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Multipliers between some function spaces on groups

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Let G be a nondiscrete locally compact abelian group with dual group Γ. For $1 \le p \le \infty$, denote by $A_p(G)$ the space of integrable functions on G whose Fourier transforms belong to $L_p(\Gamma)$. We investigate multipliers from $A_p(G)$ to $A_q(G)$. If G is compact and $2 < p_1, p_2 < \infty$, we show that multipliers of $A_{p_1}(G)$ and multipliers of $A_{p_2}(G)$ are different, provided $p_1 \ne p_2$. For compact G, we also exhibit a relationship between $l_r(\Gamma)$ and the multipliers from $A_p(G)$ to $A_q(G)$. If G is a compact nonabelian group we observe that the spaces $A_p(G)$ behave in the same way as in the abelian case as far as the multiplier problems are concerned.

Introduction

Let G be a locally compact abelian group. Throughout this paper G will be nondiscrete and, unless otherwise stated, $1 \le p < \infty$. Let $A_p(G) = \{f \in L_1(G) \mid \hat{f} \in L_p(\Gamma)\}$. For $f \in A_p(G)$, we define $\|f\|_{A_p} = \|f\|_1 + \|\hat{f}\|_p$. If G is compact and nonabelian, $A_p(G)$ is defined in an analogous way (see [4]). Under convolution as multiplication $A_p(G)$ is a commutative semi-simple Banach algebra.

Let X and Y be translation invariant topological linear spaces of

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functions or measures defined on G for which it is possible to define Fourier or Fourier-Stieltjes transforms. A continuous linear transformation T from X into Y is called a multiplier if T commutes with translations. Let T be a linear transformation from X to Y. Suppose there exists a function ϕ on Γ such that $(Tf)^\wedge = \phi\hat{f}$, for each $f \in X$. Such a T commutes with translations, and in many cases T is continuous. Consequently, such a T would define a multiplier from X to Y. The collection of all multipliers from X into Y will be denoted by $M(X, Y)$. The set of all functions ϕ on Γ which define elements $T \in M(X, Y)$ in the above manner will be denoted by $M_X^Y(\Gamma)$. We shall write $M(X, X) = M(X)$ and $M_X^X(\Gamma) = M_X(\Gamma)$. If G is infinite, compact, nonabelian then $M(X, Y)$ and $M_X^Y(\Sigma)$ are defined similarly, where Σ is the dual object of G.

If G is a noncompact locally compact abelian group, then $M_{A_p}(\Gamma) = M(G)^\wedge$ (see [7], 204-207), and if G is a compact abelian group, then $M_{A_p}(\Gamma) = l_\infty(\Gamma)$, provided $1 \le p \le 2$. Thus we see that $M_{A_p}(\Gamma)$ need not depend on the index p. The situation is not so simple for compact G when $2 < p < \infty$. In Section 1, we show that $M_{A_{p_1}}(\Gamma) \neq M_{A_{p_2}}(\Gamma)$ for compact G and $2 < p_1, p_2 < \infty$, if $p_1 \neq p_2$.

$M(A_p(G))$ and $M(L_1, A_p)$ have been studied in some detail (compare [6], [7]). However, a systematic study of $M(A_p, A_q)$ has not been made so far. It is very easy to see that $M_{A_p}^{A_q}(\Gamma) = M_{A_p}(\Gamma)$ if $p \le q$ (see [7]), and $M_{A_p}^{L_1}(\Gamma) = M_{A_p}(\Gamma)$. In Section 2, we study $M_{A_p}^{A_q}(\Gamma)$ for $q < p$.

Multipliers from $L_1(G)$ to $A_p(G)$ have been studied in detail in [6] for abelian groups. The methods of [6] do not appear to extend to non-abelian groups. In Section 3, we determine multipliers from $L_1(G)$ to $A_p(G)$ for a compact nonabelian group G. Our method works for abelian groups also. In fact, it is simpler for abelian groups.

