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SUPERLINEAR ELLIPTIC EQUATION FOR FULLY NONLINEAR OPERATORS WITHOUT GROWTH RESTRICTIONS FOR THE DATA

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Abstract We deal with existence and uniqueness of the solution to the fully nonlinear equation

$$-F(D^2u) + |u|^{s-1}u = f(x) \quad \text{in } \mathbb{R}^n,$$

where $s > 1$ and f satisfies only local integrability conditions. This result is well known when, instead of the fully nonlinear elliptic operator F, the Laplacian or a divergence-form operator is considered. Our existence results use the Alexandroff–Bakelman–Pucci inequality since we cannot use any variational formulation. For radially symmetric f, and in the particular case where F is a maximal Pucci operator, we can prove our results under fewer integrability assumptions, taking advantage of an appropriate variational formulation. We also obtain an existence result with boundary blow-up in smooth domains.

Keywords: Pucci operator; superlinear elliptic problem; boundary blow-up; local data

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1. Introduction

The problem we study in this paper is the solvability of the differential equation

$$-F(D^2u) + |u|^{s-1}u = f(x) \quad \text{in } \mathbb{R}^N, \tag{1.1}$$

when F is a fully nonlinear, uniformly elliptic operator, $s > 1$ and f has only local integrability properties, but without assuming any growth condition at infinity.

When $F(D^2u)$ is replaced by the Laplace operator, Brezis showed in [**3**] that whenever $s > 1$ one can find a (unique) solution to the above problem assuming only local integrability of f. This very weak assumption is sufficient when the nonlinearity is increasing and superlinear, as in the case of $|u|^{s-1}u$ with $s > 1$. This result was extended to the case of a general quasilinear operator, including the p-Laplace operator, and to parabolic equations by Boccardo *et al*. in [**1**] and [**2**], respectively (see also [**16**], where more general nonlinearities are considered). In all these works, the existence of the solution is obtained by using in a crucial way the variational structure of the equation by choosing appropriate test functions to obtain *a priori* estimates.

On the operator F we assume uniform ellipticity, that is

$$M^-_{\lambda,\Lambda}(M-N) \leqslant F(M) - F(N) \leqslant \mathcal{M}^+_{\lambda,\Lambda}(M-N) \quad \text{for all } N, M \in S_N$$

and $F(0) = 0$. Here $0 < \lambda \leqslant \Lambda$, $\mathcal{M}^+_{\lambda,\Lambda}$ and $\mathcal{M}^-_{\lambda,\Lambda}$ are the extremal Pucci operators as defined in [**4**] and S_N is the set of $N \times N$ symmetric matrices. We assume throughout that F satisfies this condition. Whenever no confusion arises we will simply write $\mathcal{M}^+$ and $\mathcal{M}^-$, omitting the parameters. In order to find a solution to (1.1), we have to work in the viscosity solution framework and we cannot use test functions and integration by parts to derive *a priori* estimates. The use of the viscosity theory forces us to work in the $L^N(\mathbb{R}^N)$ framework and, indeed, the presence of the $|u|^{s-1}u$ term in the equation allows us also to prove the existence of a unique L^N-viscosity for (1.1) whenever $f \in L^N_{\mathrm{loc}}(\mathbb{R}^N)$. Since there is no available theory for the viscosity solution when $f \in L^1_{\mathrm{loc}}(R^N)$, at this point we cannot expect to obtain results under this weaker condition as they do in [**3**]. However, in view of our results in § 3 for the radially symmetric case, one may expect to find solutions when f has less than L^N-integrability, but at this point we are not able to do it. Our first theorem is the following.

Theorem 1.1. *Assume that $s > 1$. For every function $f \in L^N_{\mathrm{loc}}(\mathbb{R}^N)$, (1.1) possesses a unique solution in the L^N-viscosity sense and if $f \geqslant 0$ a.e., then $u(x) \geqslant 0$ for all $x \in \mathbb{R}^N$.*

The formal definition of the solution is given in § 2.

