be conducted in future. It does, however, still provide some interesting insights. The findings from our work can be used by those currently reviewing the actuarial education programs in Australia. The broad results indicate that around 50% of students found their Part I courses covered technical material useful in their employment, and well over 50% of graduates considered non-technical training as very important for their readiness for employment. The Part II results indicated a much higher satisfaction with the content than the Part I courses in preparing students for employment. The Part III courses were not rated as highly as the Part I and II courses and the teaching quality was particularly criticised. Overall, the results indicated that the graduates surveyed found their Part I and II courses valuable but wanted greater training in the non-technical skills needed in employment, particularly communication skills, and while they found the Part III courses applicable to their employment, they expressed dissatisfaction with the education process.

EILKHIS, LEN. *Stress testing for insurers*. 87-88. Prudential Standards LPS and GPS 110 (Capital Adequacy) have positioned the ICAAP at the centre of the insurer's capital management framework. As part of the ICAAP, insurers are required to undertake stress testing of their risk exposures to evaluate the impacts of exceptional but plausible events. Within an insurance context these techniques include sensitivity analysis, scenario analysis, and reverse stress testing. In this note I discuss how insurers can obtain value from the stress testing exercise.

EVANS, JOHN; ROYER, CAROL. *A survey of human capital risk management in Australian insurers*. 77-80. Human capital risk permeates all other risks in a financial institution as was evidenced by the global financial crisis and subsequent events, but anecdotal evidence suggests this overriding risk was not well understood. The authors applied for and were successful in obtaining a grant from the Actuaries Institute to survey Australian insurers as to how they identified and managed human capital risk. Discussions were held with 8 major insurers, a representative of the Actuaries Institute Risk Management Practice Committee and a representative of the prudential regulator, APRA. The general types of questions asked are set out in Appendix A. The project identified that, while some aspects of human capital risk such as human resource management and occupational health and safety compliance, were well understood, the most important result of this survey was the identification that the linkages between human capital risks and other institutional risks was not well understood, which is why events like the Libor and fixed interest scandal in the UK could easily recur with significant reputational damage for the institution involved.

FERGUSSON, KEVIN; PLATEN, ECKHARD. *Hedging long-dated interest rate derivatives for Australian pension funds and life insurers*. 29-44. Many pension funds and life insurers seek to hedge their exposure to low interest rates using long-dated interest rate derivatives. This paper extends an approach of Platen and Heath 2006 [Platen, E., & Heath, D. (2006). A benchmark approach to quantitative finance] to price and hedge long-dated interest rate derivatives using a combination of Australian cash, bonds and equities and under a variety of market models. The results show the models under which the lowest cost hedge is achieved.

HUYNH, ALEX; BROWNE, BRIDGET; BRUHN, AARON. *Catastrophic mortality bonds: analysing basis risk and hedge effectiveness*. 45-62. Life insurers are exposed to catastrophic mortality risk. Catastrophic mortality bonds are a recent market innovation that provide an alternative risk management tool to address this risk. However there is little in the way of published studies that examines their effectiveness, given that they are subject to basis risk arising from the use of country-level general population mortality in their construction. By constructing a typical mortality risk portfolio and calibrating a bond for this portfolio, the hedge effectiveness of the instrument is analysed under a wide variety of circumstances. We find that, on a stand-alone basis, hedge effectiveness may be too low to be acceptable to small to medium insurers. However, effectiveness of the bond increases when used in combination with surplus reinsurance and/or when pooling is used to increase portfolio size.

KNOX, DAVID. *The fallacy of using superannuation taxation expenditures*. 81-84. The taxation of superannuation has been subject to much speculation and commentary in recent times. This discussion is prompted every year when the Commonwealth Treasury produces the Tax Expenditures Statement, which provides details of more than 350 tax expenditures. The latest Statement, published in January 2013, showed the largest projected tax expenditures for 2012-13 are for superannuation and owner-occupied housing, with the superannuation tax expenditure shown as $31.9 billion with an expected growth to $44.8 billion in 2015-16. Naturally such significant numbers attract considerable attention from interested parties who may be aiming to increase government revenue or improve equity within the community. This paper will show that the Treasury approach, which ignores Government savings from reduced Age Pension costs, is fundamentally flawed and, if used to develop long-term retirement income policy, is likely to lead to sub-optimal outcomes for individuals, households and the government.

KNOX, DAVID. *Long-term personal injury insurance and the NDIS*. 85-86. The actuarial profession has made a significant contribution to the prudential governance of long-tailed statutory insurance classes for the past 30 years. Our work in general insurance business began in earnest during the 1970s and accelerated after the work of Sir Owen Woodhouse introduced a new way of thinking about compulsory workers compensation and personal injury insurance in general. The Woodhouse principles focused very much on equity and durable outcomes for the injured person rather than common law retribution against negligent parties, usually in the form of lump sum monetary settlement. The Woodhouse principles are: Community responsibility; Comprehensive entitlement; Complete rehabilitation; Real compensation; Administrative efficiency. This thinking transformed the nature of many of Australia's accident compensation schemes during the 1980s, leading to longer term entitlement to weekly benefits and treatment support, and forcing actuaries to take a much longer tailed modelling approach to statutory liabilities. The notion of 'the scheme actuary' emerged for workers compensation and some transport accident schemes. As these schemes matured it became obvious that short-term claims management targeted at the bulk of injuries was often not appropriate for people with more

complex injuries. In particular the most severe injuries triggered the idea of special management teams. The Victorian Transport Accident Commission established its major injury division, and the NZ ACC established a national serious injury service. More generally, Australia's insurance ministers called for a report into a national long term care scheme for 'catastrophic injury', and New South Wales continued this work in establishing its Lifetime Care and Support Scheme in 2006 – which covers predominantly people who sustain major spinal cord injury and brain injury in a motor vehicle accident, and need lifetime support for basic activities of daily living. From this model it was not a huge leap to develop a similar approach to the lifetime support planning and service delivery for people with all types of disability – this thinking underpins the National Disability Insurance Scheme (NDIS).

MULQUINEY, PETER; MILLER, HUGH. *A topic of interest – how to extrapolate the yield curve in Australia*. 13-28. This paper addresses key practical questions concerning the setting of interest rate assumptions in actuarial work. In particular, we look at the situation where the liabilities being valued extend beyond the term of available market instruments, and so the yield curve must be extrapolated. We find that, in Australia, the yield curve up to 2 years before the longest-dated bond can be estimated reliably. We look at international evidence and conclude there is reasonable evidence of reversion to a flat long term forward rate. However, the evidence suggests that the rate of reversion is slow with a term of about 40 years being the minimum point to reversion, and 60 years the central estimate. We also propose hedging strategies for long-dated liabilities and show how these give further insight into appropriate rates.

SERVICE, DAVID A; CUMPSTON, RICHARD; SARJEANT, HUGH. *Measuring risk using household microsimulation models*. 5-12. The more we know about a problem the more chance we have of designing effective solutions. Models are tools for using our existing and emerging knowledge, and take many forms. In particular, microsimulation models enable the measurement of risks where the only robust approach is to use scenarios. Microsimulation allows scenarios to be modelled in significant detail, which enables reliable risk measurement and subsequent management. Models with fine geographic subdivisions can be used for many purposes, for example the selection of locations for infrastructure such as age-care facilities, and the simulation of risks for mortgage insurers. The limited geographic subdivisions of census unit record samples have led to the recent development of a method to create synthetic persons and dwellings from census tabulations. This method has been applied to Australian and New Zealand census data. Census data have been supplemented by survey data, for example on income, expenditure, wealth, housing values, mortgages and superannuation. Models of 123 diseases have been used to impute and project disease conditions. Five specific applications of household microsimulation are discussed.

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European Actuarial Journal

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BEN SALAH, ZIED. *On a generalization of the expected discounted penalty function to include deficits at and beyond ruin*. 219-246. In this paper we propose an extended concept of the

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expected discounted penalty function (EDPF) that takes into account new ruin-related random variables. In the well-known EDPF introduced in seminal papers by HU Gerber and ESW Shiu (Insurance: Mathematics & Economics (1997) 21:129-137; North American Actuarial Journal (1998) 2(1):48-78) and HU Gerber and B Landry (Insurance: Mathemtics & Economics (1998) 22:263-276), we consider the expectation of a sequence of discounted penalty functions of new record minima reached by a claim of the risk process after ruin (and before recovery). Inspired by results of M Huzak *et al.* (Annals of Applied Probability (2004) 14(3):1378-1397) and developments in fluctuation theory for spectrally negative Lévy processes, we provide a characterization for this extended EDPF in a setting involving a cumulative claims modeled by a subordinator, and Brownian perturbation. We illustrate how the extended EDPF can be used to compute the expected discounted value of capital injections for the Brownian perturbed risk model.

BIERBAUM, JÜRGEN; BARTEL, HOLGER; DENNSTEDT, NILS; DILLMAN, TOBIAS; ENGEL, WOLFGANG; KELLER, MARCUS; MUSIALIK, KAROL; PAULS, THORSTEN; QUAPP, NORBERT; WINTER, JENS. *Practical valuation of long-term guarantees in inactive financial markets*. 101-124. In this paper we address certain issues in the valuation of long-term (insurance) guarantees from a practical point of view. These issues arise, because markets are incomplete or inactive. We provide a general, arbitrage-free valuation approach in this context and discuss the following topics in more detail: extrapolation of interest rates, treatment of interest rate spreads and counter-cyclical measures in distorted markets. By deriving solutions from a practitioner's point of view this paper aims to contribute to an effective regulation under Solvency II. Moreover, we hope to provide a starting point for an in-depth academic analysis of these problems.

BONNIN, FRANÇOIS; PLANCHET, FRÉDÉRIC; JUILLARD, MARC. *Best estimate calculations of savings contracts by closed formulas: application to the ORSA*. 181-196. In this paper we present an analytical approximation of the best estimate of a savings contract. This approximation aims to provide a framework for robust and justifiable calculation of the own risk solvency assessment [ORSA] avoiding the complexity of direct approaches.

EDER, ARMIN; KEILER, SEBASTIAN; PICHL, HANNES. *Interest rate risk and the Swiss Solvency Test*. 77-99. In this paper, we present a new approach to measuring interest rate risk within the Swiss Solvency Test (SST), which overcomes the shortcomings of the standard model. The standard model of the SST is based on more interest rate risk factors than are actually needed to capture interest rate risk, it allows for significantly negative interest rates and it tends toward procyclical solvency capital requirements. Our new approach treats interest rate risk with direct reference to the underlying term structure model and interprets its parameters as a canonical choice of the relevant interest rate risk factors. In this way, the number of interest rate risk factors is substantially reduced and interest rate risk measurement is linked to the term structure model itself. The consideration of empirical interest rate data and the acceptance of the economical impossibility of persistently negative interest rates significantly below the cost of holding cash motivate the introduction of a truncated Gaussian process to simulate innovation in the future development of the parameters of the underlying term structure model. In a natural way this leads to mean-reverting interest rate behaviour and to countercyclical solvency capital requirements.

ELING, MARTIN; PANKOKE, DAVID. *Discontinued business in non-life insurance: an empirical test of the market development in the German-speaking countries*. 31-48. Although every company has discontinued business, its active management is a relatively new topic in practice and an entirely new field of study in academia. Based on a survey of 85 non-life insurers from

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Germany, Switzerland, Austria, and Luxembourg, we empirically test the market development and find indication that Swiss insurers seem to have more experience with the active management of discontinued business than insurers in other countries. We explain this phenomenon by that country's more advanced solvency capital requirements that better reflect the risk of discontinued business activities. We thus conclude that with the introduction of Solvency II, active management of discontinued business will become more important since insurers will have to hold higher equity capital for discontinued business portfolios. We illustrate this fact within a numerical example which shows that 23% of the Solvency II non-life premiums and reserve risk can be traced back to discontinued business.

GOVORUN, MARIA; LATOUCHE, GUY. *Modeling the effect of health: phase-type approach*. 197-218. The health of an individual is an important factor that affects the financial results of life insurance business. We focus on factoring health to different actuarial applications by using phase-type methods. In particular, we estimate the distribution of the discounted profits and losses (below, "P&L") of a pension fund, where new pensioners arrive each year. To model P&L we consider both deterministic and stochastic arrivals of new pensioners. The properties of the phase-type method allow us to valuate the financial impact of living longer due to population selection and due to health improvements.

HENRIKSEN, LARS FREDERIK BRANDT; NIELSEN, JEPPE WOETMANN; STEFFENSEN, MOGENS; SVENSSON, CHRISTIAN. *Markov chain modeling of policyholder behavior in life insurance and pension*. 1-29. We calculate reserves regarding expected policyholder behavior. The behavior is modeled to occur incidentally similarly to insurance risk. The focus is on multi-state modelling of insurance risk and behavioral risk in terms of free policy risk and surrender risk. We discuss valuation techniques in the cases where behavior is modeled to occur independently or dependently of insurance risk, respectively. Ordinary differential equations make it easier to work with dependence between insurance risk and behavior risk. We analyze the effects of the underlying behavioral assumptions for two contracts. For a "new" contract with low technical interest rate relative to the market interest rate, we obtain the lowest reserve by counting in dependence. For an "old" contract with high technical interest rate relative to the market interest rate, the picture is more blurred, depending on assumptions on reactivation (recovery) and independence.

SCHMIDT, JAN-PHILIPP. *Market-consistent valuation of long-term insurance contracts: valuation framework and application to German private health insurance*. 125-153. In this paper we derive a market-consistent value for long-term insurance contracts, with a focus on long-term health insurance contracts as found, e.g., in the German private health insurance industry. To this end, we first set up a health insurance company model and, second, conduct a simulation study to calculate the present value of future profits and the time value of financial options and guarantees from a portfolio of private health insurance policies. Our analysis quantifies the impact of investment results and underwriting surpluses on shareholder profits with respect to profit sharing rules and premium adjustment mechanisms. In contrast to the valuation of life insurance contracts with similar calculation techniques the results indicate that the time value of financial options and guarantees of German private health insurance contracts is substantially smaller in typical parameter settings.

STEFANOVITS, DAVID; WÜTHRICH, MARIO V. *Hedging of long term zero-coupon bonds in a market model with reinvestment risk*. 49-75. We present a computational methodology to value

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and hedge long term zero-coupon bonds trading in short and medium term ones. For this purpose we develop a discrete time stochastic yield curve model with limited availability of maturity dates at a fixed time point and newly issued bonds at future time points. This involves reinvestment risk and there is no perfect hedging strategy available for long term liabilities. We calibrate the model to market data and describe optimal hedging strategies under a given risk tolerance. These considerations provide a natural extrapolation of the yield curve beyond the last liquid maturity date, and a framework which allows to value long term insurance liabilities, for instance, under Solvency 2. Moreover, we determine the optimal trading strategy replicating the liabilities under the given risk tolerance.

VALDEZ, EMILIANO A. *Empirical investigation of insurance claim dependencies using mixture models.* 155-179. For several years now, there continues to be attention in the modeling of insurance and other similar type of risks, such as the risk of credit default, to incorporate the presence of dependencies. Some of the early papers appearing in the literature demonstrate that for a typical portfolio of such risks, ignoring dependencies can have a direct impact on the tail or extremes of the resulting portfolio loss distribution. The tail of the loss distribution is something not to be ignored by the actuary or the risk manager. To date, in spite of this growing number of papers in the literature on dependence modeling, we find that there is no known published work that provides for an empirical evidence to validate the presence of dependencies in an insurance portfolio. In this paper, we use mixture models, customarily suggested to model dependent credit default risks, to facilitate the investigation of claim dependencies. The empirical data used to calibrate these models came from a portfolio of automobile insurance policies drawn from a randomly selected insurance provider. In order to measure the presence of claim dependencies, one of the most reasonable statistic to use is the relative risk ratio, a measure that is widely popular in medical statistics and is used to gauge how the claim occurrence of a particular insurance risk induces claim of another insurance risk. Our calibration results indicate some presence of positive dependencies; relative risk is in the neighborhood of 1.4 and resulting pair-wise correlation is 0.04. The model naturally extends to capture policyholder heterogeneity through the presence of covariates by introducing mixture models with covariates as explained in this article. Not surprisingly, because the premium is the actuary's best guess of the degree of riskiness of an insurance risk, at least on an a priori basis, it provides for the single most important factor that influences the presence of claim dependencies.

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BOHNERT, ALEXANDER; GATZERT, NADINE. *Fair valuation and risk assessment of dynamic hybrid products in life insurance: a portfolio consideration.* 148-172. Dynamic hybrid products are innovative life insurance products particularly offered in the German market and intended to meet new consumer needs regarding stability and upside potential. These products are characterised by a periodical rebalancing process between the policy reserves (i.e. the premium reserve stock), a guarantee fund and an equity fund. The policy reserve thereby corresponds to the one also valid for traditional participating life insurance products. Hence, funds of dynamic

hybrids that are allocated to the policy reserves in times of adverse capital market environments earn the same policy interest rate determined for the participating life insurance policyholders and, hence, at least a guaranteed interest rate. In this paper, we study the fair valuation and risk situation of an insurer offering both dynamic hybrid and traditional participating life insurance contracts. The results reveal considerable interaction effects between the two contract types within the portfolio that strongly depend on the portfolio composition, thereby emphasising merits as well as risks associated with offering dynamic hybrids.

CHANG, VINCENT Y; TSAI, JEFFREY TZUHAO. *Quantile regression analysis of corporate liquidity: evidence from the U.S. property-liability insurance industry*. 77-89. This study analyses the determinants of corporate liquidity for the U.S. property-liability insurance industry from 2006 to 2010. Unlike previous studies using the ordinary least squares (OLS) approach, this study applies the quantile regression (QR) method. The QR method provides further insights on how insurers' liquidity level is determined, especially for the firms at the lower and the higher quantiles. We found that leverage and organisational structure have opposite effects on insurers' liquidity in the lower and the higher quantile groups. The empirical results also show that most firm-specific characteristics and macroeconomic conditions influence the insurers' liquidity, which are consistent with the findings of the OLS approach in previous studies.

DANG, HUONG. *A competing risks dynamic hazard approach to investigate the insolvency outcomes of property-casualty insurers*. 42-76. This study applies a competing risks approach and an event time dynamic estimation framework to identify the characteristics underlying different insolvency resolutions incurred to U.S. property-casualty insurers during 1998-2010. The estimated hazard model relates the time-varying probability of a specific insolvency outcome to insurers' characteristics and macroeconomic conditions. The study finds that (i) the hazards for different insolvency outcomes are neither equal nor proportionate; (ii) the model for generic insolvency events and the models for outcome-specific insolvency events feature different significant factors, which are not due to random variations; and (iii) the outcome-specific insolvency models exhibit better forecast performance than the generic insolvency model within a five-year forecast horizon. The results of the study provide regulators with early warnings of financial distress, aid them in prioritising troubled insurers and identifying the areas most likely to reveal material problems, and inform about the interventions that should be taken under specific circumstances.

FORSSTED, SARA. *Asymmetric information on risky behaviour: evidence from the automobile insurance market*. 104-145. This paper examines the Swedish automobile insurance market by accounting for policyholders' private information on risky behaviour in terms of major and minor traffic violations. Two approaches are used: A positive correlation test and a test where private information is used explicitly. The results show that there is a positive correlation, which is not affected when including private information in the regression, that policyholders with private information on risky behaviour are less likely to purchase full coverage, and that speeders follow a varying pattern. The conclusion is that it is favourable to use private information explicitly when asymmetric information is considered, rather than base the conclusion solely on the correlation test.

HSIEH, SHIU-HUI; LIU, CHUNG-TING; TZENG, LARRY Y. *Insurance marketing channel as a screening mechanism: empirical evidences from Taiwan automobile insurance market*. 90-103. Venezia, Galai and Shapira in 1999 proposed a theoretical framework that considers asymmetric

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information to explain the co-existence of the independent agent and direct underwriting systems in the insurance market. In separating equilibrium, high-risk clients tend to purchase insurance through independent agents, whereas low-risk clients prefer dealing directly with underwriters. Using a unique data set from Taiwan's automobile liability insurance, this paper tests the screening mechanism hypothesis proposed by Venezia, Galai and Shapira. The results reveal that a positive channel-claim correlation exists in the subsamples of cars aged more than three years. Significant positive channel-claim correlation indicates that high-risk policyholders prefer to purchase insurance from an independent agent, whereas those with lower risks tend to buy insurance from direct writer channels. The results support the screening mechanism hypothesis and demonstrate that marketing channel choice could serve as a screening mechanism in an insurance market characterised by asymmetric information.

KWON, W JEAN. *Human capital risk and talent management issues in the insurance market: public policy, industry and collegiate education perspectives*. 173-196. The human capital attraction and retention challenges in the insurance industry intensify due to a combining effect of demographic, social, economic and market-specific factors. In addition, there is human capital obsolescence risk. The study findings indicate that: the financial services occupation group requires high competency in cognitive abilities, social perspectives, management knowledge and communication skills, but not much in technical, physical/sensory or engineering skills; the insurance industry and most insurance companies have not attained a high reputation; and insurance is not a widely recognised profession by the public and many college students. Based on our macro- and micro-spective investigation of the industry and university education, we recommend: more public relations activities, closer work with local universities and their faculties for adoption of the field as a major; better talent recruitment, training and retention programmes; and preparedness for talent morbidity across financial services sectors and countries.

TIEN, JOSEPH J; YANG, SHARON S. *The determinants of life insurer's growth for a developing insurance market: domestic vs foreign insurance firms*. 1-24. The relationship between firm growth and firm characteristics has been a critical issue for insurers. This study examines the determinants of firm growth and tests Gibrat's law of life insurance company expansion in the context of the young but fast-growing insurance market for both domestic and foreign firms in Taiwan. By using Heckman's two-stage regression, our empirical results reveal that Gibrat's law does not hold for either domestic or foreign life insurers during the period from 1996 to 2007. Smaller life insurers achieve greater growth than larger ones in the booming economy. Furthermore, factors such as age, cross-marketing and product diversification show different effects on growth between domestic and foreign firms.

TSENG, LU-MING; SU, WEN-PIN. *Insurance salespeople's attitudes towards collusion: the case of Taiwan's car insurance industry*. 25-41. Insurance researchers believe that the increase in insurance fraud may be associated with the unethical decisions made by some insurance salespeople. However, to date, research that has empirically investigated the link between insurance salespeople and collusion is scant. Using the car insurance industry in Taiwan as an example, this paper explores the impact of the opportunity to obtain the fraudulent claim and that of the size of actual loss on car insurance salespeople's attitudes towards collusion in situations involving contract renewal and non-covered loss. The results showed that the size of actual loss and the fraud type (customer fraud vs insider fraud) may correlate with the decision-making of the car insurance salespeople. It seemed that the responders have a higher acceptance of customer fraud rather than insider fraud. Empirical research on insurance salespeople's attitudes towards

salespeople-customer collusion is very rare. This study may make some contribution to insurance research and practice.

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AKOTEY, JOSEPH OSCAR; ADJASI, CHARLES. *The impact of microinsurance on household asset accumulation in Ghana: an asset index approach*. 304-321. This paper examines the link between microinsurance services and households' asset accumulation in Ghana. Using FINSCOPE data from the nation-wide household surveys conducted by FinMark Trust, we created a household asset index using the multiple correspondence analysis approach. We employed three methods: Heckman sample selection, instrumental variable and treatment effects models to estimate the impact of microinsurance on household asset accumulation. These models control for the problem of endogeneity treatment effects and self-selection bias associated with the usage of microinsurance services. The findings show that microinsurance has a positive welfare impact in terms of household asset accumulation. This suggests that microinsurance prevents asset pawning and liquidation of essential household assets at "give away" prices. By absorbing the risk of low-income households, insurance equips them to cope effectively with risk, empowers them to escape poverty and sustains the welfare gains achieved.

DROR, DAVID M; MAJUMDAR, ATANU; PANDA, PRADEEP; JOHN, DENNY; KOREN, RUTH. *Implementing a participatory model of micro health insurance among rural poor with evidence from Nepal*. 280-303. This paper reports on two voluntary, contributory, contextualised, community-based health insurance (CBHI) schemes, launched in Dhading and Banke (Nepal) in 2011. The implementation followed a four-stage process: initiating (baseline survey), involving (awareness generation and engaging community in benefit-package-design), launch (enrolment and training of selected community members) and post-launch (viable claims ratio, settled within satisfactory time, sustainable affiliation). Both schemes were successful on four key parameters: effective planning; affiliation (grew from 0 to ~10,000) and renewals (>65 per cent); claims ratio (~50 per cent); and promptness of claim settlement (~23 days). This model succeeded in implementing CBHI with zero premium subsidies or subsidised health-care costs. The successful operation relied in large part on the fact that members trust that they can enforce this contract. Considerable insurance education and capacity development is necessary before the launch of the CBHI, and for sustainable operations as well as for scaling.

ELING, MARTIN; PRADHAN, SHAILEE; SCHMIT, JOAN T. *The determinants of microinsurance demand*. 224-263. The purpose of this article is to structure the extant knowledge on the determinants of microinsurance demand in a manner that achieves several outcomes. First is to offer a specific economic structure to the review through use of Outreville's insurance demand framework. Second is to identify key questions that arise out of structuring the material in this way. In particular, we attempt to clarify the critical open questions in microinsurance demand through use of Outreville's framework. Third, through comparison with literature on traditional insurance demand, we identify opportunities to understand not only the microinsurance market better, but also the traditional market. To achieve these outcomes, we review the academic literature on microinsurance demand published between 2000 and early 2014 and compare these results with evidence in the literature regarding traditional insurance markets. The review identifies 12 key factors affecting microinsurance demand, and further highlights that research focused on

the role of contract performance (including basis risk and quality), trust, financial literacy and informal risk-sharing mechanisms may be most fruitful in expanding microinsurance markets.