1. Multipliers of A_p

By proving the existence of sets of uniqueness for $L_p(G)$ with $1 \le p \le 2$, Figà-Talamanca and Gaudry have proved in [2] that $M(L_p(G)) \subsetneqq M(L_2(G))$ for a nondiscrete locally compact abelian group G. The authors of [2] then employ the Riesz convexity theorem to prove that $M_{L_p}(\Gamma) \cap C_0(\Gamma) \subsetneqq M_{L_q}(\Gamma) \cap C_0(\Gamma)$ for $1 \le p < q \le 2$. Price [9] has generalized these results.

In view of the above results, we were led to investigate analogous questions for A_p-multipliers. Our results are included in the following theorem.

THEOREM 1.1. *Let* G *be an infinite compact abelian group,* $1 \le q < \infty$, $2 < p < \infty$, *and* $p > q$. *Then*

(i) $M_{A_p}(\Gamma) \cap C_0(\Gamma) \subsetneqq M_{A_q}(\Gamma) \cap C_0(\Gamma)$,

(ii) $\bigcup_{p>q} M_{A_p}(\Gamma) \subsetneqq M_{A_q}(\Gamma)$.

Proof. *(i)* If $q \le 2$ then $M_{A_q}(\Gamma) = l_\infty(\Gamma)$. It is also known (see [7], p. 208) that there exists a function ϕ in $C_0(\Gamma)$ such that $\phi \notin M_{A_p}(\Gamma)$. This implies *(i)*. Let us then suppose that $q > 2$. Let $r = \frac{2q}{q-2}$. Then $r > 2$ and $q = \frac{2r}{r-2}$. By [10, Theorem 1] it follows that there exists a function f in $A_p(G)$ such that $\sum_{\gamma\in\Gamma} |\hat{f}(\gamma)|^{2r/r-2} = \infty$. It follows from [4, Theorem 35.4, Part VI] that there exists a $\psi \in l_r(\Gamma)$ such that $\psi\hat{f} \notin l_2(\Gamma)$. Hence by [4, Corollary 36.13] there exists a function $\varepsilon(\gamma) = \pm 1$ on Γ such that $\varepsilon(\gamma)\psi(\gamma)\hat{f}(\gamma)$ is not the Fourier transform of any integrable function on G. Let $\phi(\gamma) = \varepsilon(\gamma)\psi(\gamma)$. Then $\phi \notin M_{A_p}(\Gamma)$. We shall show that $\phi \in M_{A_q}(\Gamma)$, proving *(i)*. In fact we show

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that $l_r(\Gamma) \subseteq M_{A_q}(\Gamma)$. Let $\phi \in l_r(\Gamma)$, where $r = \frac{2q}{q-2}$, and $f \in A_q(G)$. Then by Hölder's inequality we get

$$\sum_{\gamma\in\Gamma} |\phi(\gamma)|^2|\hat{f}(\gamma)|^2 \le \left(\sum_{\gamma\in\Gamma} |\phi(\gamma)|^{2q/q-2}\right)^{q-2/q}\left(\sum_{\gamma\in\Gamma} |\hat{f}(\gamma)|^q\right)^{2/q} < \infty .$$

Therefore $\phi\hat{f} \in l_2(\Gamma)$, and there exists $g \in L_1(G)$ such that $\hat{g} = \phi\hat{f}$. Since $\hat{f} \in l_q(\Gamma)$ and ϕ is bounded, $\hat{g} \in l_q(\Gamma)$, and therefore $\phi\hat{f} \in (A_q(G))^\wedge$. Thus $\phi \in M_{A_q}(\Gamma)$, and the proof of *(i)* follows.