It is well known that in the case of superlinear problems one can find solutions which blow up at the boundary of a bounded domain. This has been shown for various cases of linear and nonlinear second-order elliptic operators in divergence form (see, for instance, the work by Keller [**13**], Loewner and Nirenberg [**17**], Kondrat'ev and Nikishkin [**14**], Díaz and Letelier [**11**], Díaz and Díaz [**10**], Del Pino and Letelier [**9**] and Marcus and Veron [**18**]).

In the case of fully nonlinear operators, the techniques used to prove Theorem 1.1 can also be used to prove the following theorem on the existence of solutions in a bounded set, with blow-up on the boundary. The simplest situation is the following.

Theorem 1.2. *Let $s > 1$ and let $\Omega \subset \mathbb{R}^N$ be a bounded domain in $\mathbb{R}^N$. Assume that $f \in L^N_{\mathrm{loc}}(\Omega)$ and for some $g \in L^N(\Omega)$ we have $f \geqslant g$. Then the equation*

$$-F(D^2u) + |u|^{s-1}u = f \quad \text{in } \Omega, \tag{1.2 a}$$

$$\lim_{x \to \partial\Omega} u(x) = \infty \tag{1.2 b}$$

possesses at least one solution in the L^N-viscosity sense.

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Here we only address this simple situation, but the same kind of results should also hold true under more general assumptions. Moreover, the asymptotic study of the blow-up rate, when $f \in L^N(\Omega)$ or when f itself blows up at the boundary, is an interesting problem due to the nonlinearity of the differential operator.

At this point we bring to the attention of the reader the work of Labutin [**15**], who studies the local behaviour of solutions to the same type of equations as ours, without the right-hand side, establishing removability of singularities.

In the second part of this paper we analyse the case of radially symmetric data f. Here we can prove the existence and uniqueness of solutions under weaker integrability assumptions on f but only in the particular case when F is the maximal Pucci operator. The reason for this is that in the radial case we can rewrite (1.1) as a divergence-form quasilinear ordinary differential equation, for which one can define the notion of a weak solution. In this case we are back to integration-by-parts techniques.

The comparison between radial solutions and positivity results, however, is not made directly. This is because the coefficient of the second-order derivative in the equation depends on the solution and its first derivative in a nonlinear way. Thus, when comparing two solutions, we do not have an obvious common factor for the second derivative of the difference or, if we have it, we do not control its integrability at the origin. An ad hoc argument has to be found to make a comparison in this case (see Lemma 3.6).

Theorem 1.3. *Assume that $s > 1$ and that f is a radially symmetric function satisfying*

$$\int_0^R r^{N_+-1}|f(r)|\,\mathrm{d}r < \infty \tag{1.3}$$

for all $R > 0$. Here

$$N_+ := \frac{\lambda}{\Lambda}(N-1)+1,$$

with λ and Λ being the parameters defining the Pucci operator $\mathcal{M}^+_{\lambda,\Lambda}$. Then (1.1) with $F = \mathcal{M}^+_{\lambda,\Lambda}$ has a unique weak radially symmetric solution and if f is non-negative, then u is also non-negative.

The formal definition of a radially symmetric weak solution and the proof of Theorem 1.3 are given in § 3. See also Remark 3.7, where we discuss the assumptions on f in this case.

Remark 1.4. In all our results, the power function $|u|^{s-1}u$ could be replaced with nonlinear functions which are superlinear at infinity. However, for simplicity, throughout the paper we will only deal with the pure power case [**1**, **3**, **16**]. Let us also stress that the assumption $s > 1$ is essential for our results to hold, as we can see from the discussion in [**1**].

2. The general case with $f \in L^N_{\mathrm{loc}}(\mathbb{R}^N)$

We devote this section to proving Theorem 1.1 by an approximation procedure together with a local estimate based on a truncation argument and the application of the Alexandroff–Bakelman–Pucci inequality.

We start by recalling the notion of solution suitable when the right-hand side in (1.1) is only in $L^p_{\mathrm{loc}}(\mathbb{R}^N)$. Following the work by Caffarelli *et al.* [**6**], we first notice that the framework requires $p > N - \varepsilon_0$, where $\varepsilon_0 > 0$ depends on the ellipticity constants λ and Λ. Thus, the case $p = N$, which is our framework, is covered by the theory. Even though the context of the definitions in [**6**] is much more general, for the purposes of this paper we only consider a 'semilinear' case (1.1)

$$-F(D^2u) + G(u) = f(x) \quad \text{in } \mathbb{R}^N, \tag{2.1}$$

where G is an increasing continuous odd function. According to [**6**] we have the following definition.