GATZERT, NADINE; KOSUB, THOMAS. *Insurers' investment in infrastructure: overview and treatment under Solvency II*. 351-372. The financial market environment poses serious challenges for insurance companies to provide stable returns on a long-term basis, as particular traditional asset classes are currently characterised by generally low interest rates and high volatility. Against this background, the aim of this paper is to study infrastructure investments from an insurer's perspective. In particular, based on a categorisation of different types of infrastructure investments, we provide an overview of main characteristics along with risks and chances. In addition, the treatment of different infrastructure investments under Solvency II regulations is studied, which can have a considerable impact on an insurer's asset management decisions. The study shows that the attractiveness of infrastructure investments strongly depends on the type of investment and its treatment under Solvency II and that considerable risks can be involved.

GEHRKE, ESTHER. *The insurability framework applied to agricultural microinsurance: what do we know, what can we learn?* 264-279. Agricultural microinsurance has gained increasing attention over the last years. In light of climate change, changing weather trends and more repeatedly occurring natural disasters, a wave of innovative approaches to insuring agricultural production risks, particularly index-based insurance products, have been proposed and implemented. However, the results of many of these projects were disappointing, raising the question whether microinsurance could provide viable coverage for agricultural production risks at all. This paper reviews existing theoretical and empirical research on the feasibility of agricultural microinsurance and discusses different approaches to cover agricultural productions risks. It finds that, contrary to common beliefs, many agricultural production risks, including drought, hail, floods and livestock diseases, could be effectively covered by microinsurance, although not all by index-based approaches. It makes the case for a flexible combination of different approaches to microinsurance, such as community-based and group insurance, with technological advancements such as remote-sensing techniques and index insurance.

MA, YU-LUEN; POPE, NAT; XIE, XIAOYING. *Insurer performance and intermediary remuneration: the impact of abandonment of contingent commissions*. 373-388. The advent of the 2004 Spitzer investigation subjected the use of contingent commissions to significant scrutiny in the public media. This research uses the Spitzer investigation as a point of differentiation and assesses the relationship shared by the abandonment of contingent commissions and insurer performance across the pre- and post-2004 periods. Our research defines insurer performance using efficiency measures derived through data envelopment analysis. We employ a difference-in-difference methodology to control for extraneous environmental factors across the time period under consideration, 1993-2008. Our results suggest that the decision to abandon the use of contingent commissions after 2004 is associated with decreases in insurer performance. We attribute that reduction in performance to insurers' abandonment of otherwise performance-maximising remuneration strategies in an attempt to distance themselves from any potential negative associations with the use of contingent commissions.

MIAO, HONG; RAMCHANDER, SANJAY; WANG, TIANYANG. *The response of bond prices to insurer ratings changes*. 389-413. This paper examines the impact of insurer ratings changes on bond prices. Using insurer ratings from four major rating agencies and data covering the recent financial crisis period, we document that downgrades have a strong negative price impact on bond prices, especially when the downgrades are reinforced by multiple agencies. In contrast, the

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announcement-day impact of upgrades is found to be weak. Our evidence is consistent with the predictions of structural credit risk models.

SCHMEISER, HATO; STÖRMER, TINA; WAGNER, JOËL. *Unisex insurance pricing: consumers' perception and market implications*. 322-350. The main reason for different insurance premiums and benefits is the use of different statistically proven risk factors in actuarial calculations for individuals. Basing its ruling on European Union Directive 2004/113/EC, the European Court of Justice on 1 March 2011 concluded that any gender-based discrimination is prohibited, so gender equality in the European Union (EU) must be ensured from 21 December 2012. Until then, gender-specific premium differentiation was allowed in most EU Member States for risks that are strongly linked to gender. We discuss the relevance of price differentiation criteria from the point of view of insurers, regulators and ethicists, and reflect on the degree of acceptance of such price differentiation by consumers, which is assessed empirically through an international consumer survey conducted in the United Kingdom, Germany, France, Italy and Switzerland. The perception of risk factors and of effective gender-related price differences is considered with respect to motor, annuity, term life and health insurance. Finally, we discuss possible consequences of the new regulation for the insurance industry.

TURNER, GINGER; SAID, FARAH; AFZAL, UZMA. *Microinsurance demand after a rare flood event: evidence from a field experiment in Pakistan*. 201-223. This paper examines the characteristics that determine demand for microinsurance when individuals have personal or observed experience with a rare weather event: the severe 2010 flooding in Pakistan. Using a sample of 384 individuals (192 in flood-affected and 192 non-affected villages matched using pre-flood propensity data), we combine post-flood survey data with behavioural experiments to test the impact of prior loss experience on willingness to purchase insurance. In the framed experiment setting, we allow participants to choose insurance payments through many rounds of random flood losses, testing whether experiment behaviour is significantly related to real-world experience or observation and whether individuals change insurance demand after experiencing losses. We find that 2010 flood-affected individuals demand significantly more insurance than non-affected individuals, and that both personal losses and observations of others' losses are significant determinants of demand, when controlling for location-specific flood propensity, pre-flood mitigation, information sources, post-flood assistance, exogenous changes in assets, potential migrant attrition and other household characteristics. Having prior experience with less severe floods before the 2010 event also increases insurance demand, although the effect disappears when controlling for 2010 flood. Contrary to expectation, household beliefs that insurance is non-Islamic are not found to be a significant barrier to take-up.

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DUMM, RANDY E; ECKLES, DAVID L; HALEK, MARTIN. *An examination of adverse selection in the public provision of insurance*. 127-147. Using a unique data set from Florida's residual property insurer, we test for adverse selection in the public provision of homeowners' insurance

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in Florida. We find a significant relationship between the losses and deductible choices of insureds in Florida's residual homeowners' insurance market. This relationship provides strong evidence of the existence of an adverse selection problem in Florida's residual property insurance market. While this relationship is important to Florida regulators (and taxpayers) specifically, a finding of an adverse selection problem in residual markets in general has implications more broadly for government providers of insurance as an adverse selection problem in these settings will impact the public policy debates and decisions involving these markets.

LIGON, JAMES A; THISTLE, PAUL D. *Background risk in the Principal-Agent Model.* 115-126. We examine the effect of background risk in the standard two-state, two-action principal-agent model. We analyse situations where the background risk is environmental (always present) and where the background risk is contractual (only present if the contract is accepted). With contractual background risk, expected wages always rise and the incentive scheme is flatter if the agent's preferences satisfy weak decreasing absolute risk aversion. With environmental background risk, the optimal incentive scheme becomes flatter if the agent is weakly prudent. We provide conditions under which the environmental background risk decreases the agent's expected wage.

MANIKOWSKI, PIOTR; WEISS, MARY A. *The satellite insurance market and underwriting cycles.* 148-182. This research analyses whether underwriting cycles are present in an important but often overlooked line of insurance, satellite insurance. Unlike previous underwriting cycle studies, this study uses rates-on-line and capacity devoted to satellite insurance as well as loss ratios to determine the applicability of cycles. The sample period encompasses virtually the entire history of the satellite insurance industry, 1968-2010. The results indicate that cycles are present in the minimum and average rates-on-line and in capacity, but not the loss ratio. Regression analysis is carried out on the rate-on-line and capacity variables, and the regression results support the rational expectations/ institutional intervention hypothesis and the capacity constraint (capital shock) hypothesis.

SCHROYEN, FRED. *Attitudes towards income risk in the presence of quantity constraints.* 183-209. Considering a consumer with standard preferences, I trace out how quantity constraints on markets impact on relative risk aversion and prudence. I first show how this impact decomposes into a local curvature effect and an endogenously changing risk aversion/prudence effect. Next, I calibrate both effects on relative risk aversion and prudence, using estimates on household demand for durables and labour supply. The calibrations show that commitments to durable goods have large effects on attitudes towards risk. And while small wedges between realised and desired levels of labour supply have only moderate effects, becoming full time unemployed on a 60 per cent unemployment benefit significantly raises risk aversion and prudence.

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ASIMIT, ALEXANDRU V; BADESCU, ALEXANDRU M; VERDONCK, TIM. *Optimal risk transfer under quantile-based risk measurers.* 252-265. The classical problem of identifying the

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optimal risk transfer from one insurance company to multiple reinsurance companies is examined under some quantile-based risk measure criteria. We develop a new methodology via a two-stage optimisation procedure which not only allows us to recover some existing results in the literature, but also makes possible the analysis of high-dimensional problems in which the insurance company diversifies its risk with multiple reinsurance counter-parties, where the insurer risk position and the premium charged by the reinsurers are functions of the underlying risk quantile. Closed-form solutions are elaborated for some particular settings, although numerical methods for the second part of our procedure represent viable alternatives for the ease of implementing it in more complex scenarios. Furthermore, we discuss some approaches to obtain more robust results.

BADAOUI, MOHAMED; FERNÁNDEZ, BEGOÑA. *An optimal investment strategy with maximal risk aversion and its ruin probability in the presence of stochastic volatility on investments*. 1-13. In this paper, we study an optimal investment problem of an insurance company with a Cramér-Lundberg risk process and investments portfolio consisting of a risky asset with stochastic volatility and a money market. The asset prices are affected by a correlated economic factor, modeled as diffusion process. We prove a verification theorem, in order to show that any solution to the Hamilton-Jacobi-Bellman equation solves the optimization problem. When the insurer preferences are exponential, we prove the existence of a smooth solution, and we give an explicit form of the optimal strategy, also numerical results are presented in the case of the Scott model. Finally we use the optimal strategy to get an estimate of the ruin probability in finite horizon.

BAO, JIANHAI; YUAN, CHENGGUI. *Long-term behavior of stochastic interest rate models with jumps and memory*. 266-272. The long-term interest rates, for example, determine when homeowners refinance their mortgages in mortgage pricing, play a dominant role in life insurance, decide when one should exchange a long bond to a short bond in pricing an option. In this paper, for a one-factor model, we reveal that the long-term return $t^{-\mu}\int_0^t X(s)ds$ for some $\mu \geq 1$, in which $X(t)$ follows an extension of the Cox-Ingersoll-Ross model with jumps and memory, converges almost surely to a reversion level which is random itself. Such a convergence can be applied in the determination of models of participation in the benefit or of saving products with a guaranteed minimum return. As an immediate application of the result obtained for the one-factor model, for a class of two-factor model, we also investigate the almost sure convergence of the long-term return $t^{-\mu}\int_0^t Y(s)ds$ for some $\mu \geq 1$, where $Y(t)$ follows an extended Cox-Ingersoll-Ross model with stochastic reversion level $-X(t)/(2\beta)$ in which $X(t)$ follows an extension of the square root process. This result can be applied to, e.g., how the percentage of interest should be determined when insurance companies promise a certain fixed percentage of interest on their insurance products such as bonds, life-insurance and so on.

BIAGINI, FRANCESCA; GROLL, ANDREAS; WIDENMANN, JAN. *Intensity-based premium evaluation for unemployment insurance products*. 302-316. We present a flexible premium determination method for insurance products, in particular, for unemployment insurance products. The price is determined with the real-world pricing formula and under the assumption that the employment-unemployment progress of an insured person follows an FF-doubly stochastic Markov chain. The stochastic intensity processes are estimated for the German labor market, using Cox's proportional hazards model with time-dependent covariates on a sample of integrated labor market biographies. The estimation procedure is based on a counting process framework with stochastic compensators, which we show to be naturally connected to the class of FF-doubly stochastic Markov chains. Based on the statistical analysis, the prices are computed using Monte Carlo simulations.

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BLACKBURN, CRAIG; SHERRIS, MICHAEL. *Consistent dynamic affine mortality models for longevity risk applications*. 64-73. This paper proposes and calibrates a consistent multi-factor affine term structure mortality model for longevity risk applications. We show that this model is appropriate for fitting historical mortality rates. Without traded mortality instruments the choice of risk-neutral measure is not unique and we fit it to observed historical mortality rates in our framework. We show that the risk-neutral parameters can be calibrated and are relatively insensitive of the historical period chosen. Importantly, the framework provides consistent future survival curves with the same parametric form as the initial curve in the risk-neutral measure. The multiple risk factors allow for applications in pricing and more general risk management problems. A state-space representation is used to estimate parameters for the model with the Kalman filter. A measurement error variance is included for each age to capture the effect of sample population size. Swedish mortality data is used to assess 2- and 3-factor implementations of the model. A 3-factor model specification is shown to provide a good fit to the observed survival curves, especially for older ages. Bootstrapping is used to derive parameter estimate distributions and residual analysis is used to confirm model fit. We use the Heath-Jarrow-Morton forward rate framework to verify consistency and to simulate cohort survivor curves under the risk-neutral measure.

BLANCHET, JOSE; LAM, HENRY. *A heavy traffic approach to modeling large life insurance portfolios*. 237-251. We explore a new framework to approximate life insurance risk processes in the scenario of plentiful policyholders, via a bottom-up approach. Given the insurance contract structure, we aggregate the balance of individual policy accounts, and derive an approximating Gaussian process with computable correlation structure. The methodology is borrowed from heavy traffic theory in the literature of many-server queues, and involves the so-called fluid and diffusion approximations. Our framework is different from the individual risk model in that it takes into account the time dimension and the specific policy structure including the premium payments. It is also different from classical risk theory in that it builds the risk process from micro-level contracts and parameters instead of assuming aggregated claim and premium processes outright. As a result, our approximating process behaves differently depending on the issued contract structure. We also illustrate the flexibility of our approach by formulating a finite-horizon ruin problem that incorporates actuarial reserve in the consideration.

The study of risk processes is a central topic in actuarial science. Most of the literature focuses on the calculation of ruin probability and deficits (or overshoots) at the time of ruin, as well as the optimal control of premiums, reinsurance levels, and investment allocation. These questions have been studied under a variety of stochastic settings, from the classical Cramer-Lundberg approximation to diffusion processes. The central theme is that random-walk-type models, with a negatively drifted premium process and a jump process of claims, provide a rich framework to allow plenty of extensions, modifications and problem formulations (see, for example, Asmussen and Albrecher, 2010 for the survey on ruin probability calculations, and Schmidli, 2008 for the counterpart in stochastic control problems).

In this paper, we take a different view from the existing literature. Rather than focusing on the computation of risk-related quantities, we explore the question of the construction of risk process itself. The approach we use is bottom-up: given the structure and parameters of the individual insurance contracts, how does the risk process of the insurer look like on an aggregate scale?

Naturally, the risk process under this framework is the sum of all the individual accounts i.e. the balances of policyholders who entered contract with the insurer over time. For actuaries,

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this points to the standard one-period individual and collective risk models. However, these standard models do not consider the time dimension. This in turn also restrains the power of such models to capture the specific contract structure involved e.g. the premium payments.

In this regard, our work can be seen as a generalization of the standard risk models to a process-level approximation. Of course, mere summation of all individual accounts might end up getting an unpleasant process that is hardly computable. To tackle this issue, we borrow techniques in so-called heavy traffic theory in the queueing literature. The basic idea is that under the assumption of large number of customers or policyholders, one can approximate the functionals of these policyholders' statuses using fluid and diffusion approximations. In the statistics literature, these correspond to stochastic-process versions of Law of Large Numbers and Central Limit Theorems. With the sheer scale of major insurance companies, the assumption of plentiful policyholders is sensible, and so these approximation techniques can be used. As we will see, these heavy traffic approximation would then lead to a Gaussian process that is as analyzable as many standard processes used in the current risk theory literature. In particular, the correlation structure of this Gaussian process is explicitly computable given the contract structure (see Section 4). To illustrate our argument on tractability, we formulate a finite-horizon ruin problem based on our Gaussian approximation (see Section 3).

We distinguish our contribution from the classical risk theory and standard actuarial risk models in a few ways. First, our model explains how individual insurance policies lead to certain features of the aggregate risk process. The construction of our risk process depends intricately on the premium and benefit structure of single policies. This means that different types of insurance, such as whole life insurance, term life, endowment etc. would lead to different correlation structure of our resulting Gaussian process. This is in sharp contrast to the current model in risk theory, where premium and claim processes are modeled separately, each as a drifted random walk (or its variants) and marked point process. This feature can potentially provide a framework to analyze the effect of contract structure on the firm-wide risk level. Second, our model allows naturally the incorporation of actuarial reserve in our approximation. Indeed, the finite-horizon ruin problem that we formulate in Section 3 will involve the calculation of prospective reserve. Third, since serial correlation is explicitly computable, this provides a way to capture the fluctuation of our approximating process over time, which can be potentially applicable to dynamically monitoring mismatch on the insurer's balance sheet with regard to statistical error.

In a more organized fashion, we summarize our contributions as follows:

(A) Under the assumption of large number of policyholders, we construct the fluid limit and diffusion limit for the aggregate risk processes. (As we mentioned, these correspond to functional Law of Large Numbers and Central Limit Theorem respectively in the statistics community; throughout the paper we mostly use the former terminology to align with the queueing literature, but will also use the latter interchangeably when necessary.) The risk processes that we are interested in include the insurer's cash level, liabilities, and per basis reserve level. These will be discussed in Section 2. We prove and numerically demonstrate that these risk processes can be approximated by Gaussian processes with certain correlation structures.

(B) Using the theory of Gaussian processes, we illustrate how our result can be used to approximate the ruin probabilities. We model ruin as the situation in which the liabilities surpass the assets (plus the initial capital) within a given time horizon (see Section 3.1). This highlights the flexibility of our methodology in incorporating reserve calculation, and also the dependency on the underlying insurance contracts. In particular, we apply our results to several common types of insurance.

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(C) Our diffusion approximation shows how, under the Equivalence Principle, the benefit reserve arises as the fluid limit of the empirical cash level per basis at any point in time (see Section 2). These results, we believe, provide a useful perspective into the basic concepts underlying the definition of benefit reserve; see the discussion following Theorem 1.

(D) We compute the correlation structures of our limiting processes, thereby showing their tractability. In particular, we illustrate how our approach allows to evaluate and compare the autocorrelation (as a function of time) of risk processes with different insurance types; see Section 4.

Let us emphasize that our purpose in applications such as (B) and (C) is to illustrate the concepts behind our ideas, and hence the models we are using in this paper are basic. There are certainly many practical considerations to make the model more realistic. We shall list out these generalizations and more realistic extensions that we believe are worth pursuing in Section 5.

In terms of methodology, as aforementioned, we will invoke primarily the machinery in heavy traffic theory i.e. fluid and diffusion approximations in the queueing literature. The ideas date back to Kingman, 1961 and Kingman, 1962 for single-server queues, and they still constitute an active research area among the queueing theorists (see the standard surveys of Whitt, 2002 and Billingsley, 1999 for instance). Under fairly mild assumptions, the tools significantly simplify and single out the important elements of the system dynamics of interest, and provide approximate solutions to many important performance measures (in our context, the ruin probability mentioned in (B) constitutes one such example). More precisely, the results in this paper relate to the analysis of so-called many-server queues, which have been substantially studied in recent years. In these queueing systems, customers arrive and elicit service for a random amount of time, as long as there are available servers. When the number of servers is infinite, every customer can start service right at arrival. Connecting to our work, policyholders can be thought of as customers in the queueing system. While the feature of arrivals is not our focus in this paper, the death time of policyholders is analogous to the end of service, and hence the approximation technique is translatable. Some relevant references on the topic include Pang and Whitt (2010) and Decreusefond and Moyal (2008), which focus on infinite-server models, Halfin and Whitt (1981), Kaspi and Ramanan (2010) and Reed (2009), which study finite but large number of servers in different proportion (or so-called regime) to the number of customers, Puhalskii and Reiman (2000) that study queues with multiclass customers, and Dai *et al.* (2010) on queues with reneging. The common theme of all these work is the heavy traffic technique being applicable to various features of the queues.

Finally, we discuss two papers that use similar approach and highlight our difference. One is a recent working paper by Bensusan and El Karoui (2009), who propose a microstructural approach to model population dynamics to capture mortality/longevity risk. Their motivation is different from ours: instead of building our mortality distribution microstructurally, we make common assumptions on mortality; instead, our focus is on how this mortality assumption, under the interaction with the contract structure, benefit level and premium calculation, leads to a macroscopic fluctuation of total assets, liabilities and other actuarial quantities. Secondly, we note that diffusion approximation has been invoked by Iglehart (1969) in arguing the use of Brownian motion in modeling insurance risk process. However, he maintained a Cramer-Lundberg framework by assuming compound Poisson claims and constantly drifted premium, and showed that under certain scaling their difference converges to a diffusion process. Contract structure, relation between premium and benefit, and actuarial reserve etc. were not considered in his work.

The organization of this paper is as follows. In Section 1 we lay out our model assumptions and define the key quantities that we approximate. Section 2 is devoted to the statement of our main

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result and its discussion. Section 3 relates to applications in ruin probability computations and shows some examples. Section 4 identifies the autocorrelation structure of our approximating Gaussian processes. Section 5 discusses some extensions. Appendix constitutes an appendix, which is divided into two parts. The first part discusses basic facts about heavy traffic limit theorems and gives the proof of our main result; the second part contains a discussion on the simulation methodology that is used to generate various examples in this paper.

CHEUNG, ERIC C K; FENG, RUNHUAN. *A unified analysis of claim costs up to ruin in a Markovian arrival risk model*. 98-109. An insurance risk model where claims follow a Markovian arrival process (MArP) is considered in this paper. It is shown that the expected present value of total operating costs up to default H, as a generalization of the classical Gerber-Shiu function, contains more non-trivial quantities than those covered in Cai *et al.* (2009) [J. Cai, R. Feng, G.E. Willmot (2009), On the total discounted operating costs up to default and its applications, Advances in Applied Probability, 41 (2) (2009): 495-522], such as all moments of the discounted claim costs until ruin. However, it does not appear that the Gerber-Shiu function ϕ with a generalized penalty function which additionally depends on the surplus level immediately after the second last claim before ruin (Cheung *et al.*, 2010a) [E.C.K. Cheung, D. Landriault, G.E. Willmot, J.-K. Woo, Gerber-Shiu analysis with a generalized penalty function, Scandinavian Actuarial Journal, 2010(3) (2010): 185-199] is contained in H. This motivates us to investigate an even more general function Z from which both H and ϕ can be retrieved as special cases. Using a matrix version of Dickson-Hipp operator (Feng, 2009b) [R. Feng, A matrix operator approach to the analysis of ruin-related quantities in the phase-type renewal risk model, Schweizerische Aktuarvereinigung Mitteilungen, 2009 (1-2) (2009): 71-87], it is shown that Z satisfies a Markov renewal equation and hence admits a general solution. Applications to other related problems such as the matrix scale function, the minimum and maximum surplus levels before ruin are given as well.

CHI, YICHUN; WENG, CHENGGUO. *Optimal reinsurance subject to Vajda condition*. 179-189. In this paper, we study optimal reinsurance design by minimizing the risk-adjusted value of an insurer's liability, where the valuation is carried out by a cost-of-capital approach based either on the value at risk or the conditional value at risk. To prevent moral hazard and to be consistent with the spirit of reinsurance, we follow Vajda (1962) [S Vajda, Minimum variance reinsurance, Astin Bulletin, 2 (2) (1962): 257-260] and assume that both the insurer's retained loss and the proportion paid by a reinsurer are increasing in indemnity. We analyze the optimal solutions for a wide class of reinsurance premium principles which satisfy three axioms (law invariance, risk loading and preserving convex order) and encompass ten of the eleven widely used premium principles listed in Young (2004) [V R Young, Premium principles, J. Teugels, B Sundt (editors), Encyclopedia of Actuarial Science, Vol. 3, John Wiley & Sons (2004)]. Our results show that the optimal ceded loss functions are in the form of three interconnected line segments. Further simplified forms of the optimal reinsurance are obtained for the premium principles under an additional mild constraint. Finally, to illustrate the applicability of our results, we derive the optimal reinsurance explicitly for both the expected value principle and Wang's principle.

CUI, WEI; YANG, JINGPING; WU, LAN. *Optimal reinsurance minimizing the distortion risk measure under general reinsurance premium principles*. 74-85. Recently the optimal reinsurance strategy concerning the insurer's risk attitude and the reinsurance premium principle has been an interesting topic. This paper discusses the optimal reinsurance problem with the insurer's risk measured by distortion risk measure and the reinsurance premium calculated by a general

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principle including expected premium principle and Wang's premium principle as its special cases. Explicit solutions of the optimal reinsurance strategy are obtained under the assumption that both the ceded loss and the retained loss are increasing with the initial loss. We present a new method for discussing the optimal problem. Based on our method, one can explain the optimal reinsurance treaty in the view of a balance between the insurer's risk measure and the reinsurance premium principle.

DI BERNARDINO, ELENA; RULLIÈRE, DIDIER. *Distortions of multivariate distribution functions and associated level curves: applications in multivariate risk theory*. 190-205. In this paper, we propose a parametric model for multivariate distributions. The model is based on distortion functions, i.e. some transformations of a multivariate distribution which permit to generate new families of multivariate distribution functions. We derive some properties of considered distortions. A suitable proximity indicator between level curves is introduced in order to evaluate the quality of candidate distortion parameters. Using this proximity indicator and properties of distorted level curves, we give a specific estimation procedure. The estimation algorithm is mainly relying on straightforward univariate optimizations, and we finally get parametric representations of both multivariate distribution functions and associated level curves. Our results are motivated by applications in multivariate risk theory. The methodology is illustrated on simulated and real examples.

FANG, Y; MADSEN, L. *Modified Gaussian pseudo-copula: applications in insurance and finance*. 292-301. The Gaussian copula is by far the most popular copula for modeling the association in finance and insurance risk problems. However, one major drawback of Gaussian copula is that it intrinsically lacks the flexibility of modeling the tail dependence, which real life data often exhibit. In this paper, we present the modified Gaussian copula, a pseudo-copula model that allows for both tail dependence and elliptical dependence structure. To improve model flexibility, the Gaussian copula is modified such that each correlation coefficient is not only an unknown parameter (to be modeled), but also a function of random variables. We present the characteristics of the modified Gaussian pseudo-copula and show that our modification enables the copula to capture the tail dependence adequately. The proposed modified Gaussian pseudo-copula is assessed by estimating the association on a real life insurance data and a finance data set. Furthermore, a comprehensive simulation study comparing goodness-of-fit test statistics is carried out. Both experiment results demonstrate that our Modified Gaussian pseudo-copula fits data (with or without tail dependence) generally better than other common copulas.