(ii) For $\phi \in M_{A_p}(\Gamma)$, let $\|\phi\|_{A_p}$ denote the norm of the corresponding operator $T \in M(A_p(G))$. Since $M_{A_p}(\Gamma)$ is a commutative semi-simple Banach algebra with this norm for all p, and $M_{A_p}(\Gamma) \subseteq M_{A_q}(\Gamma)$ whenever $p > q$, we get that for some constant K, $\|\phi\|_{A_q} \le K\|\phi\|_{A_p}$ for all $\phi \in M_{A_p}(\Gamma)$. Since $M_{A_p}(\Gamma) \subsetneq M_{A_q}(\Gamma)$ (from *(i)*), it follows from the open mapping theorem ([5], p. 99) that $M_{A_p}(\Gamma)$ is of first category in $M_{A_q}(\Gamma)$. Let $\{p_n\}$ be a decreasing sequence such that $p_n \to q$. Then $\bigcup_{p>q} M_{A_p}(\Gamma) = \bigcup_{n=1}^{\infty} M_{A_{p_n}}(\Gamma)$. This shows that $\bigcup_{p>q} M_{A_p}(\Gamma)$ is of first category in $M_{A_q}(\Gamma)$, and hence *(ii)* follows.

REMARK 1.2. *The assertions in the above theorem hold with obvious modifications, even if G is an infinite compact nonabelian group. The proof is exactly similar to the above and the results needed in the argument can be found in* [4, Theorem 35.4, Part VI] *and* [3, Theorem 2.b].

2. Multipliers from $A_p(G)$ to $A_q(G)$

In this section we study $M_{A_p}^{A_q}(\Gamma)$ for $1 \le q < p < \infty$. As mentioned in the introduction, if $p \le q$, then $M_{A_p}^{A_q}(\Gamma) = M_{A_p}^{A_p}(\Gamma)$ and the problem has been investigated in detail [6] and [7].

PROPOSITION 2.1. *Let* G *be a noncompact locally compact abelian group, and let* $B_r(G) = \{\mu \in M(G) : \hat{\mu} \in L_r(\Gamma)\}$ *, where* $1 \le r < \infty$ *. Then*

$$\left(B_{pq/p-q}(G)\right)^\wedge \subseteq M_{A_p}^{A_q}(\Gamma) \subsetneq \left(M(G)\right)^\wedge .$$

Proof. Let $\phi = \hat{\mu}$, $\mu \in B_{pq/p-q}(G)$. Then $\phi\hat{f} = \hat{\mu}\hat{f} = (\mu * f)^\wedge \in \left(L_1(G)\right)^\wedge$ for $f \in L_1(G)$. If $f \in A_p(G)$, then by Hölder's inequality $\phi\hat{f} \in L_q(\Gamma)$, and hence $\phi \in M_{A_p}^{A_q}(\Gamma)$. Also $M_{A_p}^{A_q}(\Gamma) \subseteq M_{A_p}(\Gamma) = \left(M(G)\right)^\wedge$. To prove that the second inclusion is proper, we observe that $\delta_0 \notin M_{A_p}^{A_q}(\Gamma)$, where δ_0 is the identity of $M(G)$. This follows from [10, Theorem 2].

Now we discuss $M_{A_p}^{A_q}(\Gamma)$. Throughout the rest of this section G will be an infinite compact abelian group.

PROPOSITION 2.2. *Let* $1 \le q < p \le 2$ *. Then* $M_{A_p}^{A_q}(\Gamma) = l_{pq/p-q}(\Gamma)$.

Proof. We observe that $\left(A_r(G)\right)^\wedge = l_r(\Gamma)$ for $1 \le r \le 2$. Therefore $\phi \in M_{A_p}^{A_q}(\Gamma)$ if and only if $\phi\psi \in l_q(\Gamma)$, $\psi \in l_p(\Gamma)$, that is if and only if $\phi \in l_{pq/p-q}(\Gamma)$; see [4, Theorem 35.4, Part VI].

PROPOSITION 2.3. *Let* $1 \le q \le 2 < p$ *. Then*

$$l_{pq/p-q}(\Gamma) \subseteq M_{A_p}^{A_q}(\Gamma) \subsetneq l_{2q/2-q}(\Gamma) .$$

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Moreover, if $r > \frac{pq}{p-q}$ *, then* $l_r(\Gamma) \not\subset \overset{A}{M}{}_{A_p}^{q}(\Gamma)$ *, and if* $r > p$ *, then* $l_{pq/p-q}(\Gamma) \not\subset \overset{A}{M}{}_{A_r}^{q}(\Gamma)$.