Definition 2.1. Assume that $f \in L^p_{\mathrm{loc}}(\mathbb{R}^N)$. Then we say that a continuous function $u : \mathbb{R}^N \to \mathbb{R}$ is an L^p-viscosity subsolution (supersolution) of (2.1) in $\mathbb{R}^N$ if for all $\varphi \in W^{2,p}_{\mathrm{loc}}(\mathbb{R}^N)$ and a point $\hat{x} \in \mathbb{R}^N$ at which $u - \varphi$ has a local maximum and minimum, respectively, one has

$$\operatorname{ess}\liminf_{x\to\hat{x}}(-F(D^2\varphi(x)) + G(u(x)) - f(x)) \leqslant 0 \tag{2.2}$$

and

$$\operatorname{ess}\limsup_{x\to\hat{x}}(-F(D^2\varphi(x)) + G(u(x)) - f(x)) \geqslant 0. \tag{2.3}$$

Moreover, u is an L^p-viscosity solution of (2.1) if it is both and L^p-viscosity subsolution and an L^p-viscosity supersolution.

In what follows we say that u is a C-viscosity (sub- or super-) solution of (2.1) when in the definition above we replace the test-function space $W^{2,p}_{\mathrm{loc}}(\mathbb{R}^N)$ by $C^2(\mathbb{R}^N)$. In this case the limits (2.2) and (2.3) become simple evaluation at $\hat{x}$, as given in [**7**].

As we mentioned above, the idea is to consider a sequence of approximate problems and then take the limit at the end. So, given $f \in L^N_{\mathrm{loc}}(\mathbb{R}^N)$, we assume that $\{f_n\}$ is a sequence of $C^\infty(\mathbb{R}^N)$ functions so that, for every bounded set Ω,

$$\lim_{n\to\infty}\int_\Omega |f_n - f|^N \,\mathrm{d}x = 0. \tag{2.4}$$

The sequence $\{f_n\}$ is easily constructed by mollification and a diagonal argument.

The following is a basic existence and regularity result we need in our construction of a solution to (1.1).

Lemma 2.2. *For every $n \in \mathbb{N}$ there is a solution $u_n \in C^1(B_n)$ of the equation*

$$-F(D^2u_n) + \frac{1}{n}u_n + |u_n|^{s-1}u_n = f_n(x) \quad \text{in } B_n, \tag{2.5}$$

where $B_n = B(0,n)$ is the ball centred at 0 and with radius n.

Proof. We observe that there is a constant M_n so that

$$-M_n^s \leqslant f_n(x) \leqslant M_n^s \quad \text{for all } x \in B_n$$

and then $v_- = -M_n$ and $v_+ = M_n$ are the subsolution and subsolution of (2.5), respectively. Then we can use the existence theorem [**7**, Theorem 4.1] for viscosity solutions of (2.5) to find u_n a C-viscosity solution. We observe that the hypotheses of Theorem 4.1 are fully satisfied by our operator, which is proper and satisfies the other hypothesis with $\gamma = 1/n$ [**7**]. Noticing that u_n solves the equation

$$F(D^2 u_n) = g_n \tag{2.6}$$

for the continuous function $g_n(x) = u_n(x)/n + |u_n(x)|^s u_n(x) - f_n(x)$, we have $u_n \in C^{1,\beta}(B_n)$, for certain $\beta > 0$, by applying the regularity theory of Caffarelli [**5**]. □

Our next lemma is a version of Kato's inequality for C-viscosity solutions of (2.1) with continuous right-hand side.