GUILLÉN, MONTSERRAT; SARABIA, JOSÉ MARÍA; PRIETO, FAUSTINO. *Simple risk measure calculations for sums of positive random variables*. 273-280. Closed-form expressions for basic risk measures, such as value-at-risk and tail value-at-risk, are given for a family of statistical distributions that are specially suitable for right-skewed positive random variables. This is useful for risk aggregation in many insurance and financial applications that model positive losses, where the Gaussian assumption is not valid. Our results provide a direct and flexible parametric approach to multivariate risk quantification, for sums of correlated positive loss distributions, that can be readily implemented in a spreadsheet.

HABERMAN, STEVEN; RENSHAW, ARTHUR. *Modelling and projecting mortality improvement rates using a cohort perspective*. 150-168. We investigate the feasibility of defining, modelling and projecting of (scaled) mortality improvement rates along cohort years-of-birth, that is, using a

cohort perspective. This is in contrast to the approach in the literature which has considered mortality improvement rates that are defined by reference to changes in mortality rates over successive calendar years, that is, using a period perspective. In this paper, we offer a comparison of the 2 parallel approaches to modelling and forecasting using mortality improvement rates. Comparisons of simulated life expectancy and annuity value predictions (mainly by the cohort method) using the England & Wales population mortality experiences for males and females under a variety of controlled data trimming exercises are presented and comparisons are also made between the parallel cohort and period based approaches.

HAO, XUEMIAO; LI, XUAN; SHIMIZU, YASUTAKA. *Finite-time survival probability and credit default swaps pricing under geometric Lévy markets*. 14-23. We study the first-passage time over a fixed threshold for a pure-jump subordinator with negative drift. We obtain a closed-form formula for its survival function in terms of marginal density functions of the subordinator. We then use this formula to calculate finite-time survival probabilities in a structural model for credit risk, and thus obtain a closed-form pricing formula for a single-name credit default swap (CDS). This pricing formula is well calibrated on market CDS quotes. In particular, it explains why the par CDS credit spread is not negligible when the maturity becomes short.

HASHORVA, ENKELEJD; LI, JINZHU. *ECOMOR and LCR reinsurance with gamma-like claims*. 206-215. Assuming that the claim sizes of an insurance company have a common distribution with gamma-like tail, we study the asymptotic tail behaviour of the reinsured amounts under the ECOMOR and LCR reinsurance treaties, respectively. Our novel results include a precise asymptotic expansion for the tail probability of the reinsured amounts under the ECOMOR treaty and tight asymptotic bounds for the LCR case. As a by-product we derive a precise asymptotic expansion for the tail of the product of independent regularly varying random variables.

JEVTIC, PETAR; LUCIANO, ELISA; VIGNA, ELENA. *Mortality surface by means of continuous time cohort models*. 122-133. The authors study and calibrate a cohort-based model which captures the characteristics of a mortality surface with a parsimonious, continuous-time factor approach. The model allows for imperfect correlation of the mortality intensity across generations. It is implemented on UK data for the period 1900-2008. Calibration by means of stochastic search and the Differential Evolution optimization algorithm proves to yield robust and stable parameters. We provide in-sample and out-of-sample, deterministic as well as stochastic forecasts. Calibration confirms that correlation across generations is smaller than one.

KIM, JOSEPH H T; JEON, YONGHO. *Credibility theory based on trimming*. 36-47. The classical credibility theory proposed by Bühlmann has been widely used in general insurance applications. In this paper we propose a credibility theory via truncation of the loss data, or the trimmed mean. The proposed framework contains the classical credibility theory as a special case and is based on the idea of varying the trimming threshold level to investigate the sensitivity of the credibility premium. After showing that the trimmed mean is not a coherent risk measure, we investigate some related asymptotic properties of the structural parameters in credibility. Later a numerical illustration shows that the proposed credibility models can successfully capture the tail risk of the underlying loss model, thus providing a better landscape of the overall risk that insurers assume.

LI, YONGWU; LI, ZHONGFEI. *Optimal time-consistent investment and reinsurance strategies for mean-variance insurers with state dependent risk aversion*. 86-97. In this paper, we study an

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insurer's optimal time-consistent strategies under the mean-variance criterion with state dependent risk aversion. It is assumed that the surplus process is approximated by a diffusion process. The insurer can purchase proportional reinsurance and invest in a financial market which consists of one risk-free asset and multiple risky assets whose price processes follow geometric Brownian motions. Under these, we consider two optimization problems, an investment-reinsurance problem and an investment-only problem. In particular, when the risk aversion depends dynamically on current wealth, the model is more realistic. Using the approach developed by Björk and Murgoci (2009) [Björk, T, Murgoci, A (2009), A general theory of Markovian time inconsistent stochastic control problems. Working Paper, Stockholm School of Economics], the optimal time-consistent strategies for the two problems are derived by means of corresponding extension of the Hamilton-Jacobi-Bellman equation. The optimal time-consistent strategies are dependent on current wealth, this case thus is more reasonable than the one with constant risk aversion.

LÓPEZ-DÍAZ, MARÍA CONCEPCIÓN; LÓPEZ-DÍAZ, MIGUEL. *A note on the family of extremality stochastic orders*. 230-236. The family of extremality stochastic orders was introduced in Laniado *et al*. (2012) [H Laniado, RE Lillo, F Pellerey, J Romo, Portfolio selection through an extremality stochastic order. Insurance: Mathematics and Economics 51, 1-9], as an extension of the upper and lower orthant orders, having important applications in the research of optimal allocations of wealth among risks in single period portfolio problems. In this paper we analyze some properties of such a family of stochastic orders, namely we prove that any extremality stochastic order is generated by a partial order on the Euclidean space and the class of upper quadrant sets of the partial order, showing that all the extremality orders are order-isomorphic. The above analysis will lead to the determination of the maximal generator of each extremality order by means of the maximal generator of the upper orthant order. Moreover we introduce a new family of stochastic orders which arises from the previous construction.

MENG, HUI; SIU, TAK KUEN; YANG, HAILIANG. *Optimal dividends with debts and nonlinear insurance risk processes*. 110-121. The optimal dividend problem is a classic problem in corporate finance though an early contribution to this problem can be traced back to the seminal work of an actuary, Bruno De Finetti, in the late 1950s. Nowadays, there is a leap of literature on the optimal dividend problem. However, most of the literature focus on linear insurance risk processes which fail to take into account some realistic features such as the nonlinear effect on the insurance risk processes. In this paper, we articulate this problem and consider an optimal dividend problem with nonlinear insurance risk processes attributed to internal competition factors. We also incorporate other important features such as the presence of debts, constraints in regular control variables, fixed transaction costs and proportional taxes. This poses some theoretical challenges as the problem becomes a nonlinear regular-impulse control problem. Under some suitable hypotheses for the value function, we obtain the structure of the value function using its properties, without guessing its structure, which is widely used in the literature. By solving the corresponding Hamilton-Jacobi-Bellman (HJB) equation, closed-form solutions to the problem are obtained in various cases.

QU, ZHIHUI; CHEN, YU. *Approximations of the tail probability of the product of dependent extremal random variables and applications*. 169-178. In this paper, we investigate the tail probability of the product $X\prod_{i=1}^{n} Y_i$, where $(X, Y_1, \ldots, Y_n)$ follows a multivariate Sarmanov distribution. An explicit asymptotic formula is established for the tail probability of the product when XX belongs to the Fréchet, Gumbel, or Weibull max-domain of attraction.

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As applications, we consider a discrete-time risk model with dependent insurance and financial risks, and obtain the asymptotic behavior for the (in)finite-time ruin probabilities.

ROBERT, CHRISTIAN Y. *Market Value Margin calculations under the Cost of Capital approach within a Bayesian chain ladder framework*. 216-229. n the Solvency II framework, insurance companies need to calculate the Best Estimate valuation of Liabilities (BEL) and the Market Value Margin (MVM) for non-hedgeable insurance-technical risks. The Cost-of-Capital approach defines the MVM as the present value of the current and future Solvency Capital Requirement (SCR) of the non-hedgeable risks to protect against adverse developments in the run-off of the insurance liabilities. However the SCR at time tt itself depends on the increase in the MVM between tt and t + 1t + 1. Hence there exists an intricate circularity dependency between both quantities. In this paper we present exact and accurate approximate analytic formulas for MVMs within a Bayesian log-normal chain ladder framework.

SANDERS, LISANNE; DE WAEGENAERE, ANJA; NIJMAN, THEO E. *When can insurers offer products that dominate delayed old-age pension benefit claiming?* 134-149. It is common practice for public pension schemes to offer individuals the option to delay benefit claiming until after the normal retirement age, and increase the annual benefit level as a result. Existing literature shows that for non-liquidity constrained individuals, delaying benefit claiming for a number of years after retirement is optimal from a utility perspective in a wide variety of cases. In this paper we focus on non-liquidity constrained individuals who wish to defer pension benefits, and investigate the attractiveness of an alternative deferral strategy. The alternative deferral strategy consists of claiming benefits immediately, and using them to buy deferred annuities from an insurance company. We first determine conditions under which the accrual offered by the public pension scheme for delaying benefit claiming is less than actuarially fair from the viewpoint of an insurer who uses the prevailing term structure of interest rates to determine the expected present value of missed and additional benefits. Actuarial unfairness can be generated by, e.g., age-independent accrual rates or slow adjustments of the accrual rates to changes in interest rates. We find that, in particular for men, the degree of actuarial unfairness is such that there is ample room for insurers to profitably offer annuity products that make the alternative deferral strategy preferred to deferring benefit claiming. If individuals choose to strategically exploit these alternative deferral strategies, this will affect benefit claiming behavior in public pension schemes, which in turn affects long run program costs.

TSAI, CARY CHI-LIANG; CHUNG, SAN-LIN. *Actuarial applications of the linear hazard transform in mortality immunization*. 48-63. In this paper, we apply the linear hazard transform to mortality immunization. When there is a change in mortality rates, the respective surplus (negative reserve) changes for life insurance and annuity policies lead to oppositive sign changes, which provides mortality hedging strategies with a portfolio of life insurance and annuity policies. We first show that by the strategy of matching duration of the weighted surplus at time 0, the surplus changes at time 0 for both portfolios [PTP] (the n-year term life insurance and the n-year pure endowment) and [PWA] (the n-payment whole life insurance and the n-year deferred whole life annuity) in response to a proportional or parallel shift in the underlying force of mortality are always negative. Next, we prove that the term life insurance, the whole life insurance and the deferred whole life annuity cannot always form a feasible portfolio (feasibility means that all the weights of the product members of a portfolio are positive) by the strategy of matching two durations or one duration and one convexity of the weighted surplus at time 0. Finally, numerical examples including figures and tables are exhibited for illustrations.

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WEI, J; WONG, K C; YAM, S C P; YUNG, S P. *Markowitz's mean-variance asset-liability management with regime switching: a time-consistent approach*. 281-291. In this article, we provide the first study in the time consistent solution of the mean-variance asset-liability management (MVALM). The framework is even considered under a continuous time Markov regime-switching setting. Using the extended Hamilton-Jacobi-Bellman equation (HJB) (see Björk and Murgoci (2010) [Björk, T, Murgoci, A (2010), A general theory of Markovian time inconsistent stochastic control problems. Working Paper, Stockholm School of Economics]), we show that the time consistent equilibrium control is state dependent in the sense that it depends on the uncontrollable liability process, which is in substantial contrast with the time consistent solution of the similar problem in Björk and Murgoci (2010), in which it is independent of the state. Finally, we give a numerical comparison between our work with the corrected version (as obtained here) of pre-commitment strategy in Chen *et al.* (2008) [P. Chen, H. Yang, G. Yin (2008), Markowitzs mean-variance asset-liability management with regime switching: a continuous-time model, Insurance: Mathematics and Economics, 43(3) (2008): 456-465].

ZHANG, ZHIMIN; YANG, HAILIANG. *Nonparametric estimate of the ruin probability in a pure-jump Lévy risk model*. 24-35. In this paper, we propose a nonparametric estimator of ruin probability in a Lévy risk model. The aggregate claims process X = {Xt, = 0}X = {Xt, = 0} is modeled by a pure-jump Lévy process. Assume that high-frequency observed data on XX are available. The estimator is constructed based on the Pollaczek-Khinchin formula and Fourier transform. Risk bounds as well as a data-driven cut-off selection methodology are presented. Simulation studies are also given to show the finite sample performance of our estimator.

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ANDERSSON, HANS; LAGERÅS, ANDREAS N. *Optimal bond portfolios with fixed time to maturity*. 429-438. We study interest rate models where the term structure is given by an affine relation and in particular where the driving stochastic processes are so-called generalized Ornstein-Uhlenbeck processes. For many institutional investors it is natural to consider investment in bonds where the time to maturity of the bonds in the portfolio is kept fixed over time. We show that the return and variance of such a portfolio of bonds which are continuously rolled over, also called rolling horizon bonds, can be expressed using the cumulant generating functions of the background driving Lévy processes associated with the OU processes. This allows us to calculate the efficient mean-variance portfolio. We exemplify the results by a case study on euro swap rates. We also show that if the short rate, in a risk-neutral setting, is given by a linear combination of generalized OU processes, the implied term structure can be expressed in terms of the cumulant generating functions. This makes it possible to quite easily see what kind of term structures can be generated with a particular short rate dynamics.

CHEUNG, ERIC C K. *Moments of discounted aggregate claim costs until ruin in a Sparre Andersen risk model with general interclaim times*. 343-354. In the context of a Sparre Andersen risk model with arbitrary interclaim time distribution, the moments of discounted aggregate claim costs until ruin are studied. Our analysis relies on a novel generalization of the so-called discounted density which further involves a moment-based component. More specifically, while the usual discounted density contains a discount factor with respect to the time of ruin, we propose to incorporate powers of the sum until ruin of the discounted (and possibly transformed) claims into the density.

Probabilistic arguments are applied to derive defective renewal equations satisfied by the moments of discounted aggregate claim costs until ruin. Detailed examples concerning the discounted aggregate claims and the number of claims until ruin are studied upon assumption on the claim severities. Numerical illustrations are also given at the end.

CHEUNG, KA CHUN; LO, AMBROSE. *Characterizations of counter-monotonicity and upper comonotonicity by (tail) convex order*. 334-342. In this paper, we characterize counter-monotonic and upper comonotonic random vectors by the optimality of the sum of their components in the senses of the convex order and tail convex order respectively. In the first part, we extend the characterization of comonotonicity by Cheung (2010) [K C Cheung (2010), Characterizing a comonotonic random vector by the distribution of the sum of its components, Insurance: Mathematics and Economics 47 (2010): 130-136] and show that the sum of two random variables is minimal with respect to the convex order if and only if they are counter-monotonic. Three simple and illuminating proofs are provided. In the second part, we investigate upper comonotonicity by means of the tail convex order. By establishing some useful properties of this relatively new stochastic order, we prove that an upper comonotonic random vector must give rise to the maximal tail convex sum, thereby completing the gap in Nam *et al.* (2011)'s characterization [H S Nam, Q Tang, F Yang (2011), Characterization of upper comonotonicity via tail convex order, Insurance: Mathematics and Economics 48 (2011): 368-373]. The relationship between the tail convex order and risk measures along with conditions under which the additivity of risk measures is sufficient for upper comonotonicity is also explored.

DIMITROVA, DIMITRINA S; HABERMAN, STEVEN; KAISHEV, VLADIMIR K. *Dependent competing risks: cause elimination and its impact on survival*. 464-477. The dependent competing risks model of human mortality is considered, assuming that the dependence between lifetimes is modelled by a multivariate copula function. The effect on the overall survival of removing one or more causes of death is explored under two alternative definitions of removal, ignoring the causes and eliminating them. Under the two definitions of removal, expressions for the overall survival functions in terms of the specified copula (density) and the net (marginal) survival functions are given. The net survival functions are obtained as a solution to a system of non-linear differential equations, which relates them through the specified copula (derivatives) to the crude (sub-) survival functions, estimated from data. The overall survival functions in a model with four competing risks, cancer, cardiovascular diseases, respiratory diseases and all other causes grouped together, have been implemented and evaluated, based on cause-specific mortality data for England and Wales published by the Office for National Statistics, for the year 2007. We show that the two alternative definitions of removal of a cause of death have different effects on the overall survival and in particular on the life expectancy at birth and at age 65, when one, two or three of the competing causes are removed. An important conclusion is that the eliminating definition is better suited for practical use in competing risks' applications, since it is more intuitive, and it suffices to consider only positive dependence between the lifetimes which is not the case under the alternative ignoring definition.

DONG, A X D; CHAN, J S K. *Bayesian analysis of loss reserving using dynamic models with generalized beta distribution*. 355-365. A Bayesian approach is presented in order to model long tail loss reserving data using the generalized beta distribution of the second kind (GB2) with dynamic mean functions and mixture model representation. The proposed GB2 distribution provides a flexible probability density function, which nests various distributions with light and heavy tails, to facilitate accurate loss reserving in insurance applications. Extending the mean

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functions to include the state space and threshold models provides a dynamic approach to allow for irregular claims behaviors and legislative change which may occur during the claims settlement period. The mixture of GB2 distributions is proposed as a mean of modeling the unobserved heterogeneity which arises from the incidence of very large claims in the loss reserving data. It is shown through both simulation study and forecasting that model parameters are estimated with high accuracy.

GRIFFIN, PHILIP S; MALLER, ROSS A; ROBERTS, DALE. *Finite time ruin probabilities for tempered stable insurance risk processes*. 478-489. We study the probability of ruin before time t for the family of tempered stable Lévy insurance risk processes, which includes the spectrally positive inverse Gaussian processes. Numerical approximations of the ruin time distribution are derived via the Laplace transform of the asymptotic ruin time distribution, for which we have an explicit expression. These are benchmarked against simulations based on importance sampling using stable processes. Theoretical consequences of the asymptotic formulae indicate that some care is needed in the choice of parameters to avoid exponential growth (in time) of the ruin probabilities in these models. This, in particular, applies to the inverse Gaussian process when the safety loading is less than one.

GUILLOU, ARMELLE; LOISEL, STÉPHANE; STUPFLER, GILLES. *Estimation of the parameters of a Markov-modulated loss process in insurance*. 388-404. We present a new model of loss processes in insurance. The process is a couple (N,L) where NN is a univariate Markov-modulated Poisson process (MMPP) and LL is a multivariate loss process whose behavior is driven by NN. We prove the strong consistency of the maximum likelihood estimator of the parameters of this model and present an EM algorithm to compute it in practice. The method is illustrated with simulations and real sets of insurance data.

GZYL, HENRYK; NOVI-INVERARDI, PIER-LUIGI; TAGLIANI, ALDO. *Determination of the probability of ultimate ruin by maximum entropy applied to fractional moments*. 457-463. In this work we present two different numerical methods to determine the probability of ultimate ruin as a function of the initial surplus. Both methods use moments obtained from the Pollaczek-Kinchine identity for the Laplace transform of the probability of ultimate ruin. One method uses fractional moments combined with the maximum entropy method and the other is a probabilistic approach that uses integer moments directly to approximate the density.

LI, SHENGGUO; PENG, JIN; ZHANG, BO. *The uncertain premium principle based on the distortion function*. 317-324. In this paper, we discuss the premium principle in uncertain environment. First, the net premium principle for uncertain risks is presented within the framework of uncertainty theory. With the help of distortion function, a new uncertain premium principle is derived from the uncertain net premium. Some properties of uncertain distortion premium principle are proved. Furthermore, a sufficient and necessary condition that an uncertain premium principle is an uncertain distortion premium principle has been characterized. Finally, some examples are given to illustrate the calculations of the uncertain distortion premium.

MALINOVSKII, VSEVOLOD K. *Rationale of underwriters' pricing conduct on competitive insurance market*. 325-333. Underwriters' desire to show a good annual review is known to be a rationale of the aggressive pricing conduct. On the competitive insurance market, it impacts the global insurance processes and can lead to the competition-originated underwriting cycles. Applying Lundberg's model of the annual probability mechanism of insurance, we model the

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influence of a price reduction on migration and consequently on the company's annual expansion, revenue and solvency.

MEYRICKE, RAMONA; SHERRIS, MICHAEL. *The determinants of mortality heterogeneity and implications for pricing annuities*. 379-387. Standard annuities are offered at one price to all individuals of the same age and gender. Individual mortality heterogeneity exposes insurers to adverse selection since only relatively healthy lives are expected to purchase annuities. As a result standard annuities are priced assuming above-average longevity, making them expensive for many individuals. In contrast underwritten annuity prices reflect individual risk factors based on underwriting information, as well as age and gender. While underwriting reduces heterogeneity, mortality risk still varies within each risk class due to unobservable individual risk factors, referred to as frailty. This paper quantifies the impact of heterogeneity due to underwriting factors and frailty on annuity values. Heterogeneity is quantified by fitting Generalized Linear Mixed Models to longitudinal data for a large sample of US males. The results show that heterogeneity remains after underwriting and that frailty significantly impacts the fair value of both standard and underwritten annuities. We develop a method to adjust annuity prices to allow for frailty.

PENG, XINGCHUN; HU, YIJUN. *Optimal proportional reinsurance and investment under partial information*. 416-428. In this paper, we study the optimal proportional reinsurance and investment strategy for an insurer that only has partial information at its disposal, under the criterion of maximizing the expected utility of the terminal wealth. We assume that the surplus of the insurer is governed by a jump diffusion process, and that reinsurance is used by the insurer to reduce risk. In addition, the insurer can invest in financial markets. We give a characterization for the optimal strategy within a non-Markovian setting. Malliavin calculus for Lévy processes is used for the analysis.

PICHLER, ALOIS. *The natural Banach space for version independent risk measures*. 405-415. Risk measures, or coherent measures of risk, are often considered on the space L^∞, and important theorems on risk measures build on that space. Other risk measures, among them the most important risk measure – the Average Value-at-Risk – are well defined on the larger space L^1 and this seems to be the natural domain space for this risk measure. Spectral risk measures constitute a further class of risk measures of central importance, and they are often considered on some L^p space. But in many situations this is possibly unnatural, because any L^p with $p>p_0$, say, is suitable to define the spectral risk measure as well. In addition to that, risk measures have also been considered on Orlicz and Zygmund spaces. So it remains for discussion and clarification, what the natural domain to consider a risk measure is? This paper introduces a norm, which is built from the risk measure, and a new Banach space, which carries the risk measure in a natural way. It is often strictly larger than its original domain and obeys the key property that the risk measure is finite valued and continuous on that space in an elementary and natural way.

VATAMIDOU, E; ADAN, I J B F; VLASIOU, M; ZWART, B. *Corrected phase-type approximations of heavy-tailed risk models using perturbation analysis*. 366-378. Numerical evaluation of performance measures in heavy-tailed risk models is an important and challenging problem. In this paper, we construct very accurate approximations of such performance measures that provide small absolute and relative errors. Motivated by statistical analysis, we assume that the claim sizes are a mixture of a phase-type and a heavy-tailed distribution and with the aid of perturbation analysis we derive a series expansion for the performance measure under consideration. Our proposed approximations consist of the first two terms of this series expansion, where the first

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term is a phase-type approximation of our measure. We refer to our approximations collectively as corrected phase-type approximations. We show that the corrected phase-type approximations exhibit a nice behavior both in finite and infinite time horizon, and we check their accuracy through numerical experiments.

ZHU, JINXIA; CHEN, FENG. *Dividend optimization for regime-switching general diffusions*. 439-456. We consider the optimal dividend distribution problem of a financial corporation whose surplus is modeled by a general diffusion process with both the drift and diffusion coefficients depending on the external economic regime as well as the surplus itself through general functions. The aim is to find a dividend payout scheme that maximizes the present value of the total dividends until ruin. We show that, depending on the configuration of the model parameters, there are two exclusive scenarios: (i)the optimal strategy uniquely exists and corresponds to paying out all surpluses in excess of a critical level (barrier) dependent on the economic regime and paying nothing when the surplus is below the critical level; (ii)there are no optimal strategies.

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AFONSO, LOURDES B; CARDOSO, RUI M R; EGIDIO DOS REIS, ALFREDO D. *Dividend problems in the dual risk model*. 906-918. We consider the compound Poisson dual risk model, dual to the well known classical risk model for insurance applications, where premiums are regarded as costs and claims are viewed as profits. The surplus can be interpreted as a venture capital like the capital of an economic activity involved in research and development. Like most authors, we consider an upper dividend barrier so that we model the gains of the capital and its return to the capital holders. By establishing a proper and crucial connection between the two models we show and explain clearly the dividends process dynamics for the dual risk model, properties for different random quantities involved as well as their relations. Using our innovative approach we derive some already known results and go further by finding several new ones. We study different ruin and dividend probabilities, such as the calculation of the probability of a dividend, distribution of the number of dividends, expected and amount of dividends as well as the time of getting a dividend. We obtain integro-differential equations for some of the above results and also Laplace transforms. From there we can get analytical results for cases where solutions and/or inversions are possible, in other cases we may only get numerical ones. We present examples under the two cases.