Proof. Let $\phi \in l_{pq/p-q}(\Gamma)$ and $f \in A_p(G)$. Then by Hölder's inequality $\phi\hat{f} \in l_q(\Gamma) = A_q(G)^\wedge$. Thus $l_{pq/p-q}(\Gamma) \subseteq \overset{A}{M}{}_{A_p}^{q}(\Gamma)$ and also, since $\overset{A}{M}{}_{A_p}^{q}(\Gamma) \subseteq \overset{A}{M}{}_{A_2}^{q}(\Gamma) = l_{2q/2-q}(\Gamma)$, we get

$l_{pq/p-q}(\Gamma) \subseteq \overset{A}{M}{}_{A_p}^{q}(\Gamma) \subseteq l_{2q/2-q}(\Gamma)$. To prove that $\overset{A}{M}{}_{A_p}^{q}(\Gamma) \subsetneq l_{2q/2-q}(\Gamma)$, it now suffices to show that for $r > \frac{pq}{p-q}$, $l_r(\Gamma) \not\subset \overset{A}{M}{}_{A_p}^{q}(\Gamma)$. Now $r > \frac{pq}{p-q}$ implies that $p > \frac{rq}{r-q}$. Then, by [*10*, Theorem 1], it follows that there exists $f \in A_p(G)$ such that $\sum_{\gamma\in\Gamma} |\hat{f}(\gamma)|^{rq/r-q} = \infty$. Hence by [*4*, Theorem 35.4, Part VI] there exists a function ψ in $l_r(\Gamma)$ such that $\psi\hat{f} \notin l_q(\Gamma)$. This shows that $l_r(\Gamma) \not\subset \overset{A}{M}{}_{A_p}^{q}(\Gamma)$. To complete the proof of the proposition we shall now show that $l_{pq/p-q}(\Gamma) \not\subset \overset{A}{M}{}_{A_p}^{q}(\Gamma)$ if $r > p$. By [*10*, Theorem 1] there exists a function f in $A_r(G)$ such that $\sum_{\gamma} |\hat{f}(\gamma)|^p = \infty$. Again [*4*, Theorem 35.4, Part VI] implies that for some ψ in $l_{pq/p-q}(\Gamma)$, $\psi\hat{f} \notin l_q(\Gamma)$; that is $\psi \notin \overset{A}{M}{}_{A_r}^{q}(\Gamma)$. This completes the proof of the proposition.

Let us now consider the case when $2 < q < p < \infty$. In this situation we shall show that $l_{pq/p-q}(\Gamma)$ is not contained in $\overset{A}{M}{}_{A_p}^{q}(\Gamma)$.

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LEMMA 2.4. *Let* $2 < q \le p < \infty$ *and* $1 \le r < \infty$. *Then* $l_r(\Gamma) \subseteq M_{A_p}^{A_q}(\Gamma)$ *if and only if* $l_r(\Gamma) \subseteq M_{A_p}^{A_2}(\Gamma)$.

Proof. Since $M_{A_p}^{A_2}(\Gamma) \subseteq M_{A_p}^{A_q}(\Gamma)$, the 'if' part of the lemma follows. Suppose then $l_r(\Gamma) \not\subseteq M_{A_p}^{A_2}(\Gamma)$. We shall show that $l_r(\Gamma) \not\subseteq M_{A_p}^{A_q}(\Gamma)$. If $l_r(\Gamma) \not\subseteq M_{A_p}^{A_2}(\Gamma)$, then there exists $\psi \in l_r(\Gamma)$ and $f \in A_p(G)$ such that $\psi\hat{f} \notin A_2 = l_2(\Gamma)$. As in the proof of Theorem 1.1 there exists a function $\varepsilon(\gamma) = \pm 1$ on Γ such that $\varepsilon(\gamma)\psi(\gamma)\hat{f}(\gamma) \notin \left(L_1(G)\right)^\wedge$. Then the function $\varepsilon(\gamma)\psi(\gamma)$ belongs to $l_r(\Gamma)$, but it does not belong to $M_{A_p}^{A_q}(\Gamma)$. This completes the proof of the lemma.