Lemma 2.3. *Assume $\Omega \subset \mathbb{R}^N$ and $u, v, f : \Omega \to \mathbb{R}$ are continuous functions and let $H(x) = G(u(x)) - G(v(x))$. If $u - v$ is a C-viscosity solution of equation*

$$-F(D^2(u-v)) + H(x) \leqslant f \qquad \text{in } \Omega, \tag{2.7}$$

then $(u-v)^+$ is a C-viscosity solution of

$$-F(D^2(u-v)^+) + H^+ \leqslant f^+ \quad \text{in } \Omega. \tag{2.8}$$

Proof. If $x \in \Omega$ satisfies $u(x) - v(x) > 0$ or $u(x) - v(x) < 0$, then obviously $u - v$ satisfies (2.8) at x. If $u(x) - v(x) = 0$, then we choose a test function φ so that $(u-v)^+ - \varphi$ has a local maximum at x, but then $(u - v) - \varphi$ has a local maximum at x and then we may use (2.7) to obtain

$$-F(D^2\varphi(x)) \leqslant f^+$$

so that (2.8) is satisfied in x, since $H(x) = 0$. □

Now we give a generalization of Kato's inequality [**12**] for C-viscosity solutions of (2.1).

Lemma 2.4. *If we assume that $u, f : \Omega \to \mathbb{R}$ are continuous functions and u is a C-viscosity solution of equation*

$$-F(D^2 u) + G(u) = f \qquad \text{in } \Omega, \tag{2.9}$$

then $|u|$ satisfies

$$-\mathcal{M}^+(D^2|u|) + G(|u|) \leqslant |f| \quad \text{in } \Omega \tag{2.10}$$

in the C-viscosity sense.

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Proof. We first use $v=0$ in Lemma 2.3 to get that u^+ is a subsolution with f^+ as right-hand side, and then observe that

$$-\mathcal{M}^+(D^2(-u))+G(-u)\leqslant f^-,$$

since $F\leqslant\mathcal{M}^+$ and $\mathcal{M}^-\leqslant\mathcal{M}^+$, which yields that u^- is a subsolution with f^- as right-hand side. We conclude that $|u|=\max\{u^+,u^-\}$ satisfies (2.10). □

The following lemma contains the crucial local estimate for solutions of (2.5). This result was proved by Brezis [**3**] in the context of the Laplacian and says that solutions have local estimates independent of the global behaviour of f. The approach in [**3**] (see also [**1**]) is to use suitable test functions and integration by parts. This cannot be done here, since the differential operator does not have divergence form. For this result the fact that $s>1$ is essential.

Lemma 2.5. *Let $s>1$ and let g be continuous in $\Omega\subset\mathbb{R}^N$, an open set. Suppose that $g\geqslant 0$ in Ω and u is a $C^1(\Omega)$ non-negative C-viscosity solution of*

$$-\mathcal{M}^+(D^2u)+\frac{1}{n}u+|u|^{s-1}u\leqslant g\quad\text{in }\Omega.$$

Then, for all $R>0$ and $R'>R$ such that $B_{R'}\subset\Omega$,

$$\sup_{B_R}u\leqslant C(1+\|g\|_{L^N(B_{R'})}),\tag{2.11}$$

where $C=C(s,R,R',N,\lambda,\Lambda)$ does not depend on g or n.

Proof. Let $\xi(x)=(R')^2-|x|^2$ and $\beta=2/(s-1)$ and consider $v=\xi^\beta u$. Now we want to find the equation satisfied by v. Suppose that $v-\varphi$ has a local maximum, $v(\hat{x})=\varphi(\hat{x})$, $Dv(\hat{x})=D\varphi(\hat{x})$ and $\varphi\in C^2$. Then $u-\xi^{-\beta}\varphi$ has a local maximum at $\hat{x}$. Therefore, $\xi^{-\beta}\varphi$ is a test function for u and so

$$-\mathcal{M}^+(D^2\varphi)+\frac{1}{n}\varphi+\xi^{-2}|\varphi|^{s-1}\varphi\leqslant\xi^\beta g+\mathrm{I}+\mathrm{II}+\mathrm{III},\tag{2.12}$$

where

$$\mathrm{I}:=-\beta\xi^{-1}v\mathcal{M}^-(D^2\xi),\tag{2.13}$$

$$\mathrm{II}:=\beta(\beta+1)\xi^{-2}v\mathcal{M}^+(D\xi\otimes D\xi),\tag{2.14}$$

$$\mathrm{III}:=-\beta\xi^{-1}\mathcal{M}^-(D\xi\otimes D\varphi+D\varphi\otimes D\xi).\tag{2.15}$$