ASIMIT, ALEXANDRU V; BADESCU, ALEXANDRU M; CHEUNG, KA CHUN. *Optimal reinsurance in the presence of counterparty default risk*. 690-697. The optimal reinsurance arrangement is identified whenever the reinsurer counterparty default risk is incorporated in a one-period model. Our default risk model allows the possibility for the reinsurer to fail paying in full the promised indemnity, whenever it exceeds the level of regulatory capital. We also investigate the change in the optimal solution if the reinsurance premium recognises or not the default in payment. Closed form solutions are elaborated when the insurer's objective function is set via some well-known risk measures. It is also discussed the effect of reinsurance over the policyholder welfare. If the insurer is Value-at-Risk regulated, then the reinsurance does not increase the policyholder's exposure for any possible reinsurance transfer, even if the reinsurer may default in

paying the promised indemnity. Numerical examples are also provided in order to illustrate and conclude our findings. It is found that the optimal reinsurance contract does not usually change if the counterparty default risk is taken into account, but one should consider this effect in order to properly measure the policyholders exposure. In addition, the counterparty default risk may change the insurer's ideal arrangement if the buyer and seller have very different views on the reinsurer's recovery rate.

BAI, LIHUA; CAI, JUN; ZHOU, MING. *Optimal reinsurance policies for an insurer with a bivariate reserve risk process in a dynamic setting*. 664-670. Assume that an insurer has two dependent lines of business. The reserves of the two lines of business are modeled by a two-dimensional compound Poisson risk process or a common shock model. To protect from large losses and to reduce the ruin probability of the insurer, the insurer applies a reinsurance policy to each line of business, thus the two policies form a two-dimensional reinsurance policy. In this paper, we investigate the two-dimensional reinsurance policy in a dynamic setting. By using the martingale central limit theorem, we first derive a two-dimensional diffusion approximation to the two-dimensional compound Poisson reserve risk process. We then formulate the total reserve of the insurer by a controlled diffusion process and reduce the problem of optimal reinsurance strategies to a dynamic control problem for the controlled diffusion process. Under this setting, we show that a two-dimensional excess-of-loss reinsurance policy is an optimal form that minimizes the ruin probability of the controlled diffusion process. By solving a HJB equation with two dependent controls, we derive the explicit expressions of the optimal two-dimensional retention levels of the optimal two-dimensional excess-of-loss reinsurance policy and the minimized ruin probability. The results show that optimal dynamic two-dimensional retention levels are constant and the optimal retention levels are related by a deterministic function. We also illustrate the results by numerical examples.

BRECHMANN, EIKE CHRISTIAN; HENDRICH, KATHARINA; CZADO, CLAUDIA. *Conditional copula simulation for systemic risk stress testing*. 722-732. Since the financial crisis of 2007-2009 there is an active debate by regulators and academic researchers on systemic risk, with the aim of preventing similar crises in the future or at least reducing their impact. A major determinant of systemic risk is the interconnectedness of the international financial market. We propose to analyze interdependencies in the financial market using copulas, in particular using flexible vine copulas, which overcome limitations of the popular elliptical and Archimedean copulas. To investigate contagion effects among financial institutions, we develop methods for stress testing by exploiting the underlying dependence structure. New approaches for Archimedean and, especially, for vine copulas are derived. In a case study of 38 major international institutions, 20 insurers and 18 banks, we then analyze interdependencies of CDS [credit default swap] spreads and perform a systemic risk stress test. The specified dependence model and the results from the stress test provide new insights into the interconnectedness of banks and insurers. In particular, the failure of a bank seems to constitute a larger systemic risk than the failure of an insurer.

BURREN, DANIEL. *Insurance demand and welfare-maximizing risk capital – Some hints for the regulator in the case of exponential preferences and exponential claims*. 551-568. We propose two models to analyze welfare-maximizing capital requirements for insurance companies considering that capital is costly and therefore affecting the premium. Within a continuous-time model, we derive insurance demand and welfare as a function of personal wealth, the insurance company's wealth, and the claims process, and compare them to their counterparts in a static model. Besides discussing welfare-maximizing capital, we provide some new insights on insurance demand.

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CASTAÑER, ANNA; CLARAMUNT, M MERCÈ; LEFÈVRE, CLAUDE. *Survival probabilities in bivariate risk models, with application to reinsurance*. 632-642. This paper deals with an insurance portfolio that covers two interdependent risks. The central model is a discrete-time bivariate risk process with independent claim increments. A continuous-time version of compound Poisson type is also examined. Our main purpose is to develop a numerical method for determining non-ruin probabilities over a finite-time horizon. The approach relies on, and exploits, the existence of a special algebraic structure of Appell type. Some applications in reinsurance to the joint risks of the cedent and the reinsurer are presented and discussed, under a stop-loss or excess of loss contract.

CHEN, PING; YAM, S C P. *Optimal proportional reinsurance and investment with regime-switching for mean-variance insurers*. 871-883. Following the framework of Promislow and Young (2005) [S D Promislow, V R Young, Minimizing the probability of ruin when claims follow Brownian motion with drifts, North American Actuarial Journal (2005), 9: 109-128], this paper considers an optimal investment-reinsurance problem of an insurer facing a claim process modeled by a Brownian motion with drift under the Markowitz mean-variance criterion. The market modes are divided into a finite number of regimes. All the key parameters change according to the value of different market modes. The insurer chooses to purchase proportional reinsurance to reduce the underlying risk. In addition to reinsurance, we suppose that the insurer is allowed to invest its surplus in a financial market consisting of a risk-free asset (bond or bank account) and a risky asset whose price process is modeled by a geometric Brownian motion. We investigate the feasibility of the problem, obtain an analytic expression for the optimal strategy, delineate the efficient frontier and demonstrate our results with numerical examples.

CHEUNG, KA CHUN; LO, AMBROSE. *General lower bounds on convex functionals of aggregate sums*. 884-896. The determination of the dependence structure giving rise to the minimal convex sum in a general Fréchet space is a practical, yet challenging problem in quantitative risk management. In this article, we consider the closely related problem of finding lower bounds on three kinds of convex functionals, namely, convex expectations, Tail Value-at-Risk and the Haezendonck-Goovaerts risk measure, of a sum of random variables with arbitrary distributions. The sharpness of the lower bounds on the first two types of convex functionals is characterized via the extreme negative dependence structure of mutual exclusivity. Compared to existing results in the literature, our new lower bounds enjoy the advantages of generality and analytic tractability.

COSTABILE, MASSIMO. *Analytical valuation of periodical premiums for equity-linked policies with minimum guarantee*. 597-600. We consider the problem of computing fair periodical premiums of equity-linked policies with a minimum guarantee. The policy payoff at maturity may be decomposed into two components: a fixed part representing the guaranteed payment and a European call option written on the equity reference fund. The deemed periodical contributions into the reference fund may be considered as negative dividends paid by the reference fund and the fair value of the policy may be derived through a closed-form formula by mimicking the valuation of an option written on an underlying security that pays fixed dividends. Numerical results show that the proposed model computes accurate values.

DEELSTRA, GRISELDA; RAYÉE, GRÉGORY. *Pricing variable annuity guarantees in a local volatility framework*. 650-663. In this paper, we study the price of Variable Annuity Guarantees, particularly those of Guaranteed Annuity Options (GAO) and Guaranteed Minimum Income Benefit (GMIB), in the settings of a derivative pricing model where the underlying spot (the fund) is

locally governed by a geometric Brownian motion with local volatility, while interest rates follow a Hull-White one-factor Gaussian model. Notwithstanding the fact that in this framework, the local volatility depends on a particularly complex expectation where no closed-form expression exists and it is neither directly related to European call prices or other liquid products, we present in this contribution a method based on Monte Carlo Simulations to calibrate the local volatility model. We further compare the Variable Annuity Guarantee prices obtained in three different settings, namely the local volatility, the stochastic volatility and the constant volatility models all combined with stochastic interest rates and show that an appropriate volatility modeling is important for these long-dated derivatives. More precisely, we compare the prices of GAO, GMIB Rider and barrier types GAO obtained by using the local volatility, stochastic volatility and constant volatility models.

DURANTE, FABRIZIO; FERNÁNDEZ-SÁNCHEZ, JUAN; SEMPI, CARLO. *Multivariate patchwork copulas: a unified approach with applications to partial comonotonicity*. 897-905. We present a general view of patchwork constructions of copulas that encompasses previous approaches based on similar ideas (ordinal sums, gluing methods, piecing-together, etc.). Practical applications of the new methodology are connected with the determination of copulas having specified behaviour in the tails, such as upper comonotonic copulas.

DUTANG, CHRISTOPHE; LEFÈVRE, CLAUDE; LOISEL, STÉPHANE. *On an asymptotic rule $A + B/u$ for ultimate ruin probabilities under dependence by mixing*. 774-785. The purpose of this paper is to point out that an asymptotic rule [A + B/u] for the ultimate ruin probability applies to a wide class of dependent risk processes, in continuous or discrete time. That dependence is incorporated through a mixing model in the individual claim amount distributions. Several special mixing distributions are examined in detail and some close-form formulas are derived. Claim tail distributions and the dependence structure are also investigated.

ELING, MARTIN; HOLDER, STEFAN. *The value of interest rate guarantees in participating life insurance contracts: status quo and alternative product design*. 491-503. We compare cliquet-style interest rate guarantees used in German participating life insurance contracts across different economic environments. These guarantees are proportional to the average market interest rate at contract inception and typically set at 60% of the 10-year rolling average of government bond yields. Currently, however, in the face of prolonged low interest rates and stricter solvency regulation, the continued viability of this type of product is in question. A discussion of alternative guarantee designs is thus highly relevant. To this end, we perform a comparative analysis of contracts sold in different interest rate environments with regard to the guarantee value and show that the current practice of proportional guarantees leads to higher guarantee values the lower the market interest rate. We also observe an increased interest rate sensitivity. Additionally, alternative product designs that mitigate the interest rate dependency of the guarantee value are illustrated and assessed from the policyholder perspective.

FARD, FARZAD ALAVI; SIU, TAK KUEN. *Pricing participating products with Markov-modulated jump-diffusion process: an efficient numerical PIDE approach*. 712-721. We propose a model for the valuation of participating life insurance products under a generalized jump-diffusion model with a Markov-switching compensator. The Esscher transform is employed to determine an equivalent martingale measure in the incomplete market. The results are further manipulated through the utilization of the change of numeraire technique to reduce the dimensions of the pricing formulation. This paper is the first that extends the technique for a generalized

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jump-diffusion process with a Markov-switching kernel-biased completely random measure, which nests a number of important and popular models in finance. A numerical analysis is conducted to illustrate the practical implications.

GAJEK, LESLAW; KRAJEWSKA, ELZBIETA. *A new immunization inequality for random streams of assets, liabilities and interest rates*. 624-631. In this paper, we investigate the problem of immunization of insurers' surplus when liabilities are financed by a stream of assets. The term structure of interest rates is assumed to be random, as are the streams of assets and liabilities. A new inequality for changes in the portfolio surplus in response to changes in the term structure of interest rates is proven. A comparison with other immunization inequalities shows that it gives better lower bounds for a wide variety of scenarios. The inequality is sharp in the sense that the lower bound is attainable for some interest rate perturbations. Whenever net insurance premiums are considered, it is factorized into a product of two terms: one depending only on the change of interest rates, and the other depending only on the portfolio structure. Hence the second term may be treated as a measure of the interest rate risk. We call it L2L2-measure, because it is related to the second order distance between assets and liabilities. Explicit formulas for this measure for portfolios of some life products vs streams of net premiums are given. Applications to the Merton's, Vasicek's and simple log-normal models of interest rate are also provided.

GAN, GUOJUN. *Application of data clustering and machine learning in variable annuity valuation*. 795-801. The valuation of variable annuity guarantees has been studied extensively in the past four decades. However, almost all the studies focus on the valuation of guarantees embedded in a single variable annuity contract. How to efficiently price the guarantees for a large portfolio of variable annuity contracts has not received enough attention. This paper fills the gap by introducing a novel method based on data clustering and machine learning to price the guarantees for a large portfolio of variable annuity contracts. Our test results show that this method performs very well in terms of accuracy and speed.

GERBER, HANS U; SHIU, ELIAS S W; YANG, HAILIANG. *Valuing equity-linked death benefits in jump diffusion models*. 615-623. The paper is motivated by the valuation problem of guaranteed minimum death benefits in various equity-linked products. At the time of death, a benefit payment is due. It may depend not only on the price of a stock or stock fund at that time, but also on prior prices. The problem is to calculate the expected discounted value of the benefit payment. Because the distribution of the time of death can be approximated by a combination of exponential distributions, it suffices to solve the problem for an exponentially distributed time of death. The stock price process is assumed to be the exponential of a Brownian motion plus an independent compound Poisson process whose upward and downward jumps are modeled by combinations (or mixtures) of exponential distributions. Results for exponential stopping of a Lévy process are used to derive a series of closed-form formulas for call, put, lookback, and barrier options, dynamic fund protection, and dynamic withdrawal benefit with guarantee. We also discuss how barrier options can be used to model lapses and surrenders.

GOECKE, OSKAER. *Pension saving schemes with return smoothing mechanism*. 678-689. The smoothing of capital market returns is possible if the pension plan allows for some kind of intergenerational risk transfer. This can be realized if the total of assets of the pension fund is not fully allocated to individual saving accounts but part of the assets is allocated to a collective reserve (unallocated fund). High capital returns are then used to feed the collective reserve while poor capital market returns (or even losses) are compensated by withdrawals from the

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collective reserve. Traditional with-profit (or participation) life insurance contracts are basically designed in this way; however in most cases the smoothing process is quite opaque and leaves room for opportunistic management decisions. We introduce a continuous time model to discuss two questions: firstly, what kind of benefit do pension savers draw from a return smoothing mechanism and secondly, how should the smoothing mechanism be steered in order to maximize the benefit for the savers. We will derive limit distributions for the smoothed return process and discuss the risk return profile of smoothed pension schemes.

GOECKE, OSKAR. *Pension saving schemes with return smoothing mechanism*. 678-689. The smoothing of capital market returns is possible if the pension plan allows for some kind of intergenerational risk transfer. This can be realized if the total of assets of the pension fund is not fully allocated to individual saving accounts but part of the assets is allocated to a collective reserve (unallocated fund). High capital returns are then used to feed the collective reserve while poor capital market returns (or even losses) are compensated by withdrawals from the collective reserve. Traditional with-profit (or participation) life insurance contracts are basically designed in this way; however in most cases the smoothing process is quite opaque and leaves room for opportunistic management decisions. We introduce a continuous time model to discuss two questions: firstly, what kind of benefit do pension savers draw from a return smoothing mechanism and secondly, how should the smoothing mechanism be steered in order to maximize the benefit for the savers. We will derive limit distributions for the smoothed return process and discuss the risk return profile of smoothed pension schemes.

HATZOPOULOS, P; HABERMAN, STEVEN. *Corrigendum to 'Common mortality modeling and coherent forecasts. An empirical analysis of worldwide mortality data'* 919.

HE, LIN; LIANG, ZONGXIA. *Optimal investment strategy for the DC plan with the return of premiums clauses in a mean-variance framework*. 643-649. In this paper, we study the optimal investment strategy in the DC pension plan during the accumulation phase. During the accumulation phase, a pension member contributes a predetermined amount of money as premiums and the management of the pension plan invests the premiums in equities and bonds to increase the value of the accumulation. In practice, most of the DC pension plans have return of premium clauses to protect the rights of the plan members who die during the accumulation phase. In the model, the members withdraw their premiums when they die and the difference between the premium and the accumulation (negative or positive) is distributed among the survival members. From the surviving members' point of view, when they retire, they want to maximize the fund size and to minimize the volatility of the accumulation. We formalize the problem as a continuous-time mean-variance stochastic optimal control problem. The management of the pension plan chooses the optimal investment strategy, i.e., the proportions invested in equities and bonds, to maximize the mean-variance utility of the pension member at the time of retirement. Using the variational inequalities methods in Björk and Murgoci (2009) [T Björk, A Murgoci (2009), A general theory of Markovian time inconsistent stochastic control problems. Working Paper. Stockholm School of Economics], we transform the mean-variance stochastic control into Markovian time inconsistent stochastic control, then establish a verification theorem, which is similar to one of He and Liang (2008) [L He, Z Liang (2008), Optimal financing and dividend control of the insurance company with proportional reinsurance policy, Insurance: Mathematics & Economics (2008) 42: 976-983] and He and Liang (2009) [L He, Z Liang (2009), Optimal financing and dividend control of the insurance company with fixed and proportional transaction costs, Insurance: Mathematics & Economics (2009) 44: 88-94] and Zeng and Li (2011) [Y Zeng, Z F Li, Optimal time consistent

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investment and reinsurance policies for mean-variance insurers, Insurance: Mathematics & Economics (2011) 49: 145-154], to find the optimal strategy and the efficient frontier of the pension member. The differences of the optimal strategies between the Pension plans with and without the return of premium clauses are studied via the Monte Carlo methods. The impacts of the risk averse level on the optimal strategies is also explored by the numerical methods.

HUANG, RACHEL J; HUANG, YI-CHIEH; TZENG, LARRY Y. *Insurance bargaining under ambiguity*. 812-820. This paper investigates the effects of an increase in ambiguity aversion and an increase in ambiguity in an insurance bargaining game with a risk-and-ambiguity-neutral insurer and a risk-and-ambiguity-averse client. Both a cooperative and a non-cooperative bargaining game are examined. We show that, in both games, full coverage is optimal in the presence of ambiguity, and that the optimal premium is higher in the presence of ambiguity than in the absence of it. Furthermore, the optimal premium will increase with both the degree of ambiguity aversion and an increase in ambiguity.

HUDECOVÁ, SÁRKA; PEŠTA, MICHAL. *Modeling dependencies in claims reserving with GEE*. 786-794. A common approach to the claims reserving problem is based on generalized linear models (GLM), where the claims in different origin and development years are assumed to be independent variables. If this is violated, the classical techniques may provide incorrect predictions of the claims reserves or even misleading estimates of the prediction error. In this article, the application of generalized estimating equations (GEE) for the estimation of the claims reserves is shown. Claim triangles are handled as panel data, where the claim amounts within the same accident year are dependent. Various correlation structures are introduced within the GEE framework. Model selection criteria are proposed. An estimate for the mean square error of prediction for the claims reserves is derived in a nonstandard way and its advantages are discussed. Real data examples are provided as an illustration of the potential benefits of the presented approach and a simulation study is included.

JANG, JIWOOK; DASSIOS, ANGELOS. *A bivariate shot noise self-exciting process for insurance*. 524-532. In this paper, we study a bivariate shot noise self-exciting process. This process includes both externally excited joint jumps, which are distributed according to a shot noise Cox process, and two separate self-excited jumps, which are distributed according to the branching structure of a Hawkes process with an exponential fertility rate, respectively. A constant rate of exponential decay is included in this process as it can play a role as the time value of money in economics, finance and insurance applications. We analyse this process systematically for its theoretical distributional properties, based on the piecewise deterministic Markov process theory developed by Davis (1984) [M H A. Davis (1984), Piecewise deterministic Markov processes: a general class of non diffusion stochastic models, Journal of the Royal Statistical Society, Series B, 46: 353-388], and the martingale methodology used by Dassios and Jang (2003) [A Dassios, J Jang (2003), Pricing of catastrophe reinsurance & derivatives using the Cox process with shot noise intensity, Finance & Stochastics 7(1): 73-95]. The analytic expressions of the Laplace transforms of this process and the moments are presented, which have the potential to be applicable to a variety of problems in economics, finance and insurance. In this paper, as an application of this process, we provide insurance premium calculations based on its moments. Numerical examples show that this point process can be used for the modelling of discounted aggregate losses from catastrophic events.

JEON, YONGHO; KIM, JOSEPH H T. *A gamma kernel density estimation for insurance loss data*. 569-579. Fitting insurance loss data can be challenging because of their non-negativity,

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asymmetry, skewness, and possible multi-modality. Though many parametric models have been used in the actuarial literature, these difficulties call for more flexible models for actuarial applications. In this paper, we propose a new class of gamma kernel density estimators (GKDEs) based on the gamma density. We prove that the density of the proposed model converges to that of any loss random variable which is non-negative and continuous, and establish its rate of convergence, under some technical conditions. The proposed model has several advantages over the existing gamma kernel class by Chen (2000) [S Chen (2000), Probability density function estimation using gamma kernels, Annals of the Institute of Statistical Mathematics 52 (3): 471-480] in that it is a valid density for any finite sample and has standard distributional quantities, such as the moments, the conditional tail moments, and the compound distribution with GKDE claim amounts, in analytic form. The model is also a competing model of the Erlang mixture by Lee and Lin (2010) [S. Lee, X.S. Lin, Modeling and evaluating insurance losses via mixtures of Erlang distributions, North American Actuarial Journal, 14(1): 107] in its flexibility, but with a straightforward implementation and optimization. As numerical examples, we fit the gamma kernel density estimator to actual insurance data and find that the proposed model gives adequate results compared to the Erlang mixture and the Phase-type models.

JIN, ZHUO; YIN, G; WU, FUKE. *Optimal reinsurance strategies in regime-switching jump diffusion models: stochastic differential game formulation and numerical methods*. 733-746. This work develops a stochastic differential game model between two insurance companies who adopt the optimal reinsurance strategies to reduce the risk. The surplus is modeled by a regime-switching jump diffusion process. A single payoff function is imposed, and one player devises an optimal strategy to maximize the expected payoff function, whereas the other player is trying to minimize the same quantity. Using dynamic programming principle, the upper and lower values of the game satisfy a coupled system of nonlinear integro-differential Hamilton-Jacobi-Isaacs (HJI) equations. Moreover, the existence of the saddle point for this game problem is verified. Because of the jumps and regime-switching, closed-form solutions are virtually impossible to obtain. Our effort is devoted to designing numerical methods. We use Markov chain approximation techniques to construct a discrete-time controlled Markov chain to approximate the value functions and optimal controls. Convergence of the approximation algorithms is proved. Examples are presented to illustrate the applicability of the numerical methods.

KRÄMER, NICOLE; SILVESTRINI, DANIEL; CZADO, CLAUDIA. *Total loss estimation using copula-based regression models*. 829-839. We present a joint copula-based model for insurance claims and sizes. It uses bivariate copulae to accommodate for the dependence between these quantities. We derive the general distribution of the policy loss without the restrictive assumption of independence. We illustrate that this distribution tends to be skewed and multi-modal, and that an independence assumption can lead to substantial bias in the estimation of the policy loss. Further, we extend our framework to regression models by combining marginal generalized linear models with a copula. We show that this approach leads to a flexible class of models, and that the parameters can be estimated efficiently using maximum-likelihood. We propose a test procedure for the selection of the optimal copula family. The usefulness of our approach is illustrated in a simulation study and in an analysis of car insurance policies.

LIN, TZULING; TSAI, CARY CHI-LIANG. *On the mortality/longevity risk hedging with mortality immunization*. 580-596. In this paper, we define the mortality durations and convexities of the prices of life insurance and annuity products with respect to an instantaneously proportional change and an instantaneously parallel shift, respectively, in μs (the forces of mortality), ps

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(the one-year survival probabilities) and *qs* (the one-year death probabilities), and further derive them as magnitude-free closed-form formulas. Then we propose several duration/convexity matching strategies to determine the weights of two or three products in an insurance portfolio. With the stochastic mortality models, we evaluate the Value-at-Risk (VaR) values and the hedge effectiveness of the surpluses at time zero for the underlying portfolio with these matching strategies. Illustrated numerical examples demonstrate that the duration/convexity matching strategies with respect to an instantaneously proportional change in μs and *qs* can significantly hedge the mortality/longevity risks.

LIU, YONG-JUN; ZHANG, WEI-GUO. *Fuzzy portfolio optimization model under real constraints*. 704-711. This paper discusses a multi-objective portfolio optimization problem for practical portfolio selection in fuzzy environment, in which the return rates and the turnover rates are characterized by fuzzy variables. Based on the possibility theory, fuzzy return and liquidity are quantified by possibilistic mean, and market risk and liquidity risk are measured by lower possibilistic semivariance. Then, two possibilistic mean-semivariance models with real constraints are proposed. To solve the proposed models, a fuzzy multi-objective programming technique is utilized to transform them into corresponding single-objective models and then a genetic algorithm is designed for solution. Finally, a numerical example is given to illustrate the application of our models. Comparative results show that the designed algorithm is effective for solving the proposed models.

MAEGEBIER, ALEXANDER. *Valuation and risk assessment of disability insurance using a discrete time trivariate Markov renewal reward process*. 802-811. In disability insurance, the impact of the duration since the inception of disability on future recovery and mortality rates has been modeled by bivariate Markov renewal processes and the associated semi-Markov process, but these processes do not incorporate potential dependences between the durations in two successive states. Thus, the aim of this paper is to introduce a discrete time trivariate Markov renewal reward model, an associated formula for higher moments and a corresponding simulation that include the potential dependence between the durations, i.e. the inter-arrival times, in two successive states. The proposed model is compared with two alternative models that do not include this dependence.

MAGNI, CARLO ALBERTO. *Generalized Makeham's formula and economic profitability*. 747-756. This paper generalizes Makeham's formula, allowing for varying interest rates and for a non-flat structure of valuation rates. An average interest rate (AIR) is introduced, as well as an average valuation rate (AVR), both of which exist and are unique for any asset. They can be computed either as principal-weighted arithmetic means or as interest-weighted harmonic means of period rates. Economic profitability of an asset or a portfolio of assets is captured by the spread between AIR and AVR, which has the same sign as the Net Present Value. This makes (i) AIR a more reliable tool for valuation and decision than the venerable Internal Rate of Return, and (ii) AVR a natural generalization of the cost-of-capital notion.

PELSSER, ANTOON A J; LAEVEN, ROGER J A. *Optimal dividends and ALM under unhedgeable risk*. 515-523. In this paper we develop a framework for optimal investment decisions for insurance companies in the presence of (partially) unhedgeable risk. The perspective that we choose is from an insurance company that maximises the stream of dividends paid to its shareholders. The policy instruments that the company has are the dividend policy and the investment policy. Using stochastic control theory, we derive simultaneously the optimal investment policy and the optimal dividend policy, taking the insurance risks to be given. We study the trade off between investing in the optimal hedge portfolio and the fully diversified portfolio.

