ON THE ACCURACY OF SPECTRAL CLASSIFICATIONS
OF MAIN-SEQUENCE STARS

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Abstract. By examining the observed dispersion in (colour, spectral type) relations, classification errors have been derived from the data of nearby stars. The comparisons of the colour deviations observed in spectral regions of large variations of colour with type with the deviations in regions of small variations give the following standard errors in units of a tenth of a spectral class: For K dwarfs ± 0.6 (MK), ± 1.2 (Mt. Wilson), ± 0.7 (Kuiper); for early M dwarfs ± 0.9: (MK), ± 0.7 (Mt. Wilson), ± 0.5: (Kuiper); and for late M dwarfs $+0.7$ (Kuiper).

In the preceding paper, Jaschek has mentioned an estimated standard error of MK classifications of ± 0.6 subclasses which has been given in a publication on mean relations between spectral types and photoelectric colours of nearby main-sequence stars (Gliese, 1971). The preliminary values of the accuracy of spectral classifications published there have been superseded by the results of a more refined investigation shown in the following. C. Jaschek has also demonstrated the method and the difficulties encountered when using the variation of colour with spectral type and the observed dispersions for deriving accidental errors in the classifications.

In the above cited publication 12 mean relations between MK, Mt. Wilson, and Kuiper types and the colours $B-V$, $U-B$, $(U-B)_{\text{Cape}}$, and $R-I$ have been derived using the data of the 'Catalogue of Nearby Stars' (Gliese, 1969) which have been supplemented by further data collected in card catalogue form:

$B-V$, MK type	$B-V$, Mt.Wilson type	$B-V$, Kuiper type
$U-B$, MK type	$U-B$, Mt.Wilson type	$U-B$, Kuiper type
$(U-B)_C$, MK type	$(U-B)_C$, Mt.Wilson type	$(U-B)_C$, Kuiper type
$R-I$, MK type	$R-I$, Mt.Wilson type	$R-I$, Kuiper type

As an example Figure 1 shows the smoothed curve of the $(U-B$, Kuiper type) relation. It is obvious that classification errors produce an additional dispersion in the colours which increases with increasing slope of the curve.

The principle of this method consists in comparisons of observed colour deviations in spectral regions of large variations of colour with type with the deviations in regions of small variations. In the $(U-B$, Kuiper type) relation deviations in the K-dwarf region should be compared with those in the G or F or A classes.

But before investigating how this method works a few remarks should be made about the spectral data used. Figure 2 shows the frequency of types from F0 to the late M dwarfs among the stars nearer than 22 pc. For K and M stars Kuiper has given numerous intermediates as K4+, K5+, and so on. The original MK system (the MKK) had a framework of only some special types; but in the mean time nearly
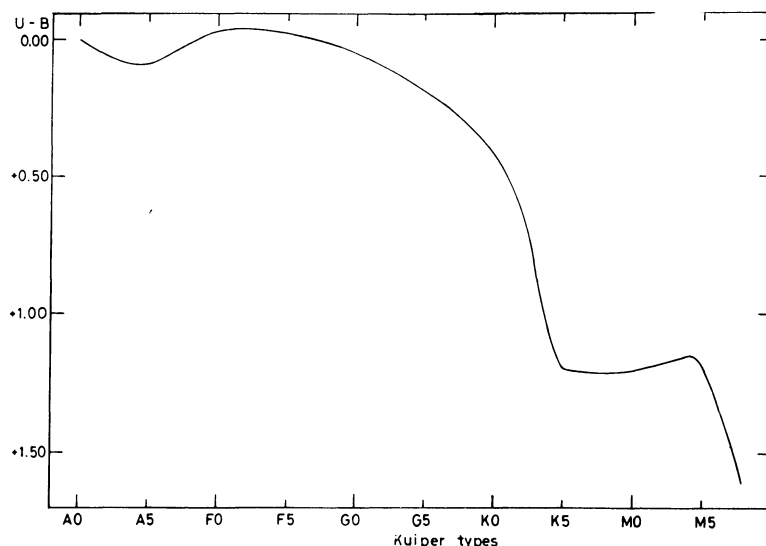
Fig. 1. $(U-B$, Kuiper type) relation of nearby main-sequence stars.

every tenth of a class is found in the literature. But even now we see the preference for the original types.

The main source for MK types has been the 'Catalogue of Stellar Spectra Classified in the Morgan-Keenan System' by Jaschek *et al.* (1964). Objects with dubious luminosity classifications have been excluded. The material used here consists only of stars lying on the main sequence. If different classifications by various observers were available the one type seeming best fitted has been used. Therefore it may be expected that the standard errors derived here will be somewhat smaller than those found by Jaschek and Jaschek (1966).