So v satisfies the equation

$$-\mathcal{M}^+(D^2v)+\frac{1}{n}\xi^{-2}v+v|v|^{s-1}\leqslant\xi^\beta g+\mathrm{I}+\mathrm{II}+\mathrm{III}\tag{2.16}$$

in $B(R')$ in the C-viscosity sense. Here in I, II and III we replace $D\varphi$ by Dv.

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In what follows we write $\Omega^+ = \{x \in \Omega \mid v(x) > 0\}$. Consider the contact set for the function v, which is defined as

$$\Gamma_v^+ = \{x \in B_{R'} \mid \exists p \in \mathbb{R}^N \text{ with } v(y) \leqslant v(x) + \langle p, y - x\rangle \text{ for all } y \in B_{R'}\}.$$

We observe that $\Gamma_v^+ \subset \Omega^+ \cap B_{R'}$ and that if $\bar{v}$ is the concave envelope of v in $\bar{B}_{R'}$, then for $x \in B_{R'}$ we have $v(x) = \bar{v}(x)$ if and only if $x \in \Gamma_v^+$. The function $\bar{v}$, being concave, satisfies

$$\bar{v}(y) \leqslant v(x) + \langle Dv(x), y - x\rangle$$

for all $x \in \Gamma_v^+$ and $y \in \bar{B}_{R'}$. Choosing adequately $y \in \partial B_{R'}$ we obtain

$$|Dv(x)| \leqslant \frac{v(x)}{R' - |x|} \quad \text{for all } x \in \Gamma_v^+. \tag{2.17}$$

Now we claim that the function v satisfies

$$-\mathcal{M}^+(D^2 v) + \xi^{-2} v(|v|^{s-1} - C) \leqslant \xi^\beta g \quad \text{for all } x \in \Gamma_v^+.$$

Notice that in Γ_v^+ we have $\mathrm{I}, \mathrm{II} \leqslant C\xi^{-2} v$ and

$$\mathrm{III} \leqslant c\xi^{-1}|Dv| \leqslant c(R' + |x|)\xi^{-2} v \leqslant C\xi^{-2} v,$$

where we used (2.17). Here c and C are constants depending on R' and s. Therefore, the claim follows.

Now we define $w = \max\{v - C^{1/(p-1)}, 0\}$ in $B_{R'}$ and we observe that $\Gamma_w^+ \subset \Gamma_v^+$ and $\Gamma_w^+ \subset \{x \in B_{R'} \mid w > 0\}$. Consequently,

$$-\mathcal{M}^+(D^2 w) \leqslant \xi^\beta g \quad \text{a.e. in } \Gamma_w^+.$$

Thus, from the Alexandroff–Bakelman–Pucci inequality (see, for example, [**4**]),

$$\sup_{B_{R'}} w \leqslant C\|\xi^\beta g\|_{L^N(B_{R'})},$$

but then

$$c \sup_{B_R} u \leqslant \sup_{B_{R'}} v \leqslant \sup_{B_{R'}} w + C^{1/(p-1)} \leqslant C(1 + \|g\|_{L^N(B_{R'})}),$$

where c and C represent generic constants depending only on s, R, R', N, λ and Λ but not on g or n, as desired. □

Remark 2.6. Observe that in this estimate the constant C does not even depend on the possibly arbitrary values of u on $\partial\Omega$. This fact is very important in the study of solutions of this equation having blow-up on the boundary of Ω, as we see in Theorem 1.2.