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We show next how the pricing of unhedgeable risk can also be embedded in our framework. Finally, we derive the distribution of the time of bankruptcy and demonstrate its usefulness in calibrating the model.

PUCCETTI, GIOVANNI; WANG, BIN; WANG, RUODU. *Complete mixability and asymptotic equivalence of worst-possible VaR and ES estimates*. 821-828. We give a new sufficient condition for a continuous distribution to be completely mixable, and we use this condition to show that the worst-possible value-at-risk for the sum of d inhomogeneous risks is equivalent to the worst-possible expected shortfall under the same marginal assumptions, in the limit as $d \to \infty$. Numerical applications show that this equivalence holds also for relatively small dimensions d.

RASSOUL, ABDELAZIZ. *Kernel-type estimator of the conditional tail expectation for a heavy-tailed distribution*. 698-703. In this paper, we are interested in the generalization and improvement of the estimator of the conditional tail expectation (CTE) for a heavy-tailed distribution when the second moment is infinite. It is well known that classical estimators of the CTE are seriously biased under the second-order regular variation framework. To reduce the bias, many authors proposed the use of so-called second-order reduced bias estimators for both first-order and second-order tail parameters. In this work, we have generalized a kernel-type estimator, and we present a number of results on its distributional behavior and compare its performance with the performance of other estimators.

SHAPIRO, ARNOLD F. *Modeling future lifetime as a fuzzy random variable*. 864-870. A recent article by de Andrés-Sánchez and Puchades (2012) [J. de Andrés-Sánchez, L.G-V. Puchades, Using fuzzy random variables in life annuities pricing, Fuzzy Sets and Systems (2012) 188(1): 27-44] modeled life annuities as fuzzy random variables (FRVs). Their article was informative. However, it had the limitation that the FRV used to model the life annuity was not a granulated FRV. This followed because the authors assumed that the uncertainty insofar as mortality is entirely due to randomness and that the uncertainty with respect to interest rates is entirely due to fuzziness. The concern is that such a dichotomy may be problematic since, in actuality, the uncertainty of both the mortality parameter and the interest rate parameter can have both random and fuzzy features. The purpose of this article is to address the mortality portion of this dichotomy and, to this end, we model future lifetime as a FRV.

SHEN, YANG; SIU, TAK KUEN. *Stochastic differential game, Esscher transform and general equilibrium under a Markovian regime-switching Lévy model*. 757-768. In this paper, we discuss three different approaches to select an equivalent martingale measure for the valuation of contingent claims under a Markovian regime-switching Lévy model. These approaches are the game theoretic approach, the Esscher transformation approach and the general equilibrium approach. We employ the dynamic programming principle to derive the optimal strategies and the value functions in the stochastic differential game and the general equilibrium approaches, each of which lead to an equivalent martingale measure. We also compare equivalent martingale measures chosen by the three approaches. Under certain conditions, the equivalent martingale measures chosen by the stochastic differential game and the Esscher transformation approaches coincide. If the equity premium is in its equilibrium state, the equivalent martingale measures chosen by the Esscher transformation and the general equilibrium approaches are identical.

XU, MAOCHAO; MAO, TIANTIAN. *Optimal capital allocation based on the Tail Mean-Variance model*. 533-543. This paper studies capital allocation problems with the aggregate risk exceeding a

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certain threshold. We propose a novel capital allocation rule based on the Tail Mean-Variance principle. General formulas for the optimal capital allocations are proposed. Explicit formulas for optimal capital allocations are derived for multivariate elliptical distributions. Moreover, we give asymptotic allocation formulas for multivariate regular variation variables. Various numerical examples are given to illustrate the results, and real insurance data is discussed as well.

YAO, HAIXIANG; YANG, ZHOU; CHEN, PING. *Markowitz's mean-variance defined contribution pension fund management under inflation: a continuous-time model.* 851-863. In defined contribution (DC) pension schemes, the financial risk borne by the member occurs during the accumulation phase. To build up sufficient funds for retirement, scheme members invest their wealth in a portfolio of assets. This paper considers an optimal investment problem of a scheme member facing stochastic inflation under the Markowitz mean-variance criterion. Besides, we consider a more general market with multiple assets that can all be risky. By applying the Lagrange method and stochastic dynamic programming techniques, we derive the associated Hamilton-Jacobi-Bellman (HJB) equation, which can be converted into six correlated but relatively simple partial differential equations (PDEs). The explicit solutions for these six PDEs are derived by using the homogenization approach and the variable transformation technique. Then the closed-form expressions for the optimal strategy and the efficient frontier can be obtained through the Lagrange dual theory. In addition, we illustrate the results by some numerical examples.

YI, BO; LI, ZHONGFEI; VIENS, FREDERI G; ZENG, YAN. *Robust optimal control for an insurer with reinsurance and investment under Heston's stochastic volatility model.* 601-614. This paper considers a robust optimal reinsurance and investment problem under Heston's Stochastic Volatility (SV) model for an Ambiguity-Averse Insurer (AAI), who worries about model misspecification and aims to find robust optimal strategies. The surplus process of the insurer is assumed to follow a Brownian motion with drift. The financial market consists of one risk-free asset and one risky asset whose price process satisfies Heston's SV model. By adopting the stochastic dynamic programming approach, closed-form expressions for the optimal strategies and the corresponding value functions are derived. Furthermore, a verification result and some technical conditions for a well-defined value function are provided. Finally, some of the model's economic implications are analyzed by using numerical examples and simulations. We find that ignoring model uncertainty leads to significant utility loss for the AAI. Moreover we propose an alternative model and associated investment strategy which can be considered more adequate under certain finance interpretations, and which leads to significant improvements in our numerical example.

YIN, CHUANCUN; WEN, YUZHEN. *Optimal dividend problem with a terminal value for spectrally positive Lévy processes.* 769-773. In this paper we consider a modified version of the classical optimal dividend problem taking into account both expected dividends and the time value of ruin. We assume that the risk process is modeled by a general spectrally positive Lévy process before dividends are deducted. Using the fluctuation theory of spectrally positive Lévy processes we give an explicit expression of the value function of a barrier strategy. Subsequently we show that a barrier strategy is the optimal strategy among all admissible ones. Our work is motivated by the recent work of Bayraktar, Kyprianou and Yamazaki (2013a). [E Bayraktar, A Kyprianou, K Yamazaki, On optimal dividends in the dual model, Astin Bulletin (2013) 43(3): 359-372]

ZENG, XUDONG; LUO, SHANGZHEN. *Stochastic Pareto-optimal reinsurance policies.* 671-677. We model reinsurance as a stochastic cooperation game in a continuous-time framework.

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Employing stochastic control theory and dynamic programming techniques, we study Pareto-optimal solutions to the game and derive the corresponding Hamilton-Jacobi-Bellman (HJB) equation. After analyzing the HJB equation, we show that the Pareto-optimal policies may be classified into either unlimited excess of loss functions or proportional functions based on different premium share principles. To illustrate our results, we solve several examples for explicit solutions.

ZHANG, HONGZHONG; LEUNG, TIM; HADJILIADIS, OLYMPIA. *Stochastic modeling and fair valuation of drawdown insurance*. 840-850. This paper studies the stochastic modeling of market drawdown events and the fair valuation of insurance contracts based on drawdowns. We model the asset drawdown process as the current relative distance from the historical maximum of the asset value. We first consider a vanilla insurance contract whereby the protection buyer pays a constant premium over time to insure against a drawdown of a pre-specified level. This leads to the analysis of the conditional Laplace transform of the drawdown time, which will serve as the building block for drawdown insurance with early cancellation or drawup contingency. For the cancellable drawdown insurance, we derive the investor's optimal cancellation timing in terms of a two-sided first passage time of the underlying drawdown process. Our model can also be applied to insure against a drawdown by a defaultable stock. We provide analytic formulas for the fair premium and illustrate the impact of default risk.

ZHAO, HUI; RONG, XIMIN; ZHAO, YONGGAN. *Optimal excess-of-loss reinsurance and investment problem for an insurer with jump-diffusion risk process under the Heston model*. 504-514. In this paper, we study the optimal excess-of-loss reinsurance and investment problem for an insurer with jump-diffusion risk model. The insurer is allowed to purchase reinsurance and invest in one risk-free asset and one risky asset whose price process satisfies the Heston model. The objective of the insurer is to maximize the expected exponential utility of terminal wealth. By applying stochastic optimal control approach, we obtain the optimal strategy and value function explicitly. In addition, a verification theorem is provided and the properties of the optimal strategy are discussed. Finally, we present a numerical example to illustrate the effects of model parameters on the optimal investment-reinsurance strategy and the optimal value function.

ZHU, LINGJIONG. *Ruin probabilities for risk processes with non-stationary arrivals and subexponential claims*. 544-550. In this paper, we obtain the finite-horizon and infinite-horizon ruin probability asymptotics for risk processes with claims of subexponential tails for non-stationary arrival processes that satisfy a large deviation principle. As a result, the arrival process can be dependent, non-stationary and non-renewal. We give three examples of non-stationary and non-renewal point processes: Hawkes process, Cox process with shot noise intensity and self-correcting point process. We also show some aggregate claims results for these three examples.

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AHCAN, ALES; MEDVED, DARKO; OLIVIERI, ANNAMARIA; PITACCO, ERMANNO. *Forecasting mortality for small populations by mixing mortality data*. 12-27. In this paper we address the problem of projecting mortality when data are severely affected by random fluctuations, due in particular to a small sample size, or when data are scanty. Such situations may emerge when dealing with small populations, such as small countries (possibly previously part of a larger country), a specific geographic area of a (large) country, a life annuity portfolio or a pension

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fund, or when the investigation is restricted to the oldest ages. The critical issues arising from the volatility of data due to the small sample size (especially at the highest ages) may be made worse by missing records; this is the case, for example, of a small country previously part of a larger country, or a specific geographic area of a country, given that in some periods mortality data could have been collected just at an aggregate level. We suggest to 'replicate' the mortality of the small population by mixing appropriately the mortality data obtained from other populations. We design a two-step procedure. First, we obtain the average mortality of 'neighboring' populations. Three alternative approaches are tested for the assessment of the average mortality; conversely, the identification and the weight of the neighboring populations are obtained through (standard) optimization techniques. Then, following a sort of credibility approach, we mix the original mortality data of the small population with the average mortality of the neighboring populations. In principle, the approach described in the paper could be adopted for any population, whatever is its size, aiming at improving mortality projections through information collected from other groups. Through backtesting, we show that the procedure we suggest is convenient for small populations, but not necessarily for large populations, nor for populations not showing noticeable erratic effects in data. This finding can be explained as follows: while the replication of the original data implies the increase of the size of the sample, it also involves a smoothing of data, with a possible loss of specific information relating to the group referred to. In the case of small populations showing major erratic movements in mortality data, the advantages gained from the larger sample size overcome the disadvantages of the smoothing effect.

BAYRAKTAR, ERHAN; KYPRIANOU, ANDREAS E; YAMAZAKI, KAZUTOSHI. *Optimal dividends in the dual model under transaction costs*. 133-143. We analyze the optimal dividend payment problem in the dual model under constant transaction costs. We show, for a general spectrally positive Lévy process, an optimal strategy is given by a (c_1, c_2)-policy that brings the surplus process down to c_1 whenever it reaches or exceeds c_2 for some $0 \leq c_1 < c_2$. The value function is succinctly expressed in terms of the scale function. A series of numerical examples are provided to confirm the analytical results and to demonstrate the convergence to the no-transaction cost case, which was recently solved by Bayraktar *et al.* (2013) [E. Bayraktar, A. Kyprianou, K. Yamazaki, On optimal dividends in the dual model, Astin Bulletin (2013) 43(3): 359-273].

BELLINI, FABIO; KLARM, BERNHARD; MÜLLER, ALFRED; GIANIN, EMANUELA ROSAZZA. *Generalized quantiles as risk measures*. 41-48. In the statistical and actuarial literature several generalizations of quantiles have been considered, by means of the minimization of a suitable asymmetric loss function. All these generalized quantiles share the important property of elicitability, which has received a lot of attention recently since it corresponds to the existence of a natural backtesting methodology. In this paper we investigate the case of *M*-quantiles as the minimizers of an asymmetric convex loss function, in contrast to Orlicz quantiles that have been considered in Bellini and Rosazza Gianin (2012) [F. Bellini, E. Rosazza Gianin, Haezendonck-Goovaerts risk measures and Orlicz quantiles, Insurance: Mathematics & Economics (2012), 51: 107-114]. We discuss their properties as risk measures and point out the connection with the zero utility premium principle and with shortfall risk measures introduced by Föllmer and Schied (2002) [H. Föllmer, A. Schied, Convex measures of risk and trading constraints, Finance and Stochastics (2002) 6: 429-447]. In particular, we show that the only *M*-quantiles that are coherent risk measures are the expectiles, introduced by Newey and Powell (1987) [W. Newey, J. Powell, Asymmetric least squares estimation and testing, Econometrica (1987) 55: 819-847] as the minimizers of an asymmetric quadratic loss function. We provide their dual and Kusuoka

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representations and discuss their relationship with CVaR. We analyze their asymptotic properties for a $\alpha \to 1$ and show that for very heavy tailed distributions expectiles are more conservative than the usual quantiles. Finally, we show their robustness in the sense of lipschitzianity with respect to the Wasserstein metric.

BERNARD, CAROLE; JIANG, XIAO; WANG, RUODU. *Risk aggregation with dependence uncertainty*. 93-108. Risk aggregation with dependence uncertainty refers to the sum of individual risks with known marginal distributions and unspecified dependence structure. We introduce the admissible risk class to study risk aggregation with dependence uncertainty. The admissible risk class has some nice properties such as robustness, convexity, permutation invariance and affine invariance. We then derive a new convex ordering lower bound over this class and give a sufficient condition for this lower bound to be sharp in the case of identical marginal distributions. The results are used to identify extreme scenarios and calculate bounds on Value-at-Risk as well as on convex and coherent risk measures and other quantities of interest in finance and insurance. Numerical illustrations are provided for different settings and commonly-used distributions of risks.

BOUDREAULT, MATHIEU; COSSETTE, HÉLÈNE; MARCEAU, ÉTIENNE. *Risk models with dependence between claim occurrences and severities for Atlantic hurricanes*. 123-132. In the line of Cossette *et al.* (2003) [H Cossette, T Duchesne, É. Marceau, Modelling catastrophes and their impact on insurance portfolios, North American Actuarial Journal (2003) 7: 1-22], we adapt and refine known Markovian-type risk models of Asmussen (1989) and Lu and Li (2005) [Y. Lu, S. Li, On the probability of ruin in a Markov-modulated risk model, Insurance: Mathematics & Economics (2005) 37: 522-532] to a hurricane risk context. These models are supported by the findings that El Niño/Southern Oscillation (as well as other natural phenomena) influence both the number of hurricanes and their strength. Hurricane risk is thus broken into three components: frequency, intensity and damage where the first two depend on the state of the Markov chain and intensity influences the amount of damage to an individual building. The proposed models are estimated with Florida hurricane data and several risk measures are computed over a fictitious portfolio.

CHEN, AN; UZELAC, FILIP. *A risk-based premium: what does it mean for DB plan sponsors?* 1-11. This paper develops a risked-based premium calculation model for the insurance provided by the Pension Benefit Guaranty Corporation (PBGC). It takes account of the pension fund's and the plan sponsor's investment policy and extends Chen (2011) [A. Chen, A risk-based model for the valuation of pension insurance, Insurance: Mathematics & Economics (2011) 49(3): 401-409] by considering distress termination triggered by the sponsor's underfunding. We empirically illustrate our theoretical pricing formula for the 100 biggest American DB sponsoring companies. Our result clearly casts doubt on the current practice where about 70% of the PBGC premiums charged are flat. We observe that the funding ratio and the leverage are the main risk factors in a risk-based premium calculation.

CHEN, XU; XIAO, TING; YANG, XIANG-QUN. *A Markov-modulated jump-diffusion risk model with randomized observation periods and threshold dividend strategy*. 76-83. This paper considers a Markov-modulated jump-diffusion risk model with randomized observation periods and threshold dividend. A second order integro-differential system of equations that characterizes the expected discounted dividend payments is obtained. As a closed-form solution does not exist, a numerical procedure based on the sinc function approximation through a collocation method is proposed. Finally, an example illustrating the procedure is presented.

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CHEUNG, KA CHUN; RONG, YIAN; YAM, S C P. *Borch's Theorem from the perspective of comonotonicity*. 144-151. This short note revisits the classical Theorem of Borch on the characterization of Pareto optimal risk exchange treaties under the expected utility paradigm. Our objective is to approach the optimal risk exchange problem by a new method, which is based on a Breeden-Litzenberger type integral representation formula for increasing convex functions and the theory of comonotonicity. Our method allows us to derive Borch's characterization without using Kuhn-Tucker theory, and also without the need of assuming that all utility functions are continuously differentiable everywhere. We demonstrate that our approach can be used effectively to solve the Pareto optimal risk-sharing problem with a positivity constraint being imposed on the admissible allocations when the aggregate risk is positive.

CHEUNG, KA CHUN; DHAENE, JAN; LO, AMBROSE; TANG, QIHE. *Reducing risk by merging counter-monotonic risks*. 58-65. In this article, we show that some important implications concerning comonotonic couples and corresponding convex order relations for their sums cannot be translated to counter-monotonicity in general. In a financial context, it amounts to saying that merging counter-monotonic positions does not necessarily reduce the overall level of risk. We propose a simple necessary and sufficient condition for such a merge to be effective. Natural interpretations and various characterizations of this condition are given. As applications, we develop cancelation laws for convex order and identify desirable structural properties of insurance indemnities that make an insurance contract universally marketable, in the sense that it is appealing to both the policyholder and the insurer.

DE-PAZ, ALBERT; MARÍN-SOLANO, JESÚS; NAVE, JUAN M; ROCH, ORIOL. *Consumption, investment and life insurance strategies with heterogeneous discounting*. 66-75. In this paper we analyze how the optimal consumption, investment and life insurance rules are modified by the introduction of a class of time-inconsistent preferences. In particular, we account for the fact that an agent's preferences evolve along the planning horizon according to her increasing concern about the bequest left to her descendants and about her welfare at retirement. To this end, we consider a stochastic continuous time model with random terminal time for an agent with a known distribution of lifetime under heterogeneous discounting. In order to obtain the time-consistent solution, we solve a non-standard dynamic programming equation. For the case of CRRA [constant relative risk aversion] and CARA [constant absolute risk aversion] utility functions we compare the explicit solutions for the time-inconsistent and the time-consistent agent. The results are illustrated numerically.

GÓMEZ DÉNIZ, EMILIO; SORDO, MIGUEL A; CALDERÍN-OJEDA, ENRIQUE. *The Log-Lindley distribution as an alternative to the beta regression model with applications in insurance*. 49-57. In this paper a new probability density function with bounded domain is presented. The new distribution raises from the generalized Lindley distribution proposed by Zakerzadeh and Dolati (2010) [H. Zakerzadeh, A. Dolati, Generalized Lindley distribution, Journal of Mathematical Extension (2010) 3(2): 13-25]. This new distribution that depends on two parameters can be considered as an alternative to the classical beta distribution. It presents the advantage of not including any special function in its formulation. After studying its most important properties, some useful results regarding insurance and inventory management applications are obtained. In particular, in insurance, we suggest a special class of distorted premium principles based on this distribution and we compare it with the well-known power dual premium principle. Since the mean of the new distribution can be normalized to give a simple parameter, this new model is appropriate to be used as a regression model when the response is

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bounded, being therefore an alternative to the beta regression model recently proposed in the statistical literature.

GUAN LIM, KIAN; LIANG, ZONGXIA. *Viscosity solution and impulse control of the diffusion model with reinsurance and fixed transaction costs*. 109-122. We consider an optimal impulse control problem on reinsurance, dividend and reinvestment of an insurance company. To close reality, we add fixed and proportional transaction costs to this problem. The value of the company is associated with expected present value of net dividends pay out minus the net reinvestment capitals until ruin time. We focus on non-cheap proportional reinsurance. We prove that the value function is a unique solution to associated Hamilton-Jacobi-Bellman equation, and establish the regularity property of the viscosity solution under a weak assumption. We solve the non-uniformly elliptic equation associated with the impulse control problem. Finally, we derive the value function and the optimal strategy of the control problem.

YANG, CHEN; SENDOVA, KRISTINA P. *The ruin time under the Sparre-Andersen dual model*. 28-40. In this paper, we study the Sparre-Andersen dual risk model in which the times between positive gains are independently and identically distributed and have a generalized Erlang-nn distribution. An important difference between this model and some other models such as the Erlang-nn dual risk model is that the roots to the generalized Lundberg's equation are not necessarily distinct. Hence, we derive an explicit expression for the Laplace transform of the ruin time, which involves multiple roots. Also, we apply our approach for obtaining the expected discounted dividends when the threshold-dividend strategy discussed by Ng (2009) [A. Ng, On a dual model with a dividend threshold, Insurance: Mathematics and Economics (2009) 44(2): 315-324] is implemented under the Sparre-Andersen model with Erlang-nn distribution of the inter-event times. In particular, we derive an explicit form of the expected discounted dividends when jump sizes are exponential.

YAO, HAIXIANG; LAI, YONGZENG; MA, QINGHUA; JIAN, MINJIE. *Asset allocation for a DC pension fund with stochastic income and mortality risk: a multi-period mean-variance framework*. 84-92. This paper investigates an asset allocation problem for defined contribution pension funds with stochastic income and mortality risk under a multi-period mean-variance framework. Different from most studies in the literature where the expected utility is maximized or the risk measured by the quadratic mean deviation is minimized, we consider synthetically both to enhance the return and to control the risk by the mean-variance criterion. First, we obtain the analytical expressions for the efficient investment strategy and the efficient frontier by adopting the Lagrange dual theory, the state variable transformation technique and the stochastic optimal control method. Then, we discuss some special cases under our model. Finally, a numerical example is presented to illustrate the results obtained in this paper.

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AHN, JAE YOUN; SHYAMALKUMAR, NARIANKADU D. *Asymptotic theory for the empirical Haezendonck-Goovaerts risk measure*. 78-90. Haezendonck-Goovaerts risk measures is a recently introduced class of risk measures which includes, as its minimal member, the Tail Value-at-Risk (T-VaR) – T-VaR arguably the most popular risk measure in global insurance regulation. In applications often one has to estimate the risk measure given a random sample from an

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unknown distribution. The distribution could either be truly unknown or could be the distribution of a complex function of economic and idiosyncratic variables with the complexity of the function rendering indeterminable its distribution. Hence statistical procedures for the estimation of Haezendonck-Goovaerts risk measures are a key requirement for their use in practice. A natural estimator of the Haezendonck-Goovaerts risk measure is the Haezendonck-Goovaerts risk measure of the empirical distribution, but its statistical properties have not yet been explored in detail. The main goal of this article is to both establish the strong consistency of this estimator and to derive weak convergence limits for this estimator. We also conduct a simulation study to lend insight into the sample sizes required for these asymptotic limits to take hold.

AVANZI, BENJAMIN; TU, VINCENT; WONG, BERNARD. *On optimal periodic dividend strategies in the dual model with diffusion*. 210-224. The dual model with diffusion is appropriate for companies with continuous expenses that are offset by stochastic and irregular gains. Examples include research-based or commission-based companies. In this context, Bayraktar *et al.* (2013a) [E. Bayraktar, A.E. Kyprianou, K. Yamazaki, On optimal dividends in the dual model, ASTIN Bulletin (2013) 43(3): 359-372] show that a dividend barrier strategy is optimal when dividend decisions are made continuously. In practice, however, companies that are capable of issuing dividends make dividend decisions on a periodic (rather than continuous) basis. In this paper, we consider a periodic dividend strategy with exponential inter-dividend-decision times and continuous monitoring of solvency. Assuming hyperexponential gains, we show that a periodic barrier dividend strategy is the periodic strategy that maximizes the expected present value of dividends paid until ruin. Interestingly, a 'liquidation-at-first-opportunity' strategy is optimal in some cases where the surplus process has a positive drift. Results are illustrated.

BALLESTRA, LUCA VINCENZO; PACELLI, GRAZIELLA. *Valuing risky debt: a new model combining structural information with the reduced-form approach*. 261-271. A new model of credit risk is proposed in which the intensity of default is described by an additional stochastic differential equation coupled with the process of the obligor's asset value. Such an approach allows us to incorporate structural information as well as to capture the effect of external factors (e.g. macroeconomic factors) in a both parsimonious and economically consistent way. From the practical standpoint, the proposed model offers great flexibility and allows us to obtain credit spread curves of many different shapes, including double humped term structures. Furthermore, an approximate closed-form solution is derived, which is accurate, easy to implement, and allows for an efficient calibration to realized credit spreads. Numerical experiments are presented showing that the novel approach provides a very satisfactory fitting to market data and outperforms the model developed by Madan and Unal (2000) [D.B. Madan, H. Unal, A two-factor hazard rate model for pricing risky debt and the term structure of credit spreads, Journal of Financial Quantitative Analysis (2000) 35: 43-65].

BERNARD, CAROLE; MACKAY, ANNE; MUEHLBEYER, MAX. *Optimal surrender policy for variable annuity guarantees*. 116-128. This paper proposes a technique to derive the optimal surrender strategy for a variable annuity (VA) as a function of the underlying fund value. This approach is based on splitting the value of the VA into a European part and an early exercise premium following the work of Kim and Yu (1996) [I.J. Kim, G.G. Yu, An alternative approach to the valuation of American options and applications, Review of Derivatives Research (1996) 1(1): 61-85] and Carr *et al.* (1992) [P. Carr, R. Jarrow, R. Myneni, Alternative characterizations of American put options, Mathematical Finance (1992) 2(2): 87-106]. The technique is first applied to the simplest VA with GMAB (path-independent benefits) and is then shown to be possibly

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generalized to the case when benefits are path-dependent. Fees are paid continuously as a fixed percentage of the fund value. Our approach is useful to investigate the impact of path-dependent benefits on surrender incentives.

