large an area as the one illustrated. With the introduction of an electronic computer into the Meteorological Office it was decided to do all the calculations on the computer, and a programme was devised which, without any effort other than the insertion of appropriate constants as data into the computer, prints out, in order:

(i) the intercepts along the borders of the map of each longitude,
(ii) the distances from the border along each longitude of each intersecting latitude,
(iii) the distances along each appropriate border of the bearings from any point within the chart whose coordinates are given.

Item (i) requires the calculation of six trigonometrical functions plus a division, one or two multiplications and a few additions and subtractions for each intercept. Item (ii) requires the calculation of two trigonometrical functions, one division, two or three multiplications, and a few additions and subtractions for each latitude. Item (iii) requires a small preliminary calculation including six trigonometrical functions, one division, twenty-eight multiplications and eight additions or subtractions (all this is accomplished within $\frac{1}{15}$ sec.), and then, for each bearing, two more trigonometrical functions, twelve multiplications, one division and eight additions or subtractions.

The slowest part of the programme is the actual printing out of the solution which is limited by the speed of the punch which transforms the answer as obtained in the machine to teleprinter tape. The output is thus limited (even printing to only four-figure accuracy or less than half that used by the machine and with the punch operating at full speed) to three numbers a second. The teleprinter on which the results are printed is five times as slow as the punch (even when working at full speed). Thus the coordinates for the chart illustrated took some 40–45 minutes to punch out, plus another 15 minutes for the direction roses. The printed solution was obtained from the teleprinter in a little under 5 hours. With the high-speed printer now available it should be possible to keep up with the computer, and print the whole solution out in some 15–20 minutes. This compares with an estimated time of calculation, using logarithmic and trigonometrical tables, of about three months.

The solution is quite general as to the position of the tangential point of the projection plane, but the chart must not cover so large an area of the Earth's surface that latitudes 60° south of the tangential point are included within it. The stations for which direction roses are required must lie within the borders of the chart.

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Detection of Gross Instrumentation Errors

from J. B. Parker

MR. J. F. GREEN, in an article on gyro failure (this *Journal*, **13**, 78) draws attention to a principle that is of direct importance whenever navigators are dependent, for their information, on an equipment which, though normally serviceable, can develop a gross error. His principle is the triplication of instrumentation: the agreement between the two supposed serviceable equipments is a control on the possible malfunctioning of the third.

As Mr. Green states in his argument, the basic problem is to decide on action given three readings x_1, x_2 and x_3, each independently drawn from the same underlying probability distribution.

In work of this nature the choice of the probability distribution is a difficult one and in my opinion represents an unsatisfactory feature of the treatment. Bench and air tests are generally adequate to establish that part of the performance that is characteristic of normal operating conditions, and this is generally expressed in terms of a single quantity called the standard deviation. One is, however, never sure of the rate of incidence, let alone the magnitude, of the errors prevailing when the equipment is malfunctioning. If one has to choose a mathematical model for this, I think Mr. Green's is probably the best one but first we should ask ourselves how far we can go without it.

This involves a slight change of ground. The argument now runs as follows. Take as the hypothesis that the equipments are all correctly functioning and let us seek a criterion based on x_1, x_2, x_3 themselves which will either refute or confirm this hypothesis. An obvious criterion is the difference between the two extremes: if this is high the hypothesis is refuted. In this case we must reject one of the readings—obviously the discordant one. If there is no reason to doubt the hypothesis the best value to take is the mean of the three readings.

The problem of assessing a three-position line fix is an interesting parallel. For normal practical purposes the incentre is a good choice (this *Journal*, **5**, 248). The size of the cocked hat, however, is a good control of the consistency of the observations and if the radius of the inscribed circle exceeds some quite definite value the hypothesis that all observations are sound should be discarded. The rules for this procedure do not require any assumptions to be made about the likely blunder rate or about the distribution of errors that occur when a blunder has been made.

In Mr. Green's problem, unlike the cocked hat problem, the bad reading can in general be isolated and ignored. It is possible to put forward, in his case, the following strategy:

(a) If some criterion, C, is satisfied, take the mean of x_1, x_2 and x_3.
(b) If the criterion is not satisfied, reject the rogue and take the mean of the other two.

Before specifying C (Mr. Green in fact goes a long way to defining a C in his 'Scheme III'), a few remarks comparing 'median steering'—that is, selecting the middle one of x_1, x_2 and x_3 and sticking by it—with the strategy suggested here may be of interest. The median approach is attractively simple (always choose the middle observation) whilst the suggested method is philosophically objectionable, for two quite different lines of action result depending on whether the criterion is just met or just fails to be met. Nevertheless, median steering leads to the throwing away of data. Whether statistical accuracy is preferable to a simplified procedure is a matter for experience to decide.