COROLLARY 2.5. *If* $1 \le p \le 4$, *then* $l_p(\Gamma) \subseteq M_{A_p}(\Gamma)$, *but if* $4 < p < \infty$ *then* $l_4(\Gamma) \not\subseteq M_{A_p}(\Gamma)$ *and* $l_p(\Gamma) \not\subseteq M_{A_4}(\Gamma)$.

Proof. If $1 \le p \le 4$ then $p \le \frac{2p}{p-2}$ and by Proposition 2.3, $l_p(\Gamma) \subseteq l_{2p/p-2}(\Gamma) \subseteq M_{A_p}^{A_2}(\Gamma) \subseteq M_{A_p}(\Gamma)$, provided $p > 2$. If $p \le 2$, then $M_{A_p}(\Gamma) = l_\infty(\Gamma) \supseteq l_p(\Gamma)$. If $4 < p < \infty$ then $4 > \frac{2p}{p-2}$ and hence by Proposition 2.3, $l_4(\Gamma) \not\subseteq M_{A_p}^{A_2}(\Gamma)$ and by the lemma above, it follows that $l_4(\Gamma) \not\subseteq M_{A_p}(\Gamma)$. Also if $4 < p < \infty$ then $p > \frac{4.2}{4-2} = 4$ and hence $l_p(\Gamma) \not\subseteq M_{A_4}^{A_2}(\Gamma)$ and $l_p(\Gamma) \not\subseteq M_{A_4}(\Gamma)$ as before.

COROLLARY 2.6. *If* $2 < q < p < \infty$ *then* $l_{pq/p-q}(\Gamma) \not\subseteq M_{A_p}^{A_q}(\Gamma)$.

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Proof. Since $q > 2$, $\frac{pq}{p-q} > \frac{2p}{p-2}$ and therefore, by Proposition 2.3, $l_{pq/p-q}(\Gamma) \not\subseteq M_{A_p}^{A_2}(\Gamma)$, and hence, by Lemma 2.4, $l_{pq/p-q}(\Gamma) \not\subseteq M_{A_p}^{A_q}(\Gamma)$.

PROPOSITION 2.7. *If* $2 < q < p < \infty$ *then*

$$\left(B_{pq/p-q}(G)\right)^{\wedge} \subsetneq M_{A_p}^{A_q}(\Gamma) .$$

Proof. Let $\phi \in \left(B_{pq/p-q}(G)\right)^{\wedge}$ and $f \in A_p(G)$. Since $\phi \in \left(M(G)\right)^{\wedge}$, $\phi f \in \left(L_1(G)\right)^{\wedge}$. Since $\phi \in l_{pq/p-q}(\Gamma)$ and $\hat{f} \in l_p(\Gamma)$, it follows that $\phi f \in l_q(\Gamma)$. Therefore $\left(B_{pq/p-q}(G)\right)^{\wedge} \subseteq M_{A_p}^{A_q}(\Gamma)$. We shall now show that the inclusion is proper. Since $\frac{2p}{p-2} > 2$, there exists $f \in A_{2p/p-2}(G)$ such that $\hat{f} \notin l_2(\Gamma)$. By [4, Corollary 36.13] there exists a function $\varepsilon(\gamma) = \pm 1$ on Γ such that $\varepsilon(\gamma)\hat{f}(\gamma) \notin \left(M(G)\right)^{\wedge}$ and hence $\varepsilon(\gamma)\hat{f}(\gamma) \notin \left(B_{pq/p-q}(G)\right)^{\wedge}$. However $\varepsilon(\gamma)\hat{f}(\gamma) \in l_{2p/p-2}(\Gamma)$ and, by Proposition 2.3, $l_{2p/p-2}(\Gamma) \subseteq M_{A_p}^{A_2}(\Gamma) \subseteq M_{A_p}^{A_q}(\Gamma)$. This completes the proof.