BUCHARDT, KRISTIAN. *Dependent interest and transition rates in life insurance*. 167-179. For market consistent life insurance liabilities modelled with a multi-state Markov chain, it is of importance to consider the interest and transition rates as stochastic processes, for example in order to consider hedging possibilities of the risks, and for risk measurement. In the literature, this is usually done with an assumption of independence between the interest and transition rates. In this paper, it is shown how to valuate life insurance liabilities using affine processes for modelling dependent interest and transition rates. This approach leads to the introduction of so-called dependent forward rates. We propose a specific model for surrender modelling, and within this model the dependent forward rates are calculated, and the market value and the Solvency II capital requirement are examined for a simple savings contract.

CAI, JUN; WEI, WEI. *Some new notions of dependence with applications in optimal allocation problems*. 200-209. Dependence structures of multiple risks play an important role in optimal allocation problems for insurance, quantitative risk management, and finance. However, in many existing studies on these problems, risks or losses are often assumed to be independent or comonotonic or exchangeable. In this paper, we propose several new notions of dependence to model dependent risks and give their characterizations through the probability measures or distributions of the risks or through the expectations of the transformed risks. These characterizations are related to the properties of arrangement increasing functions and the proposed notions of dependence incorporate many typical dependence structures studied in the literature for optimal allocation problems. We also develop the properties of these dependence structures. We illustrate the applications of these notions in the optimal allocation problems of deductibles and policy limits and in capital reserves problems. These applications extend many existing researches to more general dependent risks.

CECI, CLAUDIA; COLANERI, KATIA; CRETAROLA, ALESSANDRA. *A benchmark approach to risk-minimization under partial information*. 129-146. The goal of this paper is to investigate (locally) risk-minimizing hedging strategies under the benchmark approach in a financial semimartingale market model where there are restrictions on the available information. More precisely, we characterize the optimal strategy as the integrand appearing in the Galtchouk-Kunita-Watanabe decomposition of the benchmarked contingent claim under partial information and provide its description in terms of the integrand in the classical Galtchouk-Kunita-Watanabe decomposition under full information via dual predictable projections. Finally we show how these results can be applied to unit-linked life insurance contracts.

CHEUNG, KA CHUN; LO, AMBROSE. *Characterizing mutual exclusivity as the strongest negative multivariate dependence structure*. 180-190. Mutual exclusivity is an extreme negative dependence structure that was first proposed and studied in Dhaene and Denuit (1999) [J. Dhaene, M. Denuit, The safest dependence structure among risks, Insurance: Mathematics & Economics (1999) 25: 11-21] in the context of insurance risks. In this article, we revisit this notion and present versatile characterizations of mutually exclusive random vectors via their pairwise counter-monotonic behaviour, minimal convex sum property, distributional representation and the characteristic function of the sum of their components. These characterizations highlight the role of mutual exclusivity in generalizing counter-monotonicity as the strongest negative dependence structure in a multi-dimensional setting.

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COUSIN, ARESKI; DI BERNARDINO, ELENA. *On multivariate extensions of conditional-tail-expectation*. 272-282. In this paper, we introduce two alternative extensions of the classical univariate Conditional-Tail-Expectation (CTE) in a multivariate setting. The two proposed multivariate CTEs are vector-valued measures with the same dimension as the underlying risk portfolio. As for the multivariate Value-at-Risk measures introduced by Cousin and Di Bernardino (2013) [A. Cousin, E. Di Bernardino, On multivariate extensions of value-at-risk, Journal of Multivariate Analysis (2013) 119: 32-46], the lower-orthant CTE (resp. the upper-orthant CTE) is constructed from level sets of multivariate distribution functions (resp. of multivariate survival distribution functions). Contrary to allocation measures or systemic risk measures, these measures are also suitable for multivariate risk problems where risks are heterogeneous in nature and cannot be aggregated together. Several properties have been derived. In particular, we show that the proposed multivariate CTE-s satisfy natural extensions of the positive homogeneity property, the translation invariance property and the comonotonic additivity property. Comparison between univariate risk measures and components of multivariate CTE is provided. We also analyze how these measures are impacted by a change in marginal distributions, by a change in dependence structure and by a change in risk level. Sub-additivity of the proposed multivariate CTE-s is provided under the assumption that all components of the random vectors are independent. Illustrations are given in the class of Archimedean copulas.

FARKAS, WALTER; KOCH-MEDINA, PABLO; MUNARI, COSIMO. *Capital requirements with defaultable securities*. 58-67. We study capital requirements for bounded financial positions defined as the minimum amount of capital to invest in a chosen eligible asset targeting a pre-specified acceptability test. We allow for general acceptance sets and general eligible assets, including defaultable bonds. Since the payoff of these assets is not necessarily bounded away from zero, the resulting risk measures cannot be transformed into cash-additive risk measures by a change of numéraire. However, extending the range of eligible assets is important because, as exemplified by the recent financial crisis, assuming the existence of default-free bonds may be unrealistic. We focus on finiteness and continuity properties of these general risk measures. As an application, we discuss capital requirements based on Value-at-Risk and Tail-Value-at-Risk acceptability, the two most important acceptability criteria in practice. Finally, we prove that there is no optimal choice of the eligible asset. Our results and our examples show that a theory of capital requirements allowing for general eligible assets is richer than the standard theory of cash-additive risk measures.

GUAN, GUOHUI; LIANG, ZONGXIA. *Optimal reinsurance and investment strategies for insurer under interest rate and inflation risks*. 105-115. In this paper, we investigate an optimal reinsurance and investment problem for an insurer whose surplus process is approximated by a drifted Brownian motion. Proportional reinsurance is to hedge the risk of insurance. Interest rate risk and inflation risk are considered. We suppose that the instantaneous nominal interest rate follows an Ornstein-Uhlenbeck process, and the inflation index is given by a generalized Fisher equation. To make the market complete, zero-coupon bonds and Treasury Inflation Protected Securities (TIPS) are included in the market. The financial market consists of cash, zero-coupon bond, TIPS and stock. We employ the stochastic dynamic programming to derive the closed-forms of the optimal reinsurance and investment strategies as well as the optimal utility function under the constant relative risk aversion (CRRA) utility maximization. Sensitivity analysis is given to show the economic behavior of the optimal strategies and optimal utility.

HEBERLE, JOCHEN; THOMAS, ANNE. *Combining chain-ladder claims reserving with fuzzy numbers*. 96-104. In this paper we extend the classical chain-ladder claims reserving method using fuzzy methods. Therefore, we derive new estimators for the claims development factors as well as

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new predictors for the ultimate claims. The advantage in using fuzzy numbers lies in the fact that the model uncertainty is directly included in and can be controlled by the "new" fuzzy claims development factors. We also provide an estimator for the uncertainty of the ultimate claims for single accident years and for aggregated accident years.

HUANG, YU-LIEH; TSAI, JEFFREY TZUHAO; YANG, SHARON S; CHENG, HUNG-WEN. *Price bounds of mortality-linked security in incomplete insurance market*. 30-39. This study investigates reasonable price bounds for mortality-linked securities when the issuer has only a partial hedging ability. The price bounds are established by minimizing the difference between the benchmark price and the replicating portfolio cost subject to the gain-loss ratio of excess payoff of the mortality-linked securities. In contrast to the previous studies, the assumptions of no-arbitrage pricing and utility-based pricing are not fully employed in this study because of the incompleteness of the insurance securitization market. Instead, a framework including three insurance basis assets is constructed to search for the price bounds of mortality-linked securities and use the Swiss Re mortality catastrophe bond, issued in 2003, as a numerical example. The proposed price bounds are valuable for setting bid-asked spreads and coupon premiums, and establishing trading strategies in the raising mortality securitization markets.

HYNDMAN, CODY B; WENGER, MENACHEM. *Valuation perspectives and decompositions for variable annuities with GMWB riders*. 283-290. The guaranteed minimum withdrawal benefit (GMWB) rider, as an add on to a variable annuity (VA), guarantees the return of premiums in the form of periodic withdrawals while allowing policyholders to participate fully in any market gains. GMWB riders represent an embedded option on the account value with a fee structure that is different from typical financial derivatives. We consider fair pricing of the GMWB rider from a financial economic perspective. Particular focus is placed on the distinct perspectives of the insurer and policyholder and the unifying relationship. We extend a decomposition of the VA contract into components that reflect term-certain payments and embedded derivatives to the case where the policyholder has the option to surrender, or lapse, the contract early.

KLEIN, NADJA; DENUIT, MICHEL; LANG, STEFAN; KNEIB, THOMAS. *Nonlife ratemaking and risk management with Bayesian generalized additive models for location, scale, and shape*. 225-249. Generalized additive models for location, scale and, shape define a flexible, semi-parametric class of regression models for analyzing insurance data in which the exponential family assumption for the response is relaxed. This approach allows the actuary to include risk factors not only in the mean but also in other key parameters governing the claiming behavior, like the degree of residual heterogeneity or the no-claim probability. In this broader setting, the Negative Binomial regression with cell-specific heterogeneity and the zero-inflated Poisson regression with cell-specific additional probability mass at zero are applied to model claim frequencies. New models for claim severities that can be applied either per claim or aggregated per year are also presented. Bayesian inference is based on efficient Markov chain Monte Carlo simulation techniques and allows for the simultaneous estimation of linear effects as well as of possible nonlinear effects, spatial variations and interactions between risk factors within the data set. To illustrate the relevance of this approach, a detailed case study is proposed based on the Belgian motor insurance portfolio studied in Denuit and Lang (2004) [M. Denuit, S. Lang, Nonlife ratemaking with Bayesian GAM's, Insurance: Mathematics & Economics (2004) 35: 627-647].

LI, HAO; MELNIKOV, ALEXANDER. *Polynomial extensions of distributions and their applications in actuarial and financial modeling*. 250-260. The paper deals with orthogonal

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polynomials as a useful technique which can be attracted to actuarial and financial modeling. We use Pearson's differential equation as a way for orthogonal polynomials construction and solution. The generalized Rodrigues formula is used for this goal. Deriving the weight function of the differential equation, we use it as a basic distribution density of variables like financial asset returns or insurance claim sizes. In this general setting, we derive explicit formulas for option prices as well as for insurance premiums. The numerical analysis shows that our new models provide a better fit than some previous actuarial and financial models.

LIANG, ZHIBIN; BAYRAKTAR, ERHAN. *Optimal reinsurance and investment with unobservable claim size and intensity*. 156-166. We consider the optimal reinsurance and investment problem in an unobservable Markov-modulated compound Poisson risk model, where the intensity and jump size distribution are not known but have to be inferred from the observations of claim arrivals. Using a recently developed result from filtering theory, we reduce the partially observable control problem to an equivalent problem with complete observations. Then using stochastic control theory, we get the closed form expressions of the optimal strategies which maximize the expected exponential utility of terminal wealth. In particular, we investigate the effect of the safety loading and the unobservable factors on the optimal reinsurance strategies. With the help of a generalized Hamilton-Jacobi-Bellman equation where the derivative is replaced by Clarke's generalized gradient as in Bäuerle and Rieder (2007) [N. Bäuerle, U. Rieder, Portfolio optimization with jumps and unobservable intensity process, Mathematical Finance (2007) 17: 205-224], we characterize the value function, which helps us verify that the strategies we constructed are optimal.

LIU, JINGCHEN; WOO, JAE-KYUNG. *Asymptotic analysis of risk quantities conditional on ruin for multidimensional heavy-tailed random walks*. 1-9. In this paper we consider a multi-dimensional renewal risk model with regularly varying claims. This model may be used to describe the surplus of an insurance company possessing several lines of business where a large claim possibly puts multiple lines in a risky condition. Conditional on the occurrence of ruin, we develop asymptotic approximations for the average accumulated number of claims leading the process to a rare set, and the expected total amount of shortfalls to this set in finite and infinite horizons. Furthermore, for the continuous time case, asymptotic results regarding the total occupation time of the process in a rare set and time-integrated amount of shortfalls to a rare set are obtained.

LOISEL, STÉPHANE; TRUFIN, JULIEN. *Properties of a risk measure derived from the expected area in red*. 191-199. This paper studies a new risk measure derived from the expected area in red introduced in Loisel (2005) [S. Loisel, Differentiation of some functionals of risk processes, and optimal reserve allocation, Journal of Applied Probability (2005) 42(2): 379-392]. Specifically, we derive various properties of a risk measure defined as the smallest initial capital needed to ensure that the expected time-integrated negative part of the risk process on a fixed time interval $[0, T]$ (T can be infinite) is less than a given predetermined risk limit. We also investigate the optimal risk limit allocation: given a risk limit set at a company level for the sum of the expected areas in red of all lines, we determine the way(*s*) to allocate this risk limit to the subsequent business lines in order to minimize the overall capital needs.

LUCIANO, ELISA; REGIS, LUCA. *Efficient versus inefficient hedging strategies in the presence of financial and longevity (value at) risk*. 68-77. This paper provides a closed-form Value-at-Risk (VaR) for the net exposure of an annuity provider, taking into account both mortality and interest-rate risk, on both assets and liabilities. It builds a classical risk-return frontier and shows that hedging strategies – such as the transfer of longevity risk – may increase the overall risk while decreasing

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expected returns, thus resulting in inefficient outcomes. Once calibrated to the 2010 UK longevity and bond market, the model gives conditions under which hedging policies become inefficient.

MAGNI, CARLO ALBERTO. *Arithmetic returns for investment performance measurement*. 291-300. This paper introduces new money-weighted metrics for investment performance analysis, based on arithmetic means of holding period rates weighted by the investment's market values. This approach generates rates of return which measure a fund's or portfolio's performance and a fund manager's performance. It also enables to show that the Internal Rate of Return (IRR) is a weighted mean of holding period rates associated with interim values which differ from market values, so that value additivity is violated. The manager's Arithmetic Internal Rate of Return (AIRR) is shown to be the true period equivalent of the cumulative Time Weighted Rate of Return (TWRR), whereas the period TWRR (a geometric return) provides a different ranking. The method is easily generalized for coping with varying benchmark rates. We also cope with the practical problem of estimating interim values whenever they are not available.

MALINOVSKII, VSEVOLOD K. *Annual intrinsic value of a company in a competitive insurance market*. 310-318. In this paper we analyze a measure of the insurance company's value in an extended Lundberg model which includes the effect of competition on pricing. The extended model is designed to be an integral part of a multi-year controlled risk model of a company operating on both competitive insurance and financial markets, when insureds migrate in seeking for better rates and investors migrate in seeking for higher return on investments.

MALINOVSKII, VSEVOLOD K. *Improved asymptotic upper bounds on the ruin capital in the Lundberg model of risk*. 301-309. This paper deals with ruin capital $u_{\alpha,t}(c \mid \lambda, \mu)$ in the classical Lundberg model of risk. It is defined as the initial capital needed to keep the probability of ruin within finite time t equal to a predefined value α. Considered as a decreasing function of premium rate c, the ruin capital is shown to be convex (i.e., concave downward) for $c>\lambda/\mu$ and t sufficiently large. This observation is used to construct explicit upper bounds on the ruin capital.

MATSUI, MUNEYA. *Prediction in a non-homogeneous Poisson cluster model*. 10-17. A non-homogeneous Poisson cluster model is studied, motivated by insurance applications. The Poisson center process which expresses arrival times of claims, triggers off cluster member processes which correspond to number or amount of payments. The cluster member process is an additive process. Given the past observations of the process we consider expected values of future increments and their mean squared errors, aiming at application in claims reserving problems. Our proposed process can cope with non-homogeneous observations such as the seasonality of claims arrival or the reducing property of payment processes, which are unavailable in the former models where both center and member processes are time homogeneous. Hence results presented in this paper are significant extensions toward applications.

MEYRICKE, RAMONA; SHERRIS, MICHAEL. *Longevity risk, cost of capital and hedging for life insurers under Solvency II*. 147-155. The cost of capital is an important factor determining the premiums charged by life insurers issuing life annuities. This capital cost can be reduced by hedging longevity risk with longevity swaps, a form of reinsurance. We assess the costs of longevity risk management using indemnity based longevity swaps compared to costs of holding capital under Solvency II. We show that, using a reasonable market price of longevity risk, the market cost of hedging longevity risk for earlier ages is lower than the cost of capital required under Solvency II. Longevity swaps covering higher ages, around 90 and above, have higher

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market hedging costs than the saving in the cost of regulatory capital. The Solvency II capital regulations for longevity risk generates an incentive for life insurers to hold longevity tail risk on their own balance sheets, rather than transferring this to the reinsurance or the capital markets. This aspect of the Solvency II capital requirements is not well understood and raises important policy issues for the management of longevity risk.

MOUSSA, A MBAIRADJIM; KAMDEM, JULES SADEFO; SHAPIRO, ARNOLD F; TERRAZA, M. *CAPM with fuzzy returns and hypothesis testing*. 40-57. Over the last four decades, several estimation issues of the beta have been discussed extensively in many articles. An emerging consensus is that the betas are time-dependent and their estimates are impacted by the return interval and the length of the estimation period. These findings lead to the prominence of the practical implementation of the Capital Asset Pricing Model. Our goal in this paper is two-fold. After studying the impact of the return interval on the beta estimates, we analyze the sample size effects on the preceding estimation. Working in the framework of fuzzy set theory, we first associate the returns based on closing prices with the intraperiod volatility for the representation by the means of a fuzzy random variable in order to incorporate the effect of the interval period over which the returns are measured in the analysis. Next, we use these fuzzy returns to estimate the beta via fuzzy least square method in order to deal efficiently with outliers in returns, often caused by structural breaks and regime switches in the asset prices. A bootstrap test is carried out to investigate whether there is a linear relationship between the market portfolio fuzzy return and the given asset fuzzy return. Finally, the empirical results on French stocks suggest that our beta estimates seem to be more stable than the ordinary least square (OLS) estimates when the return intervals and the sample size change.

SCHMIDT, KLAUS D. *On inequalities for moments and the covariance of monotone functions*. 91-95. Intuition based on the usual interpretation of the covariance of two random variables suggests that the inequality $\mathrm{cov}[f(X), g(X)] \geq 0$ should hold for any random variable X and any two increasing functions f and g. The inequality holds indeed, but a proof is hard to find in the literature. In this paper we provide an elementary proof of a more general inequality for moments and we present several applications in actuarial mathematics.

SHI, PENG; VALDEZ, EMILIANO A. *Multivariate negative binomial models for insurance claim counts*. 18-29. It is no longer uncommon these days to find the need in actuarial practice to model claim counts from multiple types of coverage, such as the ratemaking process for bundled insurance contracts. Since different types of claims are conceivably correlated with each other, the multivariate count regression models that emphasize the dependency among claim types are more helpful for inference and prediction purposes. Motivated by the characteristics of an insurance dataset, we investigate alternative approaches to constructing multivariate count models based on the negative binomial distribution. A classical approach to induce correlation is to employ common shock variables. However, this formulation relies on the NB-I distribution which is restrictive for dispersion modeling. To address these issues, we consider two different methods of modeling multivariate claim counts using copulas. The first one works with the discrete count data directly using a mixture of max-id copulas that allows for flexible pair-wise association as well as tail and global dependence. The second one employs elliptical copulas to join continuitized data while preserving the dependence structure of the original counts. The empirical analysis examines a portfolio of auto insurance policies from a Singapore insurer where claim frequency of three types of claims (third party property damage, own damage, and third party bodily injury) are considered. The results demonstrate the superiority of the copula-based approaches over the common shock model. Finally, we implemented the various models in loss predictive applications.

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DONNELLY, CATHERINE; GUILLÉN, MONTSERRAT; NIELSEN, JENS PERCH. *Bringing cost transparency to the life annuity market*. 14-27. The financial industry has recently seen a push away from structured products and towards transparency. The trend is to decompose products, such that customers understand each component as well as its price. Yet the enormous annuity market combining investment and longevity has been almost untouched by this development. We suggest a simple decomposed annuity structure that enables cost transparency and could be linked to any investment fund. It has several attractive features: (i) it works for any heterogeneous group; (ii) participants can leave before death without financial penalty; and (iii) participants have complete freedom over their own investment strategy.

FU, LUYANG; NG, CHEUK-YIN ANDREW. *Asymptotics for the ruin probability of a time-dependent renewal risk model with geometric Lévy process investment returns and dominatedly-varying-tailed claims*. 80-87. Consider a continuous-time renewal risk model, in which the claim sizes and inter-arrival times form a sequence of independent and identically distributed random pairs, with each pair obeying a dependence structure. Suppose that the surplus is invested in a portfolio whose return follows a Lévy process. When the claim-size distribution is dominatedly-varying tailed, asymptotic estimates for the finite- and infinite-horizon ruin probabilities are obtained.

HASHORVA, ENKELEJD; LING, CHENGXIU; PENG, ZUOXIANG. *Second-order tail asymptotics of deflated risks*. 88-101. Random deflation of risk models is an interesting topic for both theoretical and practical actuarial problems. In this paper, we investigate second-order tail asymptotics of the deflated risk $X = RS$ under the assumptions of second-order regular variation on the survival functions of the risk R and the deflator S. Our findings are applied to derive second-order expansions of Value-at-Risk. Further we investigate the estimation of small tail probability for deflated risks and then discuss the asymptotics of the aggregated deflated risk.

HUANG, H; MILEVSKY, MOSHE A; SALISBURY, T S. *Optimal initiation of a GLWB in a variable annuity: No Arbitrage approach*. 102-111. This paper offers a financial economic perspective on the optimal time (and age) at which the owner of a Variable Annuity (VA) policy with a Guaranteed Lifetime Withdrawal Benefit (GLWB) rider should initiate guaranteed lifetime income payments. We bypass issues related to utility, bequest and consumption preference by treating the VA as liquid and tradable. This allows us to use an American option pricing framework to derive a so-called optimal initiation region. Our main practical finding is that given current design parameters in which volatility (asset allocation) is restricted to less than 20%, while guaranteed payout rates (GPR) as well as bonus (roll-up) rates are less than 5%, GLWBs that are in-the-money should be turned on by the late 50s and certainly the early 60s. The exception to the rule is when a non-constant GPR is about to increase to a higher age band, in which case the optimal policy is to wait until the new GPR is hit and then initiate immediately. Also, to offer a different perspective, we invert the model and solve for the bonus (roll-up) rate that is required to justify delaying initiation at any age. We find that the required bonus is quite high and more than what is currently promised by existing products. Our methodology and results should be of interest to researchers as well as to the individuals that collectively have over $1 USD trillion in aggregate invested in these products. We conclude by suggesting that much of the non-initiation at older ages is irrational (which obviously benefits the insurance industry).

LEDWINA, TERESA; WYLUPEK, GRZEGORZ. *Validation of positive quadrant dependence*. 38-47. Quadrant dependence is a useful dependence notion of two random variables, widely applied in reliability, insurance and actuarial sciences. The interest in this dependence structure ranges from modeling it, throughout measuring its strength and investigations on how increasing the dependence effects of several reliability and economic indexes, to hypothesis testing on the dependence. In this paper, we focus on testing for positive quadrant dependence. We propose two new tests for verifying positive quadrant dependence. We prove novel results on finite sample behavior of power function of one of the proposed tests as well as evaluate and compare the two new solutions with the best existing ones, via a simulation study. These comparisons demonstrate that the new solutions are slightly weaker in detecting positive quadrant dependence modeled by classical bivariate models and outperform the best existing solutions when some mixtures, regression and heavy-tailed models have to be detected. Finally, the methods introduced in the paper are applied to real life insurance data, to assess the dependence and test them for positive quadrant dependence.

LEE, WOOJOO; AHN, JAE YOUN. *On the multidimensional extension of countermonotonicity and its applications*. 68-79. In a 2-dimensional space, Fréchet-Hoeffding upper and lower bounds define comonotonicity and countermonotonicity, respectively. Similarly, in the multidimensional case, comonotonicity can be defined using the Fréchet-Hoeffding upper bound. However, since the multidimensional Fréchet-Hoeffding lower bound is not a distribution function, there is no obvious extension of countermonotonicity in multidimensions. This paper investigates in depth a new multidimensional extension of countermonotonicity. We first provide an equivalent condition for countermonotonicity in 2-dimension, and extend the definition of countermonotonicity into multidimensions. In order to justify such extensions, we show that newly defined countermonotonic copulas constitute a minimal class of copulas. Two applications will be provided. First, we will study the relationships between multidimensional countermonotonicity and such well-known multivariate concordance measures as Kendall's tau or Spearman's rho. Second, we will give a financial interpretation of multidimensional countermonotonicity via the existing herd behavior index.

NOLDE, NATALIA; PARKER, GARY. *Stochastic analysis of life insurance surplus*. 1-13. The aim of the paper is to examine the behavior of insurance surplus over time for a portfolio of homogeneous life policies. We distinguish between stochastic and accounting surpluses and derive their first two moments. A recursive formula is proposed for calculating the distribution function of the accounting surplus. We then examine the probability that the accounting surplus becomes negative in a given insurance year. Numerical examples illustrate the results for portfolios of temporary and endowment life policies assuming a conditional AR(1) process for the rates of return.

PEŠTA, MICHAL; OKHRIN, OSTAP. *Conditional least squares and copulae in claims reserving for a single line of business*. 28-37. One of the main goals in non-life insurance is to estimate the claims reserve distribution. A generalized time series model, that allows for modeling the conditional mean and variance of the claim amounts, is proposed for the claims development. On contrary to the classical stochastic reserving techniques, the number of model parameters does not depend on the number of development periods, which leads to a more precise forecasting. Moreover, the time series innovations for the consecutive claims are not considered to be independent anymore. Conditional least squares are used to estimate model parameters and consistency of these estimates is proved. The copula approach is used for modeling the dependence

structure, which improves the precision of the reserve distribution estimate as well. Real data examples are provided as an illustration of the potential benefits of the presented approach.