Nearly all Mt. Wilson types have been taken from the *General Catalogue of Stellar Radial Velocities* (Wilson, 1953). Mostly these types agree with the classification of 1935 (Adams *et al.*) investigated by Butler and Thackeray (1940).

Kuiper's types are found in various publications, mainly in his list of stars nearer than 10 parsecs (1942) and in the *General Catalogue of Trigonometric Stellar Parallaxes* (Jenkins, 1952, 1963). Further red dwarf data were made available by private communication.

In the 12 (colour, spectral type) relations the observed standard deviations (s.d.) of the colours have been derived in various spectral regions; the s.d. result from observational errors in colour ε_c, from observational errors in spectral type ε_t, and from a 'cosmic dispersion' σ. As we are dealing with nearby stars only no further dispersion in colour by different effects of the interstellar matter will occur.

Many of the colour data used here are mean values of various measurements. Their errors are small and ε_c is nearly insignificant, namely of the order ± 0.01 or ± 0.02.

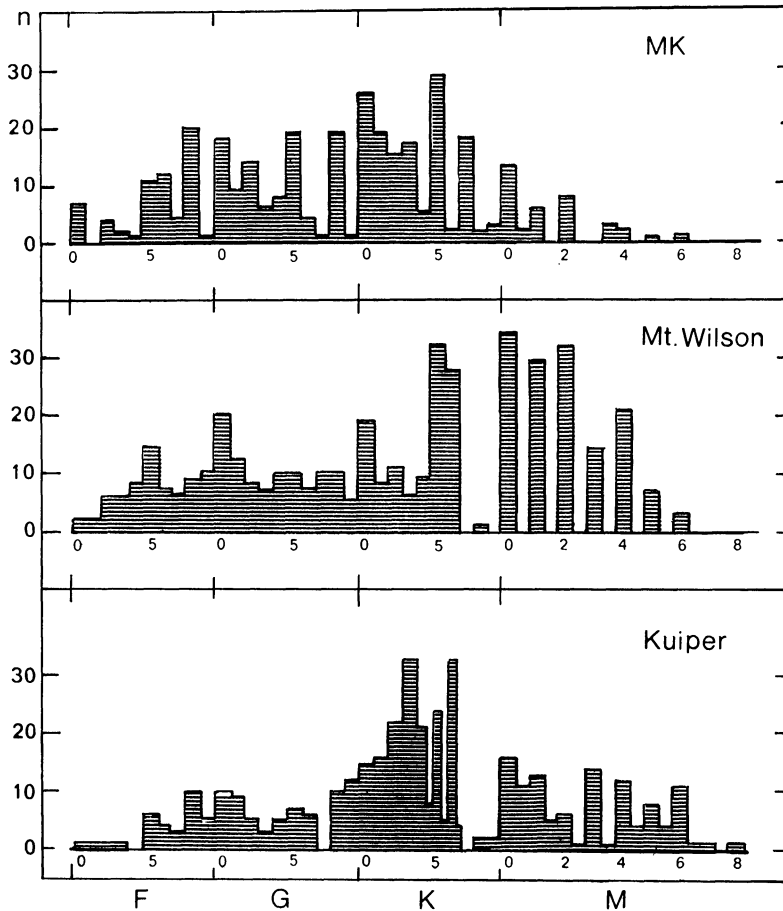The classification error, ε_t, produces an additional contribution, ε_s, to the observed

Fig. 2.   Frequencies of the various spectral types of nearby stars.

standard deviations, which is $\varepsilon_s = \varepsilon_t\, v_c$, where ε_t is given in units of a tenth of a spectral type and v_c is the variation of colour with a tenth of a spectral class: $v_c = \Delta$ colour / 0.1 spectral class.

Since there is no one to one correlation between spectral type and colour, we define σ as the dispersion which occurs even after elimination of all observational errors; that is, the true colour dispersion in a (colour, spectral type) relation.

For each spectral region (s.d.)$^2 = \varepsilon_c^2 + \varepsilon_s^2 + \sigma^2$ has been derived from the observations. Substracting an estimated value of the square of the observational error, ε_c, a quantity l is introduced by $l^2 = $ (s.d.)$^2 - \varepsilon_c^2$, or $l^2 = \sigma^2 + \varepsilon_t^2\, v_c^2$. σ and ε_t are unknown; probably both quantities vary with spectral type.

