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On matrix methods in ring theory

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Several classes of rings are studied and their structures determined by a direct method of representing rings as matrix rings. This method is one of the main contributions of the thesis. It is developed in a general form in Chapter 1.

Nonsingular rings with essential socles are investigated in Chapter 2. The main result is a characterization of these rings by an embedding into products of matrix rings over sfields and their bi-vector spaces. When the identity of the ring is finite (that is, a sum of orthogonal primitive idempotents) the embedding is onto and gives the known representation for such a ring. These results are used in Chapter 3 to determine the structure of nonsingular QF-3 rings with finite identities. It is also shown that a nonsingular left, and right, QF-3 ring has a finite identity and finite dimensional left, and right, socles which are essential.

Chapter 4 is devoted to rings whose ideals are quasi-injective (Q-rings). It proves that apart from the simple Artinian rings there is only one class of indecomposable non-local Q-rings. These are the rings of all $m \times m$-matrices, m an integer greater than one, whose non-zero entries are arbitrary elements of a sfield D along the diagonal and arbitrary elements of a null D-algebra V, whose left and right D-dimensions are both equal to one, at the places $(2, 1), \ldots, (m, m-1)$ and $(1, m)$.

These four chapters emphasize the dependence of the complexity of a ring's structure on its idempotents. It is the author's opinion that a theory of "large" rings could be developed on the structure of their sets of idempotents. This might provide a useful alternative to the traditional

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approach to studying rings by placing radical-type restrictions.

The last chapter, Chapter 5, is in some senses the most important. It determines the structure of left generalized uniserial rings and shows that it is possible, in non-trivial cases, to construct a ring from its ideal lattice.

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