In choosing C, Mr. Green has suggested that the difference between the extremes should be taken as an index for deciding whether the values are consonant one with another. This is a good, simple criterion. If the standard deviation of the basic equipment when operating normally is known to be σ, the difference between the two extremes should not exceed $3\cdot3\ \sigma$. (This is the 5 per cent point of the distribution of the range in samples of 3. In only one occasion out of every 20 will the difference exceed this value if all equipments are operating in accordance with their specified standard deviation.) If they do, the rogue will be rejected and the mean of the remaining two taken.

This criterion is the simplest but not the best. This is derived as follows. The variance of the three values x_1, x_2 and x_3 is the quantity

$$V = \tfrac{1}{2}\{x_1^2 + x_2^2 + x_3^2 - \tfrac{1}{3}(x_1 + x_2 + x_3)^2\}$$
$$= \tfrac{1}{3}\{D_1^2 + D_2^2 + D_1 D_2\}$$

where D_1 = largest minus median

D_2 = median minus smallest

V is on average the square of σ, and will only differ from this through statistical fluctuations provided all the equipments are operating satisfactorily. If $V < 6\sigma^2$ (the numerical value of 6 being the 5 per cent point of the appropriate statistical distribution) C is satisfied.

Summing up, there is a case for considering a strategy based on a rejection technique (which has been put forward as a possibility by Mr. Green in his Scheme III). Both criteria put forward here could be automized. It is suggested that such considerations should be borne in mind in all fields of instrumentation where Mr. Green's principle of triplication is a practical possibility.

Mr. J. F. Green comments:

I am in agreement with all that Mr. J. B. Parker has said, and what follows is only in amplification of several points that he has referred to in my original text.

My original paper was largely restricted to a discussion of free gyro techniques as they apply at the present day (see the 1st Theme of the Paris Convention 28 April, 1959). One has to assume then that the navigator has no automatic equipment available to process the information that he receives from the three azimuth gyros. Under these conditions the attractions of median steering as a technique are overwhelming. The correct signal is arrived at by an elementary mental process. Moreover, the probability that this steering signal is within specification is of the same order as that derived from a sophisticated automatic process. So safety requirements have been satisfied.

As Mr. Parker points out, however, accuracy has been sacrificed when selecting the median between three signals which are all within specification because some data has been rejected. Once automation is introduced the average steering error can be further reduced without prejudicing safety.

In general the extra accuracy is obtained by selecting the mean of the three signals. Safety is ensured by applying a test to the three signals before mixing them. If the test signifies a fault then further tests are necessary to identify the faulty signal in order that it may be rejected.

The ease with which these tests may be devised and the reliability with which the whole system may be engineered are probably the most significant factors affecting the final design. For instance the discriminatory mechanisms are themselves fallible and it is essential that they should fail safe.

An interesting application of this problem arises from the monitoring of the outputs of three nominally identical autopilot control channels. By such

7

redundant multiplicity it is possible to obtain the overall standard of reliability required of an autopilot for automatic landing.

The servo motor outputs from the three autopilot sub-channels are ganged mechanically to a common output arm and in this way the three output signals are averaged. Torque limiters in each arm effectively compare the torque generated by each channel with the mean torque and provided that three inequalities which are approximately of the form

$$|T_i - T_M| < K \qquad \text{where } T_i = \text{servo torque generated in } i\text{th channel}$$
$$T_M = \text{mean servo torque}$$
$$K = \text{a constant}$$

are satisfied, the system continues to operate. If one inequality is violated not only is a fault detected but also it is automatically identified and rejected by the action of the torque limiter which disengages the errant channel.

By mixing three signals in this way a high average accuracy is ensured. At the same time the torque discriminators which can only fail safe ensure maintenance of high reliability.

'Manœuvres to Ensure the Avoidance of Collision'

THE following comments have been received on Mr. E. S. Calvert's paper, published in the last number of the *Journal*. A further selection of comment, including a note by Captain F. J. Wylie, and possibly a reply by Mr. Calvert, will be published in the next number.

from D. H. Sadler, O.B.E.

THE following comments are restricted to two aspects only of Mr. Calvert's paper, namely the proposal for a new form of radar display and vector diagram, and the geometrical analysis of the collision problem. All forms of display, or of the vector geometry, are equivalent, but it is of great importance to develop one which is capable of showing immediately the changing situation; Mr. Calvert's proposed presentation is worthy of detailed study from this point of view.

An essential preliminary to any comprehensive study of collision avoidance is a complete and detailed study of the geometry of the problem. Whether this conflicts with the Collision Regulations or not, whether some manœuvres are impracticable or not, is irrelevant; as Mr. Calvert says, the geometry is universal and unchanging, and cannot be the subject of opinion. Any manœuvres or rules must be based on a clear understanding of the underlying geometry.

From this point of view Mr. Calvert has written a most important paper and one that will repay detailed study. His proposed manœuvres may constitute a complete answer to the collision problem in certain circumstances; but it will be for practical navigators to determine to what extent these manœuvres can be incorporated in future rules.