Figures 3–5 show in the 12 relations the observed l^2 (solid lines) together with the squares of the colour variation v_c^2 (dashed lines). Obviously in all diagrams there is a strong correlation.

From A to G the variation of colour with type is small. Normally, the solid lines
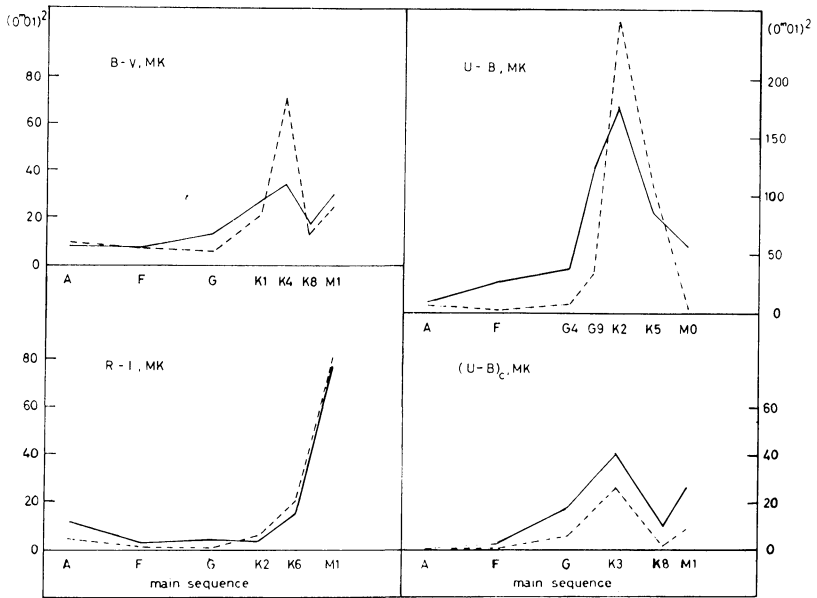
Fig. 3. Comparison between the colour dispersions observed in (colour, MK type) relations with the variation of colour with type for nearby main-sequence stars. Solid line: Squares of the dispersions observed in different spectral regions. Dashed line: Squares of the variation of colour for 0.1 spectral class. Unit for colours: 0.01.
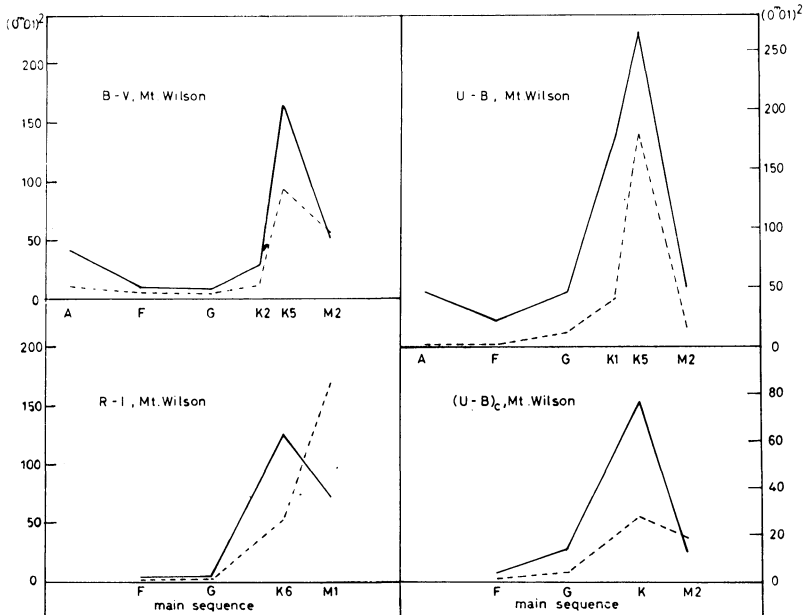