3. Multipliers from L_1 to A_p

As mentioned in the introduction, for abelian groups G, multipliers from L_1 to A_p have been investigated by Krogstad in [6]. Krogstad has shown that $M\left(L_1, A_p\right) \simeq \left(B_p(G)\right)^{\wedge}$. We shall show that the same result holds for compact nonabelian groups. As mentioned earlier, our proof differs from that of Krogstad which does not appear to extend to the non-abelian case. A similar proof for the abelian case is simpler. We shall follow the notations of [4] in dealing with the nonabelian case. Thus G will denote an infinite compact nonabelian group and Σ its dual object. $A_p(G) = \{f \in L_1(G) : \hat{f} \in \underline{\underline{C}}_p(\Sigma)\}$ and $\|f\|_{A_p} = \|f\|_1 + \|\hat{f}\|_p$.

PROPOSITION 3.1. *Let* G *be an infinite compact nonabelian group and* $1 \le q < \infty$. *Let* $\mu \in M(G)^{\wedge}$ *be such that* $\mu * L_1(G) \subset A_q(G)$; *then*

$\hat{\mu} \in \underline{\underline{C}}_q(\Sigma)$.

Proof. Suppose $\hat{\mu} \notin \underline{\underline{C}}_q(\Sigma)$. Choose a sequence $\{\psi_n\}_{n=1}^{\infty}$ of finite subsets of Σ such that

$$\sum_{\sigma\in\psi_n} d_\sigma \|\hat{\mu}(\sigma)\|_{\phi_q}^q \geq n^{3q} .$$

Now by [4, Theorem 28.53] choose a sequence $\{h_k\}$ in $L_1(G)$ such that $\|h_n\|_1 = 1$, $\hat{h}_n(\sigma) = \alpha_n(\sigma)I_{d_\sigma}$, $\alpha_n(\sigma) \geq 0$, and $\alpha_n(\sigma) > \frac{1}{2}$ for all $\sigma \in \psi_n$. Let $h = \sum_{n=1}^{\infty} \frac{h_n}{n^2}$; then $h \in L_1(G)$, $\hat{h}(\sigma) = \alpha(\sigma)I_{d_\sigma}$, and $\alpha(\sigma) \geq 0$ for all $\sigma \in \Sigma$, and $\alpha(\sigma) \geq \frac{1}{2n^2}$ for all $\sigma \in \psi_n$. Now

$$\begin{aligned}\sum_\sigma d_\sigma \|\hat{\mu}(\sigma)\hat{h}(\sigma)\|_{\phi_q}^q &= \sum_\sigma d_\sigma (\alpha(\sigma))^q \|\hat{\mu}(\sigma)\|_{\phi_q}^q \\ &\geq \sum_{\sigma\in\psi_n} d_\sigma \frac{1}{2^q n^{2q}} \|\hat{\mu}(\sigma)\|_{\phi_q}^q \\ &\geq \frac{1}{2^q} \frac{n^{3q}}{n^{2q}} = \left(\frac{n}{2}\right)^q \quad \text{for all } n ,\end{aligned}$$

a contradiction.

The proof of the following corollary is now obvious.

COROLLARY 3.2. *Let G be an infinite compact nonabelian group and $1 \leq p < \infty$. Then* $M(L_1, A_p) \simeq (B_p(G))^\wedge$.

PROPOSITION 3.3. *Let G be an infinite compact nonabelian group and $1 \leq q \leq 2$. Then*

$$M_{L_1}^{A_q}(\Sigma) = (A_q(G))^\wedge .$$

Proof. It is obvious that $(A_q(G))^\wedge \subseteq M_{L_1}^{A_q}(\Sigma)$. Conversely, $M_{L_1}^{A_q}(\Sigma) \subseteq M_{L_1}^{L_1}(\Sigma) = M(G)^\wedge$. Hence if $\hat{\mu} \in M_{L_1}^{A_q}(\Sigma)$, then $\mu * L_1(G) \subseteq A_q(G)$.

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Therefore, by Proposition 3.1, $\hat{\mu} \in \underline{\underline{C}}_q(\Sigma)$. Then it follows from [4, (34.47) (b)] that $\hat{\mu} \in A_q(G)^\wedge$. This completes the proof of the proposition.

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