WONG, TAT WING; CHIU, MEI CHOI; WONG, HOI YING. *Time-consistent mean-variance hedging of longevity risk: effect of cointegration*. 56-67. This paper investigates the time-consistent dynamic mean-variance hedging of longevity risk with a longevity security contingent on a mortality index or the national mortality. Using an HJB framework, we solve the hedging problem in which insurance liabilities follow a doubly stochastic Poisson process with an intensity rate that is correlated and cointegrated to the index mortality rate. The derived closed-form optimal hedging policy articulates the important role of cointegration in longevity hedging. We show numerically that a time-consistent hedging policy is a smoother function in time when compared with its time-inconsistent counterpart.

ZAKS, YANIV; TSANAKAS, ANDREAS. *Optimal capital allocation in a hierarchical corporate structure*. 48-55. We consider capital allocation in a hierarchical corporate structure where stakeholders at two organizational levels (e.g., board members vs line managers) may have conflicting objectives, preferences, and beliefs about risk. Capital allocation is considered as the solution to an optimization problem whereby a quadratic deviation measure between individual losses (at both levels) and allocated capital amounts is minimized. Thus, this paper generalizes the framework of Dhaene *et al.* (2012) [J. Dhaene, A. Tsanakas, E.A. Valdez, S. Vanduffel, Optimal capital allocation principles, Journal of Risk and Insurance (2012) 79(1): 1-28], by allowing potentially diverging risk preferences in a hierarchical structure. An explicit unique solution to this optimization problem is given. In several examples, it is shown how the optimal capital allocation achieves a compromise between conflicting views of risk within the organization

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ARNOLD, LUTZ G; HARTL, JOHANNES. *Corporate insurance with safety loadings: a note*. 1087-1094. In an article in this journal, Schnabel and Roumi (1989) assert that if uninsured debt is risky, a levered firm takes a casualty insurance with a positive safety loading if, and only if, the amount of debt is sufficiently high. This note shows that in marked contrast to this assertion, the correct conclusion from their model is that the firm generally takes insurance for low levels of risky debt, and it depends on the magnitude of the loading whether it also takes insurance for high levels of debt.

COX, SAMUEL H; LIN, YIJIA; TIAN, RUILIN; ZULUAGA LUIS F. *Mortality portfolio risk management*. 853-890. We provide a new method, the "MV + CVaR approach," for managing unexpected mortality changes underlying annuities and life insurance. The MV + CVaR approach optimizes the mean-variance trade-off of an insurer's mortality portfolio, subject to constraints on

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downside risk. We apply the method of moments and the maximum entropy method to analyze the efficiency of MV + CVaR mortality portfolios relative to traditional Markowitz mean-variance portfolios. Our numerical examples illustrate the superiority of the MV + CVaR approach in mortality risk management and shed new light on natural hedging effects of annuities and life insurance.

GUISO, LUIGI; JAPPELLI, TULLIO; PADULA, MARIO. *Pension wealth uncertainty*. 1057-1085. Using a representative sample of Italian investors, we measure the uncertainty of social security benefits by eliciting for each individual the subjective distribution of the replacement rate as a summary indicator of pension uncertainty. We find that pension uncertainty varies across individuals in a way that is consistent with what one would expect a priori, given different information sets and pension schemes. In particular, individuals who are a long way from retirement, and thus face more career uncertainty, report more subjective pension uncertainty. Since expectations reveal information about people's understanding of pension reforms, our findings suggest that they should also be an important determinant of how people respond to reforms.

KOFMAN, PAUL; NINI, GREGORY P. *Do insurance companies possess an informational monopoly? Empirical evidence from auto insurance*. 1001-1026. This article investigates the impact of policyholder tenure on contractual relationships in nonlife insurance markets. For a sample of auto insurance policies, we find that average risk decreases with policyholder tenure, but the effect is entirely due to the impact of observable information. We reject the hypothesis that the incumbent insurer is privately learning faster about quality of their policyholders. We highlight the importance of a public signal regarding policyholders' claims experiences and suggest alternative explanations for the unconditional relationships in the data.

LIN, YIJIA; LIU, SHEEN; YU, JIFENG. *Pricing mortality securities with correlated mortality indexes*. 921-948. This article proposes a stochastic model, which captures mortality correlations across countries and common mortality shocks, for analyzing catastrophe mortality contingent claims. To estimate our model, we apply particle filtering, a general technique that has wide applications in non-Gaussian and multivariate jump-diffusion models and models with nonanalytic observation equations. In addition, we illustrate how to price mortality securities with normalized multivariate exponential titling based on the estimated mortality correlations and jump parameters. Our results show the significance of modeling mortality correlations and transient jumps in mortality security pricing.

OWADALLY, IQBAL; HABERMAN, STEVEN; HERNÁNDEZ, DENISE GÓMEZ. *A savings plan with targeted contributions*. 975-1000. We consider a simple savings problem where contributions are made to a fund and invested to meet a future liability. The conventional approach is to estimate future investment return and calculate a fixed contribution to be paid regularly by the saver. We propose a flexible plan where contributions are systematically adjusted and targeted. We show by means of stochastic simulations that this plan has a reduced risk of a shortfall and is relatively insensitive to errors in the planner's estimate of future returns. Sensitivity analyses in terms of parameter values, stochastic return models and investment horizons are also performed.

QIAO, CHAO; SHERRIS, MICHAEL. *Pricing mortality securities with correlated mortality indexes*. 949-974. Group self-annuitization (GSA) schemes are designed to share uncertain future mortality experience including systematic improvements. Challenges for designing group pooled schemes include decreasing average payments when mortality improves significantly, decreasing numbers in the pool at older ages, and the impact of dependence from systematic mortality

improvements across different ages of members in the pool. This article uses a multiple-factor stochastic mortality model in a simulation study to show how pooling can be made more effective and to quantify the limitations of these pooling schemes arising from the impact of systematic longevity risk.

WANG, CHOU-WEN; YANG, SHARON S. *Pricing survivor derivatives with cohort mortality dependence under the Lee-Carter framework*. 1027-1056. This article introduces cohort mortality dependence in mortality modeling. We extend the classical Lee-Carter model to incorporate cohort mortality dependence by considering mortality correlations for a cohort of people born in the same year. The pattern of cohort mortality dependence is demonstrated on the basis of U.S. mortality experience. We study the effect of cohort mortality dependence on the pricing of survivor derivatives. For this purpose, a survivor floor is introduced. To understand the difference between a survivor floor and other survivor securities, the valuation formulas for survivor swaps and survivor floors are all derived in detail and the effects of cohort mortality dependence on pricing survivor derivatives are investigated numerically.

ZHANG, YANWEI; DUKIC, VANJA. *Predicting multivariate insurance loss payments under the bayesian copula framework*. 891-919. The literature of predicting the outstanding liability for insurance companies has undergone rapid and profound changes in the past three decades, most recently focusing on Bayesian stochastic modeling and multivariate insurance loss payments. In this article, we introduce a novel Bayesian multivariate model based on the use of parametric copula to account for dependencies between various lines of insurance claims. We derive a full Bayesian stochastic simulation algorithm that can estimate parameters in this class of models. We provide an extensive discussion of this modeling framework and give examples that deal with a wide range of topics encountered in the multivariate loss prediction settings.

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ALAI, DANIEL H; CHEN, HUA; CHO, DANIEL; HANEWALD, KATJA; SHERRIS, MICHAEL. *Developing equity release markets: Risk Analysis for reverse mortgages and home reversions*. 217-241. Equity release products are sorely needed in an aging population with high levels of home ownership. There has been a growing literature analyzing risk components and capital adequacy of reverse mortgages in recent years. However, little research has been done on the risk analysis of other equity release products, such as home reversion contracts. This is partly due to the dominance of reverse mortgage products in equity release markets worldwide. In this article we compare cash flows and risk profiles from the provider's perspective for reverse mortgage and home reversion contracts. An at-home/in long-term care split termination model is employed to calculate termination rates, and a vector autoregressive (VAR) model is used to depict the joint dynamics of economic variables including interest rates, house prices, and rental yields. We derive stochastic discount factors from the no arbitrage condition and price the no negative equity

guarantee in reverse mortgages and the lease for life agreement in the home reversion plan accordingly. We compare expected payoffs and assess riskiness of these two equity release products via commonly used risk measures: Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR).

ARO, HELENA. *Systematic and nonsystematic mortality risk in pension portfolios*. 59-67. We study the effects of nonsystematic and systematic mortality risks on the required initial capital in a pension plan, in the presence of financial risks. We discover that for a pension plan with few members the impact of pooling on the required capital per person is strong, but nonsystematic risk diminishes rapidly as the number of members increases. Systematic mortality risk, on the other hand, is a significant source of risk in a pension portfolio.

BIFFIS, ENRICO; BLAKE, DAVID. *Keeping some skin in the game: How to start a capital market in longevity risk transfers*. 14-21. The recent activity in pension buyouts and bespoke longevity swaps suggests that a significant process of aggregation of longevity exposures is under way, led by major insurers, investment banks, and buyout firms with the support of leading reinsurers. As regulatory capital charges and limited reinsurance capacity constrain the scope for market growth, there is now an opportunity for institutions that are pooling longevity exposures to issue securities that appeal to capital market investors, thereby broadening the sharing of longevity risk and increasing market capacity. For this to happen, longevity exposures need to be suitably pooled and tranched to maximize diversification benefits offered to investors and to address asymmetric information issues. We argue that a natural way for longevity risk to be transferred is through suitably designed principal-at-risk bonds.

BISETTI, EMILIO; FAVERO, CARLO A. *Measuring the impact of longevity risk on pension systems: The case of Italy*. 87-103. This article estimates the impact of longevity risk on pension systems by combining the prediction based on a Lee-Carter mortality model with the projected pension payments for different cohorts of retirees. We measure longevity risk by the difference between the upper bound of the total old-age pension expense and its mean estimate. This difference is as high as 4% of annual GDP over the period 2040-2050. The impact of longevity risk is sizeably reduced, but not fully eliminated, by the introduction of indexation of retirement age to expected life at retirement. Our evidence speaks in favor of a market for longevity risk and calls for a closer scrutiny of the potential redistributive effects of longevity risk.

BLAKE, DAVID; MACMINN, RICHARD; LI, JOHNNY SIU-HANG; HARDY, MARY. *Longevity risk and capital markets: The 2012-2013 update*. 1-13. Introduction to this Special Issue of the North American Actuarial Journal contains 15 contributions to the academic literature all dealing with longevity risk and capital markets. Draft versions of the articles were presented at Longevity Eight: The Eighth International Longevity Risk and Capital Markets Solutions Conference, which was held in Waterloo, Ontario, Canada, on September 7-8, 2012.

BLAKE, DAVID; BOARDMAN, TOM; CAIRNS, ANDREW. *Sharing longevity risk: Why governments should issue longevity bonds*. 258-277. Government-issued longevity bonds would allow longevity risk to be shared efficiently and fairly between generations. In exchange for paying a longevity risk premium, the current generation of retirees can look to future generations to hedge their systematic longevity risk. Longevity bonds will lead to a more secure pension savings market, together with a more efficient annuity market. By issuing longevity bonds, governments can aid the establishment of reliable longevity indices and key price points on the longevity risk term structure and help the emerging capital market in longevity-linked instruments to build on this term structure with liquid longevity derivatives.

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CHAN, WAI-SUM; LI, JOHNNY SIU-HANG; LI, JACKIE. *The CBD Mortality Indexes: Modeling and applications*. 38-58. Most extrapolative stochastic mortality models are constructed in a similar manner. Specifically, when they are fitted to historical data, one or more series of time-varying parameters are identified. By extrapolating these parameters to the future, we can obtain a forecast of death probabilities and consequently cash flows arising from life contingent liabilities. In this article, we first argue that, among various time-varying model parameters, those encompassed in the Cairns-Blake-Dowd (CBD) model (also known as Model M5) are most suitably used as indexes to indicate levels of longevity risk at different time points. We then investigate how these indexes can be jointly modeled with a more general class of multivariate time-series models, instead of a simple random walk that takes no account of cross-correlations. Finally, we study the joint prediction region for the mortality indexes. Such a region, as we demonstrate, can serve as a graphical longevity risk metric, allowing practitioners to compare the longevity risk exposures of different portfolios readily.

CHUANG, SHUO-LI; BROCKETT, PATRICK L. *Modeling and pricing longevity derivatives using stochastic mortality rates and the Esscher transform*. 22-37. The Lee-Carter mortality model provides a structure for stochastically modeling mortality rates incorporating both time (year) and age mortality dynamics. Their model is constructed by modeling the mortality rate as a function of both an age and a year effect. Recently the MBMM model (Mitchell *et al.* 2013) showed the Lee Carter model can be improved by fitting with the growth rates of mortality rates over time and age rather than the mortality rates themselves. The MBMM modification of the Lee-Carter model performs better than the original and many of the subsequent variants. In order to model the mortality rate under the martingale measure and to apply it for pricing the longevity derivatives, we adapt the MBMM structure and introduce a Lévy stochastic process with a normal inverse Gaussian (NIG) distribution in our model. The model has two advantages in addition to better fit: first, it can mimic the jumps in the mortality rates since the NIG distribution is fat-tailed with high kurtosis, and, second, this mortality model lends itself to pricing of longevity derivatives based on the assumed mortality model. Using the Esscher transformation we show how to find a related martingale measure, allowing martingale pricing for mortality/longevity risk-related derivatives. Finally, we apply our model to pricing a q-forward longevity derivative utilizing the structure proposed by Life and Longevity Markets Association.

D'AMATO, VALERIA; HABERMAN, STEVEN; PISCOPO, GABRIELLA; RUSSOLILLO, MARIA; TRAPANI, LORENZO. *Detecting common longevity trends by a multiple population approach*. 139-149. Recently the interest in the development of country and longevity risk models has been growing. The investigation of long-run equilibrium relationships could provide valuable information about the factors driving changes in mortality, in particular across ages and across countries. In order to investigate cross-country common longevity trends, tools to quantify, compare, and model the strength of dependence become essential. On one hand, it is necessary to take into account either the dependence for adjacent age groups or the dependence structure across time in a single population setting – a sort of intradependence structure. On the other hand, the dependence across multiple populations, which we describe as interdependence, can be explored for capturing common long-run relationships between countries. The objective of our work is to produce longevity projections by taking into account the presence of various forms of cross-sectional and temporal dependencies in the error processes of multiple populations, considering mortality data from different countries. The algorithm that we propose combines model-based predictions in the Lee-Carter (LC) framework with a bootstrap procedure for dependent data, and

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so both the historical parametric structure and the intragroup error correlation structure are preserved. We introduce a model which applies a sieve bootstrap to the residuals of the LC model and is able to reproduce, in the sampling, the dependence structure of the data under consideration. In the current article, the algorithm that we build is applied to a pool of populations by using ideas from panel data; we refer to this new algorithm as the Multiple Lee-Carter Panel Sieve (MLCPS). We are interested in estimating the relationship between populations of similar socioeconomic conditions. The empirical results show that the MLCPS approach works well in the presence of dependence.

HUNT, ANDREW; BLAKE, DAVID. *A general procedure for constructing mortality models*. 116-138. Recently a large number of new mortality models have been proposed to analyze historic mortality rates and project them into the future. Many of these suffer from being over-parametrized or have terms added in an ad hoc manner that cannot be justified in terms of demographic significance. In addition, poor specification of a model can lead to period effects in the data being wrongly attributed to cohort effects, which results in the model making implausible projections. We present a general procedure for constructing mortality models using a combination of a toolkit of functions and expert judgment. By following the general procedure, it is possible to identify sequentially every significant demographic feature in the data and give it a parametric structural form. We demonstrate using U.K. mortality data that the general procedure produces a relatively parsimonious model that nevertheless has a good fit to the data.

KOGURE, ATSUYUKI; LI, JACKIE; KAMIYA, SHINICHI. *A bayesian multivariate risk-neutral method for pricing reverse mortgages*. 242-257. In this article, we propose a Bayesian multivariate framework to price reverse mortgages that involve several risks in both insurance and financial sectors (e.g., mortality rates, interest rates, and house prices). Our method is a multivariate extension of the Bayesian risk-neutral method developed by Kogure and Kurachi. We apply the proposed method to Japanese data to examine the possibility for a successful introduction of reverse mortgages into Japan. The results suggest a promising future for this new market.

LIN, YIJIA; TAN, KEN SENG; TIAN, RUILIN; YU, JIFENG. *Downside risk management of a defined benefit plan considering longevity basis risk*. 68-86. To control downside risk of a defined benefit pension plan arising from unexpected mortality improvements and severe market turbulence, this article proposes an optimization model by imposing two conditional value at risk constraints to control tail risks of pension funding status and total pension costs. With this setup, we further examine two longevity risk hedging strategies subject to basis risk. While the existing literature suggests that the excess-risk hedging strategy is more attractive than the ground-up hedging strategy as the latter is more capital intensive and expensive, our numerical examples show that the excess-risk hedging strategy is much more vulnerable to longevity basis risk, which limits its applications for pension longevity risk management. Hence, our findings provide important insight on the effect of basis risk on longevity hedging strategies.

MAYHEW, LES; SMITH, DAVID. *Gender convergence in human survival and the postponement of death*. 194-216. It has been a long-accepted demographic maxim that females outlive males. Using data for England and Wales, we show that life expectancy at age 30 is converging, and continuation of this long-term trend suggests life expectancy could reach parity in 2030, resulting in considerable economic and social ramifications. The degree of parity in life expectancy is examined by comparing the historical record in four countries that show that convergence is not a new phenomenon. Contributory factors are considered including changes in male smoking habits and male employment patterns. A model is presented that considers gender differences in longevity

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using novel methods for analyzing life tables. It determines the ages from which death is being postponed, to the ages at which people now die, the relative speed at which these changes are taking place between genders, and how the changes observed are affecting survival prospects at different ages up to 2030. It finds that as life expectancy continues to rise there is accompanying convergence in modal age of death of between 92 and 93 years.

VILLEGAS, ANDRÉS M; HABERMAN, STEVEN. *On the modeling and forecasting of socioeconomic mortality differentials: An application to deprivation and mortality in England*. 168-193. In any country, mortality rates and indices such as life expectancy usually differ across subpopulations, for example, defined by gender, geographic area, or socioeconomic variables (e.g., occupation, level of education, or income). These differentials, and in particular those related to socioeconomic circumstances, pose important challenges for the design of public policies for tackling social inequalities, as well as for the design of pension systems and the management of longevity risk in pension funds and annuity portfolios. We discuss the suitability for the modeling and forecasting of socioeconomic differences in mortality of several multiple population extensions of the Lee-Carter model, including a newly introduced relative model based on the modeling of the mortality in socioeconomic subpopulations alongside the mortality of a reference population. Using England mortality data for socioeconomic subpopulations defined using a deprivation index, we show that this new relative model exhibits the best results in terms of goodness of fit and ex post forecasting performance. We then use this model to derive projections of deprivation specific mortality rates and life expectancies at pensioner ages and analyze the impact of socioeconomic differences in mortality on the valuation of annuities.

ZHOU, RUI; WANG, YUJIAO; KAUFHOLD, KAI; LI, JOHNNY SIU-HANG; TAN, KEN SENG. *Modeling period effects in multi-population mortality models: applications to Solvency II*. 150-167. Recently Cairns *et al.* introduced a general framework for modeling the dynamics of mortality rates of two related populations simultaneously. Their method ensures that the resulting forecasts do not diverge over the long run by modeling the difference in the stochastic factors between the two populations with a mean-reverting autoregressive process. In this article, we investigate how the modeling of the stochastic factors may be improved by using a vector error correction model. This extension is highly intuitive, allowing us to visualize the cross-correlations and the long-term equilibrium relation between the two populations. Another key benefit is that this extension does not require the user to assume which one of the two populations is dominant. This benefit is important because, as we demonstrate, it is not always easy to identify the dominant population, even if one population is much larger than the other. We illustrate our proposed extension with data from a pair of populations and apply it to the calculation of Solvency II risk capital.

ZHU, NAN; BAUER, DANIEL. *A cautionary note on natural hedging of longevity risk*. 104-115. In this article, we examine the so-called natural hedging approach for life insurers to internally manage their longevity risk exposure by adjusting their insurance portfolio. In particular, unlike the existing literature, we also consider a nonparametric mortality forecasting model that avoids the assumption that all mortality rates are driven by the same factor(s). Our primary finding is that higher order variations in mortality rates may considerably affect the performance of natural hedging. More precisely, although results based on a parametric single factor model – in line with the existing literature – imply that almost all longevity risk can be hedged, results are far less encouraging for the nonparametric mortality model. Our finding is supported by robustness tests based on alternative mortality models.

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HARDY, MARY R; SAUNDERS, D; ZHU, X. *Market-consistent valuation and funding of cash balance pensions*. 294-314. Cash balance pension benefits are accumulated at guaranteed crediting rates, usually based on yields on government securities. Viewed as a financial liability, the benefit is a form of interest rate derivative, which can be valued using financial models and theory. In this article, we derive the market value for a range of commonly used crediting rates, assuming the accrued benefit liability comprises the past contributions, allowing for full interest credits up to a known future retirement date. We use the Hull-White interest rate model to determine crediting rates and to determine the market value. We explore the risks associated with different crediting rate choices by evaluating the liability using market data from 1998 to 2013. We propose two other approaches to the accrued benefit. The first approach assumes the accrued benefit comprises past contributions with interest up to the valuation date, but no future interest credits. Future credits on past contributions are assumed funded through future contributions. The second method projects all contributions and interest to retirement and assumes equal units of accrual of this projected benefit in each year of service. We compare the three approaches through numerical examples.

NEVES, CÉSAR; FERNANDES, CRISTIANO; MELO, EDUARDO. *Forecasting surrender rates using elliptical copulas and financial variables*. 343-362. A multistage stochastic model to forecast surrender rates for life insurance and pension plans is proposed. Surrender rates are forecasted by means of Monte Carlo simulation after a sequence of GLM, ARMA-GARCH, and copula fitting is executed. The model is illustrated by applying it to age-specific time series of surrender rates derived from pension plans with annuity payments of a Brazilian insurer. In the GLM process, the only macroeconomic variable used as an explanatory variable is the Brazilian real short-term interest rate. The advantage of such a variable is that we can take future market expectation through the current term structure of interest rates. The GLM residuals of each age/gender group are then modeled by ARMA-GARCH processes to generate i.i.d. residuals. The dependence among these residuals is then modeled by multivariate Gaussian and Student's t copulas. To produce a conditional forecast on a stock market index, in our application we used the residuals of an ARMA-GARCH model fitted to the Brazilian stock market index (Ibovespa) returns, which generates one of the marginal distributions used in the dependence modeling through copulas. This strategy is adopted to explain the high and uncommon surrender rates observed during the recent economic crisis. After applying known simulation methods for elliptical copulas, we proceeded backwards to obtain the forecasted distributions of surrender rates by application, in the sequel, of ARMA-GARCH and GLM models. Additionally, our approach produced an algorithm able to simulate multivariate elliptical copulas conditioned on a marginal distribution. Using this algorithm, surrender rates can be simulated conditioned on stock index residuals (in our case, the residuals of the Ibovespa returns), which allows insurers and pension funds to simulate future surrender rates assuming a financial stress scenario with no need to predict the stock market index.

TAN, KEN SENG; WENG, CHENGGUO. *Empirical approach for optimal reinsurance design*. 315-342. This article proposes a novel and practical approach of addressing optimal reinsurance via an empirical approach. This method formulates reinsurance models using the observed data directly and has advantages including (1) transformation of an infinite dimensional optimization problem to a finite dimension, (2) no required explicit distributional assumption on the underlying risk, and (3) many empirical-based reinsurance models can be solved efficiently using the second-

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order conic programming. This allows insurers to incorporate many desirable objective functions and constraints while still retaining the ease of obtaining optimal reinsurance strategies. Numerical examples, including applications to actual Danish fire loss data, are provided to highlight the efficiency and the practicality of the proposed empirical models. The stability and consistency of the empirical-based solutions are also analyzed numerically.

WOODWARD, JOSHUA D. *Impacts of weather and time horizon selection on crop insurance ratemaking: a conditional distribution approach*. 279-293. An important issue in the agricultural risk and actuarial literatures is the extent to which sample period selection affects the accuracy of insurance rating. This is typically more problematic in agriculture than in other types of insurance because of the variation in weather through time. A conditional Weibull distribution approach is developed that explicitly models the interaction of weather, technology, and other variables on probabilistic yield outcomes to address this issue. Results from an application with an extensive producer-level yield dataset representing commercial-scale Illinois firms suggest that the impact of weather heterogeneity on risk estimation across reasonable samples is likely not as great as is often claimed. The results also suggest that yield risk is decreasing significantly through time and indicate the presence of trend acceleration. A rating analysis indicates that violations in the risk evolution assumptions of the rating approaches used in the Federal Crop Insurance Program – which implicitly assume increasing yield risk through time when yields trend – result in severely biased rates, with typical overstatements of 200% to 400% for Midwest corn.

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ARMERIN, FREDRIK. *An axiomatic approach to the valuation of cash flows*. 32-40. We model a stream of cash flows as an optional stochastic process, and value the cash flows by using a continuous and strictly positive linear functional. By applying a representation theorem from the general theory of stochastic processes we are able to study this valuation principle, as well as properties of the stochastic discount factor it implies. This approach to valuation is useful in the non-presence of a financial market, as is often the case when valuing cash flows arising from insurance contracts and in the application of real options.