Fig. 4. Comparison between the colour dispersions observed in (colour, Mt. Wilson type) relations with the variation of colour with type for nearby main-sequence stars. Solid line: Squares of the dispersions observed in different spectral regions. Dashed line: Squares of the variation of colour for 0.1 spectral class. Unit for colours: 0.01.
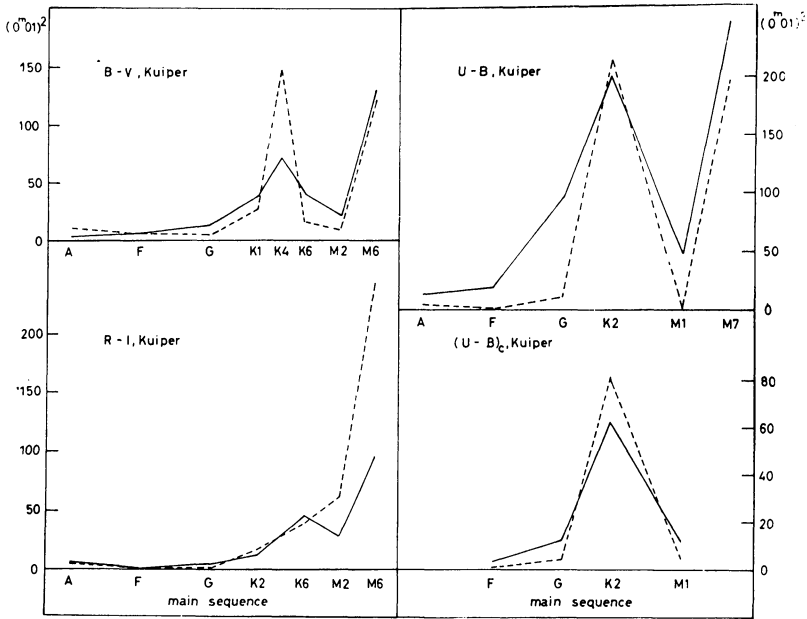
Fig. 5. Comparison between the colour dispersions observed in (colour, Kuiper type) relations with the variation of colour with type for nearby main-sequence stars. Solid line: Squares of the dispersions observed in different spectral regions. Dashed line: Squares of the variation of colour for 0.1 spectral class. Unit for colours: 0.01.

are somewhat above the dashed values–a result which seems to be caused mainly by the σ^2. This region is not suitable for deriving the errors ε_t. Large coefficients v_c^2 are in the K and M star regions.

The limited number of objects does not allow an exact separation of the various quantities contributing to the observed dispersions. A preliminary view of these diagrams shows

(1) in the MK classification, ε_t of the K types is smaller than 0.1 spectral class – except in the $[(U-B)_C$, MK type] diagram which has only low weight. For the early M dwarfs ε_t is near a tenth of a spectral class.

(2) in the Mount Wilson classification, ε_t is larger than one in class K but decreases for the early M dwarfs.

(3) Kuiper's classification shows an ε_t somewhat smaller than 0.1 spectral type.

To get precise values of the errors, differences between the quantities in regions of large variation v_c of colour with type and v_c in regions with low variations have been computed; for example

$$(l^2)_K - (l^2)_G = (\sigma^2)_K - (\sigma^2)_G + (\varepsilon_t^2)_K (v_c^2)_K - (\varepsilon_t^2)_G (v_c^2)_G .$$

In these adjacent regions $(\sigma^2)_K - (\sigma^2)_G$ will be negligible and $(\varepsilon_t)_G$ will be approximately equal to $(\varepsilon_t)_K$:

$(\varepsilon_t)_G = (\varepsilon_t)_K + y$ where y is a small quantity: $|y| \ll 1$

$(\varepsilon_t^2)_G \sim (\varepsilon_t^2)_K + 2\,(\varepsilon_t)_K\,y\,.$

In the first approximation $\varDelta\sigma^2 = [(\sigma^2)_K - (\sigma^2)_G]$ and y are supposed equal to zero:

$$(\varepsilon_t^2)_K = \frac{(l^2)_K - (l^2)_G}{(v_c^2)_{K'} - (v_c^2)_G}\,.$$

For example, the results by Butler and Thackeray (1940) show that the mean errors in the Mt. Wilson classification do not vary from G to K dwarfs. In this way approximate $(\varepsilon_t)_K$ can be derived from a comparison with the dispersions in the G region and also in other spectral regions (A, F, ...).

As an example Table I shows the approximate values of ε_t in the classification of K dwarfs (mean type K4 or K5) as derived from the dispersions in the $(B-V,$ spectral type) relations. The last columns give relative weights in an arbitrary system. The weight of such a determination depends on the number of stars in the compared regions and on the value of the denominator which should not be too small. In the MK and in the Kuiper classifications the approximate values are slightly decreasing from A to G; the same trend is observed in the $(U-B,$ MK type) and $(U-B,$ Kuiper type) relations but not in the Mt. Wilson system. Such variations can be explained by an increase of the cosmic dispersion σ from A to G whereas the true mean errors ε_t probably do not vary appreciably in this spectral region from A to K $(y\sim 0)$. In the $(R-I,$ spectral type) relations no trend in the approximate ε_t is observed.

TABLE I

Approximate values of the classification errors ε'_t of K dwarfs
(mean type: K4 or K5; unit: 0.1 spectral type; p: relative weight)

Comparison region	MK types		Mt. Wilson types		Kuiper types	
	ε'_t	p	ε'_t	p	ε'_t	p
A	± 0.66	0.5	± 1.21	0.6	± 0.71	1.0
F	0.65	2.1	1.32	4.4	0.68	4.1
G	0.57	3.5	1.32	6.1	0.64	8.3
K1	0.37	1.6	1.28	2.5	0.54	5.4
K7	0.55	0.7			0.50	4.8
M1	0.32	0.7	1.73	3.3	0.60	10.0

Corresponding series of approximate values of ε_t have been computed from the dispersions in all 12 relations and also for other spectral regions with the mean types K1, K7, M1...M6. From the trends of these ε_t the true values of the mean errors are estimated making allowance for the different weights. The combination of all results gives the values in Table II.

But these results need some qualifying remarks; the complications in this method arise from the details. Spectral types are discrete points along a continuously filled

TABLE II
Mean errors ε'_t of spectral classifications, still affected by the 'slope effect'
(Unit: 0.1 spectral type; p: relative weights)

Spectral class	Colour	MK types		Mt. Wilson types		Kuiper types	
		ε'_t	p	ε'_t	p	ε'_t	p
K	$B-V$	± 0.6	11	± 1.4	17	± 0.65	36
	$U-B$	0.7	22	1.2	35	0.85	56
	$(U-B)_C$	1.1	1	1.6	0.4	0.8	2
	$R-I$	0.8	0.5	1.6	1	1.05	2
K	mean	± 0.7	35	± 1.25	54	± 0.8	96
early M	$B-V$	± 1.0	1	± 0.8	14		
	$U-B$			1.2	1		
	$(U-B)_C$			0.7	0.1		
	$R-I$	1.0	2.5	0.7	14	± 0.6	2.5
early M	mean	± 1.0	4	± 0.8	29	± 0.6	2.5
late M	$B-V$					± 1.0	6
	$U-B$					1.0	4
	$R-I$					0.6	14
late M	mean					± 0.8	24

sequence of spectra. Type t_i is assigned to all spectra from $t_{i-1/2}$ to $t_{i+1/2}$ with the mean colours between $c_{i-1/2} = \frac{1}{2}(c_{i-1} + c_i)$ and $c_{i+1/2} = \frac{1}{2}(c_i + c_{i+1})$. Therefore, even if all spectra are classed correctly (and σ is equal to zero) a dispersion in colour will be observed. In a homogeneous filled sequence the average square of the colour deviations originating in this effect is one sixth of the square of the colour variation with a tenth of a spectral class: $\frac{1}{6} v_c^2$. This percentage is included in the ε_t in Tables I and II. Its exact elimination seems to be difficult as this dispersion has no Gaussian distribution and only a limited number of objects is available. But it may be estimated that the values of the classification errors ε_t in Table II have to be diminished by at most 0.1.

The elimination of this 'slope effect' from the results in Table II gives the true values of the standard errors in classification which are listed in Table III.

TABLE III
Standard errors ε_t of spectral classifications
(Unit: 0.1 spectral type)

Spectral class	MK	Mt. W.	Kuiper
K	± 0.6	± 1.2	± 0.7
early M	0.9:	0.7	0.5: :
late M			0.7

A comparison of the data in the three systems shows the superiority of the MK classification in the range of the K dwarfs. The value ± 0.6 agrees fairly well with the accuracy derived by C. Jaschek and M. Jaschek (1966). $\varepsilon_t = \pm 1.2$ for the Mt. Wilson system confirms the result by Butler and Thackeray (1940). It is very interesting that the early M dwarfs have been classified with smaller errors than the Mt. Wilson K dwarfs even though their apparent magnitudes normally are fainter than those of the dK stars. The relatively high accuracy of the late M types classified by Kuiper should be emphasized.

An application of this method to other luminosity classes such as subgiants or giants will be complicated by the reddening effects.

Errors in the luminosity classification cannot be investigated. C. Jaschek and M. Jaschek (1966) have given estimates for the MK classes. The nearby-star data indicate that among F and G dwarfs and subgiants the percentage of dubious luminosity classifications in the Mt. Wilson system is nearly twice that of the MK system.

Concluding this report on the accuracy of classifications derived from nearby-star data, it should be pointed out that the errors given here are not valid for MK classifications on objective prism plates.

Acknowledgements

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