CHEUNG, KA CHUN; SUNG, K C J; YAM, S C P; YUNG, S P. *Optimal reinsurance under general law-invariant risk measures*. 72-91. In recent years, general risk measures play an important role in risk management in both finance and insurance industry. As a consequence, there is an increasing number of research on optimal reinsurance decision problems using risk measures beyond the classical expected utility framework. In this paper, we first show that the stop-loss reinsurance is an optimal contract under law-invariant convex risk measures via a new simple geometric argument. A similar approach is then used to tackle the same optimal reinsurance problem under Value at Risk and Conditional Tail Expectation; it is interesting to note that,

instead of stop-loss reinsurances, insurance layers serve as the optimal solution. These two results highlight that law-invariant convex risk measure is better and more robust, in the sense that the corresponding optimal reinsurance still provides the protection coverage against extreme loss irrespective to the potential increment of its probability of occurrence, to expected larger claim than Value at Risk and Conditional Tail Expectation which are more commonly used. Several illustrative examples will be provided.

DONNELLY, CATHERINE. *Quantifying mortality risk in small defined-benefit pension schemes*. 41-57. A risk of small defined-benefit pension schemes is that there are too few members to eliminate idiosyncratic mortality risk, that is, there are too few members to effectively pool mortality risk. This means that when there are few members in the scheme, there is an increased risk of the liability value deviating significantly from the expected liability value, as compared to a large scheme. We quantify this risk through examining the coefficient of variation of a scheme's liability value relative to its expected value. We examine how the coefficient of variation varies with the number of members and find that, even with a few hundred members in the scheme, idiosyncratic mortality risk may still be significant. Next we quantify the amount of the mortality risk concentrated in the executive section of the scheme, where the executives receive a benefit that is higher than the non-executive benefit. We use the Euler capital allocation principle to allocate the total standard deviation of the liability value between the executive and non-executive sections. The results suggest that the mortality risk of the scheme should be monitored and managed within the sections of a scheme and not only on a scheme-wide basis.

GUILLEN, MONTSERRAT; NIELSEN, JENS PERCH; PEREZ-MARTIN, ANA MARIA; PETERSEN, KITT S. *Performance measurement of pension strategies: A case study of Danish life-cycle products*. 49-68. The Danish pension market of life-cycle products have expanded considerably since its introduction in the beginning of the millennium. The market is maturing and pensioners have the choice between a wide area of different products. It is therefore about time that financial insurance technology is developed to guide the performance measurement of available products. In this paper we develop a simple first version of such a method and we investigate life-cycle products recommended on the web of the four biggest commercial Danish pension companies on one day in February 2007. All considered products are outperformed by trivial benchmark products with constant stock proportion over time. Our approach is the following: for each life-cycle product we first find a trivial benchmark product with the same longterm risk and then we compare the long-term return of the two equivalent products. We primarily consider value at risk (VaR) and tail VaR as risk measures, but we also include a study where the fair value of an interest guarantee is used as risk measure. We consider both long-term mean returns and long-term median returns. We hope that our new method will be regarded as a first step toward a scientifically based ranking of the quality of pension products.

GÜNTHER, CLARA-CECILIE; TVETE, INGUNN FRIDE; AAS, KJERSTI; SANDNES, GEIR INGE; BORGAN, ØRNULF. *Modelling and predicting customer churn from an insurance company*. 58-71. Within a company's customer relationship management strategy, finding the customers most likely to leave is a central aspect. We present a dynamic modelling approach for predicting individual customers' risk of leaving an insurance company. A logistic longitudinal regression model that incorporates time-dynamic explanatory variables and interactions is fitted to the data. As an intermediate step in the modelling procedure, we apply generalised additive models to identify non-linear relationships between the logit and the explanatory variables. Both out-of-sample and out-of-time prediction indicate that the model performs well in terms of identifying

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customers likely to leave the company each month. Our approach is general and may be applied to other industries as well.

HASHORVA, ENKELEJD. *On beta-product convolutions*. 69-83. Let R be a positive random variable independent of S which is beta distributed. In this paper we are interested on the relation between R and RS. For this model we derive first some distributional properties, and then investigate the lower tail asymptotics of RS when R is regularly varying at 0, and vice-versa. Our first application concerns the asymptotic behaviour of the componentwise sample minima related to elliptical distributions. Further, we derive the lower tail asymptotics of the aggregated risk for bivariate polar distributions.

KUZNETSOV, ALEXEY; MORALES, MANUEL. *Computing the finite-time expected discounted penalty function for a family of Lévy risk processes*. 1-31. Ever since the first introduction of the expected discounted penalty function (EDPF), it has been widely acknowledged that it contains information that is relevant from a risk management perspective. Expressions for the EDPF are now available for a wide range of models, in particular for a general class of Lévy risk processes. Yet, in order to capitalize on this potential for applications, these expressions must be computationally tractable enough as to allow for the evaluation of associated risk measures such as Value at Risk (VaR) or Conditional Value at Risk (CVaR). Most of the models studied so far offer few interesting examples for which computation of the associated EDPF can be carried out to the last instances where evaluation of risk measures is possible. Another drawback of existing examples is that the expressions are available for an infinite-time horizon EDPF only. Yet, realistic applications would require the computation of an EDPF over a finite-time horizon. In this paper we address these two issues by studying examples of risk processes for which numerical evaluation of the EDPF can be readily implemented. These examples are based on the recently introduced meromorphic processes, including the beta and theta families of Lévy processes, whose construction is tailor-made for computational ease. We provide expressions for the EDPF associated with these processes and we discuss in detail how a finite-time horizon EDPF can be computed for these families. We also provide numerical examples for different choices of parameters in order to illustrate how ruin-based risk measures can be computed for these families of Lévy risk processes.

MIERZEJEWSKI, FERNANDO. *Raising and allocation capital principles as optimal managerial contracts*. 24-48. A unified framework is presented to characterise the capital structure of firms that face borrowing restrictions – which extends the classic theory of capital by incorporating elements from actuarial and agency theory. It is demonstrated that the bankruptcy and agency costs afforded by these firms can be expressed in terms of the actuarial prices of the underlying exposures. Then the optimal surplus is determined in order to maximise value – which is equivalent to minimise the cost of bankruptcy plus the opportunity cost of capital. The capital principle thus obtained explicitly depends on risk and expectations, and can be applied to allocate reserves both in financial and insurance companies. An optimal decentralised mechanism is also defined that stimulates the exchange of information inside multidivisional corporations.

ZHOU, RUI; LIA, JOHNNY SIU-HANG. *A cautionary note on pricing longevity index swaps*. 1-23. In December 2007, Goldman Sachs launched a product called QxX index swap, which is designed to allow market participants to hedge or gain exposure to longevity and mortality risks. In this paper, we offer a quantitative analysis of this brand new financial innovation. First of all, we set up a risk-neutral framework to price QxX index swaps. This framework, which is based on

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the dynamics of death rates under a two-factor stochastic mortality model in a risk-adjusted probability measure, yields prices (spreads) that are fairly close to the spreads that Goldman Sachs currently offers. We then explore the uncertainty involved in this model-based pricing framework. Specifically, we study parameter risk by using Bayesian methods and model risk by examining structural changes in mortality dynamics. Our results indicate that both model risk and parameter risk are significant. Actuaries should therefore be aware of these issues when placing a value on a longevity index swap.

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BREUER, LOTHAR; BADESCU, ANDREI L. *A generalised Gerber–Shiu measure for Markov-additive risk processes with phase-type claims and capital injections*. 93-115. In this paper we consider a risk reserve process where the arrivals (either claims or capital injections) occur according to a Markovian point process. Both claim and capital injection sizes are phase-type distributed and the model allows for possible correlations between these and the inter-claim times. The premium income is modelled by a Markov-modulated Brownian motion which may depend on the underlying phases of the point arrival process. For this risk reserve model we derive a generalised Gerber-Shiu measure that is the joint distribution of the time to ruin, the surplus immediately before ruin, the deficit at ruin, the minimal risk reserve before ruin, and the time until this minimum is attained. Numeral examples illustrate the influence of the parameters on selected marginal distributions.

CASTAÑER, ANNA; CLARAMUNT, M MERCÈ; GATHY, MAUDE; LEFÈVRE, CLAUDE; MÁRMOL, MAITE. *Ruin problems for a discrete time risk model with non-homogeneous conditions*. 83-102. This paper is concerned with a non-homogeneous discrete time risk model where premiums are fixed but non-uniform, and claim amounts are independent but non-stationary. It allows one to account for the influence of inflation and interest and the effect of variability in the claims. Our main purpose is to develop an algorithm for calculating the finite time ruin probabilities and the associated ruin severity distributions. The ruin probabilities are shown to rely on an underlying algebraic structure of Appell type. That property makes the computational method proposed quite simple and efficient. Its application is illustrated through some numerical examples of ruin problems. The well known Lundberg bound for ultimate ruin probabilities is also reexamined within such a non-homogeneous framework.

CHADJICONSTANTINIDIS, STATHIS; VRONTOS, SPYRIDON. *On a renewal risk process with dependence under a Farlie-Gumbel-Morgenstern copula*. 125-158. In this article, we consider an extension to the renewal or Sparre Andersen risk process by introducing a dependence structure between the claim sizes and the interclaim times through a Farlie-Gumbel-Morgenstern copula proposed by Cossette *et al.* (2010) [Héléne Cossette, Etienne Marceau, Fouad Marri (2010), Analysis of ruin measures for the classical compound Poisson risk model with dependence, Scandinavian Actuarial Journal (2010) 3: 221-245] for the classical compound Poisson risk model. We consider that the inter-arrival times follow the Erlang(n) distribution. By studying the roots of the generalised Lundberg equation, the Laplace transform (LT) of the expected discounted penalty function is derived and a detailed analysis of the Gerber-Shiu function is given when the initial surplus is zero. It is proved that this function satisfies a defective renewal equation and its solution is given through the compound geometric tail representation of the LT of the time to ruin.

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Explicit expressions for the discounted joint and marginal distribution functions of the surplus prior to the time of ruin and the deficit at the time of ruin are derived. Finally, for exponential claim sizes explicit expressions and numerical examples for the ruin probability and the LT of the time to ruin are given.

CHEUNG, KA CHUN; VANDUFFEL, STEVEN. *Bounds for sums of random variables when the marginal distributions and the variance of the sum are given*. 103-118. In this paper, we establish several relations between convex order, variance order, and comonotonicity. In the first part, we extend Cheung (2008b) [Cheung, K. C. (2008b). Characterization of comonotonicity using convex order. Insurance: Mathematics and Economics 43, 403-406] to show that when the marginal distributions are fixed, a sum with maximal variance is in fact a comonotonic sum. Thus the convex upper bound is achieved if and only if the marginal variables are comonotonic. Next, we study the situation where besides the marginal distributions; the variance of the sum is also fixed. Intuitively one expects that adding this information may lead to a bound that is sharper than the comonotonic upper bound. However, we show that such upper bound does not even exist. Nevertheless, we can still identify a special dependence structure known as upper comonotonicity, in which case the sum behaves like a convex largest sum in the upper tail. Finally, we investigate when the convex order is equivalent to the weaker variance order. Throughout this paper, interpretations and significance of the results in terms of portfolio risks will be emphasized.

CHIU, SUNG NOK; YIN, CHUANCUN. *On the complete monotonicity of the compound geometric convolution with applications in risk theory*. 116-124. We prove that the complete monotonicity is preserved under mixed geometric compounding, and hence show that the ruin probability, the Laplace transform of the ruin time, and the density of the tail of the joint distribution of ruin and the deficit at ruin in the Sparre Andersen model are completely monotone if the claim size distribution has a completely monotone density.

HUANG, HUNG-HSI; SHIU, YUNG-MING; WANG, CHING-PING. *Optimal insurance contract with stochastic background wealth*. 119-139. This study presents an optimal insurance contract developed endogenously when insured individuals face two mutually dependent risks, background wealth and insurable loss. If background wealth is conditionally normally distributed given insurable loss, the optimal insurance contract may be proportional coinsurance above a straight deductible for a quadratic, negative exponential, or mean-variance utility function. Additionally, when the insured has a quadratic utility or mean-variance utility, the optimal retained schedule is a function of conditional expected value of background wealth given insurable loss. Moreover, the optimal insurance contracts for quadratic and negative exponential utility functions need not to be mean-variance efficient, even when the conditional normal distribution is assumed. Finally, when a portfolio problem is considered, the calculation about the optimal insurance contract remains almost unchanged.

NADARAJAH, SARALEES; BAKAR, S A A. *New composite models for the Danish fire insurance data*. 180-187. In recent years, several composite models based on the lognormal distribution have been developed for the Danish fire insurance data. In this note, we propose new composite models based on the lognormal distribution. At least one of the newly proposed models is shown to give a better fit to the Danish fire insurance data.

SHI, PENG; VALDEZ, EMILIANO A. *Longitudinal modeling of insurance claim counts using jitters*. 159-179. Modeling insurance claim counts is a critical component in the ratemaking

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process for property and casualty insurance. This article explores the usefulness of copulas to model the number of insurance claims for an individual policyholder within a longitudinal context. To address the limitations of copulas commonly attributed to multivariate discrete data, we adopt a 'jittering' method to the claim counts which has the effect of continuitizing the data. Elliptical copulas are proposed to accommodate the intertemporal nature of the 'jittered' claim counts and the unobservable subject-specific heterogeneity on the frequency of claims. Observable subject-specific effects are accounted in the model by using available covariate information through a regression model. The predictive distribution together with the corresponding credibility of claim frequency can be derived from the model for ratemaking and risk classification purposes. For empirical illustration, we analyze an unbalanced longitudinal dataset of claim counts observed from a portfolio of automobile insurance policies of a general insurer in Singapore. We further establish the validity of the calibrated copula model, and demonstrate that the copula with 'jittering' method outperforms standard count regression models.

ZHU, JINXIA. *Optimal dividend control for a generalized risk model with investment incomes and debit interest*. 140-162. This paper investigates dividend optimization of an insurance corporation under a more realistic model, which takes into consideration refinancing or capital injections. The model follows the compound Poisson framework with credit interest for positive reserve and debit interest for negative reserve. Ruin occurs when the reserve drops below the critical value. The company controls the dividend pay-out dynamically with the objective to maximize the expected total discounted dividends until ruin. We show that the optimal strategy, is a band strategy and it is optimal to pay no dividends when the reserve is negative.

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ALAI, DANIEL H; SHERRIS, MICHAEL. *Rethinking age-period-cohort mortality trend models*. 208-227. Longevity risk arising from uncertain mortality improvement is one of the major risks facing annuity providers and pension funds. In this article, we show how applying trend models from non-life claims reserving to age-period-cohort mortality trends provides new insight in estimating mortality improvement and quantifying its uncertainty. Age, period and cohort trends are modelled with distinct effects for each age, calendar year and birth year in a generalised linear models framework. The effects are distinct in the sense that they are not conjoined with age coefficients, borrowing from regression terminology, we denote them as main effects. Mortality models in this framework for age-period, age-cohort and age-period-cohort effects are assessed using national population mortality data from Norway and Australia to show the relative significance of cohort effects as compared to period effects. Results are compared with the traditional Lee-Carter model. The bilinear period effect in the Lee-Carter model is shown to resemble a main cohort effect in these trend models. However, the approach avoids the limitations of the Lee-Carter model when forecasting with the age-cohort trend model.

BARGÈS, MATHIEU; LOISEL, STÉPHANE; VENEL, XAVIER. *On finite-time ruin probabilities with reinsurance cycles influenced by large claims*. 163-185. Market cycles play a great role in reinsurance. Cycle transitions are not independent from the claim arrival process: a large claim or a high number of claims may accelerate cycle transitions. To take this into account, a semi-Markovian risk model is proposed and analyzed. A refined Erlangization method is developed to compute the finite-time ruin probability of a reinsurance company. Numerical applications and

comparisons to results obtained from simulation methods are given. The impact of dependency between claim amounts and phase changes is studied.

BEBBINGTON, MARK; GREEN, REBECCA; LAI, CHIN-DIEW; ZITIKIS, RICARDAS. *Beyond the Gompertz law: exploring the late-life mortality deceleration phenomenon.* 189-207. Using a new distribution capable of exhibiting all the possible modes of accelerating and decelerating mortality, we conduct a systematic investigation of late-life mortality in humans. We check the insensitivity of the distribution to age cutoffs in the data relative to the logistic mortality model and propose a method to forecast evolution in the characteristic deceleration ages of the distribution. A number of data sets have been explored, with a particular emphasis on those originating from Scandinavia. Although those from Australia, Canada, and the USA are compatible with Gompertzian mortality, those from the other countries examined are not. We find in particular that the onset of mortality deceleration is being progressively delayed in Western societies but that there is evidence of mortality plateauing at earlier ages.

LENART, ADAM. *The moments of the Gompertz distribution and maximum likelihood estimation of its parameters.* 255-277. The Gompertz distribution is widely used to describe the distribution of adult deaths. Previous works concentrated on formulating approximate relationships to characterise it. However, using the generalised integro-exponential function, exact formulas can be derived for its moment-generating function and central moments. Based on the exact central moments, higher accuracy approximations can be defined for them. In demographic or actuarial applications, maximum likelihood estimation is often used to determine the parameters of the Gompertz distribution. By solving the maximum likelihood estimates analytically, the dimension of the optimisation problem can be reduced to one both in the case of discrete and continuous data. Monte Carlo experiments show that by ML estimation, higher accuracy estimates can be acquired than by the method of moments.

RABEHASAINA, LANDY; TSAI, CARY CHI-LIANG. *Ruin time and aggregate claim amount up to ruin time for the perturbed risk process.* 186-212. We consider the classical Sparre-Andersen risk process perturbed by a Wiener process, and study the joint distribution of the ruin time and the aggregate claim amounts until ruin by determining its Laplace transform. This is first done when the claim amounts follow respectively an exponential/Phase-type distribution, in which case we also compute the distribution of recovery time and study the case of a barrier dividend. Then the general distribution is considered when ruin occurs by oscillation, in which case a renewal equation is derived.

SASS, JÖRN; SEIFRIED, FRANK THOMAS. *Insurance markets and unisex tariffs: is the European Court of Justice improving or destroying welfare?* 228-254. We analyze the effects of mandatory unisex tariffs in insurance contracts, such as those required by a recent ruling of the European Court of Justice, on equilibrium insurance premia and equilibrium welfare. In a unified framework, we provide a quantitative analysis of the associated insurance market equilibria in both monopolistic and competitive insurance markets. We investigate the welfare loss caused by regulatory adverse selection and show that unisex tariffs may cause market distortions that significantly reduce overall social welfare.

SCOLLNIK, DAVID P M. *Letter to the Editor: Regarding folded models and the paper by Brazauskas and Kleefeld (2011)* 278-281. [This paper considers: Vytaras Brazauskas and Andreas Kleefeld (2011), Folded and log-folded-t distributions as models for insurance loss data, Scandinavian Actuarial Journal (2011) 1: 59-74.]

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ZHANG, ZHIMIN; YANG, HAILIANG; YANG, HU. *On a Sparre Andersen risk model perturbed by a spectrally negative Lévy process*. 213-239. In this paper, we consider a Sparre Andersen risk model perturbed by a spectrally negative Lévy process (SNLP). Assuming that the interclaim times follow a Coxian distribution, we show that the Laplace transforms and defective renewal equations for the Gerber-Shiu functions can be obtained by employing the roots of a generalized Lundberg equation. When the SNLP is a combination of a Brownian motion and a compound Poisson process with exponential jumps, explicit expressions and asymptotic formulas for the Gerber-Shiu functions are obtained for exponential claim size distribution and heavy-tailed claim size distribution, respectively.

ZHU, JINXIA. *Errata for 'Optimal dividend control for a generalized risk model with investment incomes and debit interest' online version*. 282. Errata for 'Optimal dividend control for a generalized risk model with investment incomes and debit interest' online version [Scandinavian Actuarial Journal (2013) 2: 140-162]

Available for downloading from: http://www.openathens.net/

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ADAMIC, PETER; CARON, SYLVAIN. *SC-CR Algorithms with informative masking*. 339-351. In this article, we present a significant improvement to the Self-Consistent, Competing Risks (SC-CR) Algorithms that have been published in the actuarial literature over the last several years. These algorithms were fairly flexible, admitting of any combination of partially masked risks and interval-censored failure times. However, we wish to show here that the SC-CR Algorithm can be further generalized to allow for each specific decrement to have its own distinct interval-censored range for any given individual observation. We have chosen to refer to this dynamic as 'informative masking' since additional information regarding the masking sets will be allowed to be incorporated. The enhancements will be applied to both the double-censored as well as interval-censored SC-CR Algorithms. Numerical examples that illustrate the usefulness of these enhancements will also be furnished.

CHRISTIANSEN, MARCUS C. *Safety margins for unsystematic biometric risk in life and health insurance*. 286-323. In multistate life and health insurances, the pattern of states of the policyholder is random, thus exposing the insurer to an unsystematic biometric risk. For this reason safety margins are added on premiums and reserves. But in contrast to non-life insurance, traditionally the safety margins are not chosen explicitly but implicitly in form of a valuation basis of first order. If we define the implicit margins bottom-up, we are not able to control the level of safety that we finally reach for premiums and reserves. If we use a top-down approach, that means that we directly calculate explicit margins for premiums and reserves and then choose implicit safety margins that correspond to the explicit margins, we are able to control the total portfolio risk, but we have the problem that it is unclear how to allocate the total margin to partial margins for different transitions at different ages. Although the allocation of the total margin to the partial (implicit) margins is not relevant for the total portfolio risk, we have to pay attention since it can have a great effect on the calculation of surplus. In this paper we calculate asymptotic probability distributions for premiums and reserves of second order by using the functional delta method. As a result, we can not only determine the actual level of

safety that is induced by given implicit safety margins, but we can also linearly decompose the total randomness of a portfolio to contributions that the different transition rates at different ages make to the total uncertainty. As a result we do not only get new insight into the sources of unsystematic biometric risk, but we also obtain a useful tool that allows to construct reasonable principles for the allocation of the total safety margin to implicit margins with respect to transitions and ages.

EKHULT, HANS; HÖSSJER, OLA. *Pricing catastrophe risk in life (re)insurance*. 352-367. What is the catastrophe risk a life insurance company faces? What is the correct price of a catastrophe cover? During a review of the current standard model, due to Strickler, we found that this model has some serious shortcomings. We therefore present a new model for the pricing of catastrophe excess of loss cover (Cat XL). The new model for annual claim cost C is based on a compound Poisson process of catastrophe costs. To evaluate the distribution of the cost of each catastrophe, we use the Peaks Over Threshold model for the total number of lost lives in each catastrophe and the beta binomial model for the proportion of these corresponding to customers of the insurance company. To be able to estimate the parameters of the model, international and Swedish data were collected and compiled, listing accidents claiming at least twenty and four lives, respectively. Fitting the new model to data, we find the fit to be good. Finally we give the price of a Cat XL contract and perform a sensitivity analysis of how some of the parameters affect the expected value and standard deviation of the cost and thus the price.

LANDRIAULT, DAVID; SHI, TIANXIANG. *First passage time for compound Poisson processes with diffusion: ruin theoretical and financial applications*. 368-382. In this paper, we propose to revisit Kendall's identity (see, e.g. Kendall (1957) [D G Kendall (1957). Some problems in the theory of dams. Journal of the Royal Statistical Society: Series B (1957) 19: 207-212.]) related to the distribution of the first passage time for spectrally negative Lévy processes. We provide an alternative proof to Kendall's identity for a given class of spectrally negative Lévy processes, namely compound Poisson processes with diffusion, through the application of Lagrange's expansion theorem. This alternative proof naturally leads to an extension of this well-known identity by further examining the distribution of the number of jumps before the first passage time. In the process, we generalize some results of Gerber (1990) [H U Gerber, H. U. (1990), When does the surplus reach a given target? Insurance: Mathematics and Economics (1990) 9: 115-119] to the class of compound Poisson processes perturbed by diffusion. We show that this main result is particularly relevant to further our understanding of some problems of interest in actuarial science. Among others, we propose to examine the finite-time ruin probability of a dual Poisson risk model with diffusion or equally the distribution of a busy period in a specific fluid flow model. In a second example, we make use of this result to price barrier options issued on an insurer's stock price.

LIU, JINGZHEN; YIU, KA-FAI CEDRIC; SIU, TAK KUEN; CHING, WAI-KI. *Optimal investment-reinsurance with dynamic risk constraint and regime switching*. 263-285. We study an optimal investment-reinsurance problem for an insurer who faces dynamic risk constraint in a Markovian regime-switching environment. The goal of the insurer is to maximize the expected utility of terminal wealth. Here the dynamic risk constraint is described by the maximal conditional Value at Risk over different economic states. The rationale is to provide a prudent investment-reinsurance strategy by taking into account the worst case scenario over different economic states. Using the dynamic programming approach, we obtain an analytical solution of the problem when the insurance business is modeled by either the classical Cramer-Lundberg model or its diffusion

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approximation. We document some important qualitative behaviors of the optimal investment-reinsurance strategies and investigate the impacts of switching regimes and risk constraint on the optimal strategies.

MASIELLO, ESTERINA. *On semiparametric estimation of ruin probabilities in the classical risk model*. 283-308. The ruin probability of an insurance company is a central topic in risk theory. We consider the classical Poisson risk model when the claim size distribution and the Poisson arrival rate are unknown. Given a sample of inter-arrival times and corresponding claims, we propose a semiparametric estimator of the ruin probability. We establish properties of strong consistency and asymptotic normality of the estimator and study bootstrap confidence bands. Further, we present a simulation example in order to investigate the finite sample properties of the proposed estimator.

ZADEH, AMIN HASSAN; BILODEAU, MARTIN. *Fitting bivariate losses with phase-type distributions*. 241-262. Maximum likelihood estimation and (parametric bootstrap) goodness-of-fit test are considered for bivariate phase-type distributions introduced by Assaf and colleagues. In a special case, the dependence structure of bivariate phase-type distributions is revealed. The results are used to fit a real bi-dimensional data set related to insurance losses (LOSS) and allocated loss adjustment expenses (ALAE). The fitted bivariate phase-type is used to obtain conditional quantiles and mean of ALAE for a given value of LOSS. The bivariate phase-type distribution meets all the requirements listed in the study by Klugman and Parsa.

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