Proof of Theorem 1.1 (existence). We start with a sequence of smooth functions $\{f_n\}$ such that, for every bounded set Ω, (2.4) holds. Then we use Lemma 2.2 to construct

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a sequence of solutions $\{u_n\}$ of (2.5). According to Lemmas 2.4 and 2.5, for every $0 < R < R' < n$ we have

$$\sup_{B_R} |u_n| \leqslant C(1 + \|f\|_{L^N(B'_R)}),$$

where C does not depend on f or n. With this inequality in hand we look at (2.5) and use [**4**, Proposition 4.10] to obtain, for every bounded open set Ω,

$$\|u_n\|_{C^\alpha(\Omega)} \leqslant C,$$

where C depends not on n, but only on f, Ω and the other parameters. By a diagonal procedure, we then obtain a subsequence of solutions of equation

$$-F(D^2 u_n) + c_n u_n + |u_n|^{s-1} u_n = f_n,$$

which we continue to call $\{u_n\}$, such that u_n converges uniformly over every bounded subset of $\mathbb{R}^N$. Here the equation holds in B_{1/c_n}, with $c_n \to 0$ as $n \to \infty$, and f_n has been redefined. Then using [**6**, Theorem 3.8] we conclude that u is an L^N-viscosity solution of (1.1), completing the proof of the existence part of the Theorem 1.1. □

The next lemma gives the positivity part of Theorem 1.1.

Lemma 2.7. *Assume that $s > 1$. If $f \leqslant 0$ a.e. and u solves (1.1) in the L^N-viscosity sense, then $u \leqslant 0$ in $\mathbb{R}^N$ and if $f \geqslant 0$ a.e., then $u \geqslant 0$ in $\mathbb{R}^N$.*

Proof. We proceed as in [**3**], considering the function defined by Osserman in [**19**]:

$$U(x) = \frac{CR^\beta}{(R^2 - |x|^2)^\beta} \quad \text{in } B_R, \quad R > 0,$$

where $\beta = 2/(s-1)$ and $C^{s-1} = 2\beta\Lambda \max\{N, \beta + 1\}$. Since U' and U'' are positive, we see that $\mathcal{M}^+(D^2 U) = \Lambda \Delta U$ and then a direct computation gives that

$$-\mathcal{M}^+(D^2 U) + U^s \geqslant 0 \quad \text{in } B_R. \tag{2.18}$$

From this, the equation for u and the non-positivity of f we obtain

$$-\mathcal{M}^+(D^2(u - U)) + |u|^{s-1}u - U^s \leqslant 0.$$

We observe that this inequality is in the L^N-viscosity sense. However, since f was dropped, it also holds in the C-viscosity sense. Then by Lemma 2.3 we find that

$$-\mathcal{M}^+(D^2(u - U)^+) + (|u|^{s-1}u - U^s)^+ \leqslant 0,$$

whence we get

$$-\mathcal{M}^+(D^2(u - U)^+) \leqslant 0 \quad \text{in } B_R.$$

We observe that the function $u - U$ is negative in the set $R - \delta \leqslant |x| < R$, for some sufficiently small $\delta > 0$. Then, by the Alexandroff–Bakelman–Pucci maximum principle, $(u - U)^+ = 0$, which implies that $u - U \leqslant 0$ in B_R. From here, taking the pointwise limit as $R \to \infty$, we find that $u \leqslant 0$.

In the case when $f \geqslant 0$ we proceed similarly, but rely on Lemma 2.3, with the operator $\mathcal{M}^-$, to obtain that $u + U \geqslant 0$ in B_R. From here the result follows. □

Proof of Theorem 1.1 (uniqueness). If u_1 and u_2 are solutions of (1.1), then the continuous function $w = u_1 - u_2$ satisfies

$$-\mathcal{M}^+(D^2 w) + |u_1|^{s-1}u_1 - |u_2|^{s-1}u_2 \leqslant 0$$

in the C-viscosity sense (see, for example, [**8**, Proposition 2.1]). Next we use Lemma 2.3 to obtain

$$-\mathcal{M}^+(D^2 w^+) + (|u_1|^{s-1}u_1 - |u_2|^{s-1}u_2)^+ \leqslant 0$$

and, using the fact that

$$||a|^{s-1}a - |b|^{s-1}b| \geqslant \delta|a-b|^s \quad \text{for all } a, b \in \mathbb{R}$$

for certain $\delta > 0$, we conclude that

$$-\mathcal{M}^+(D^2 w^+) + \delta(w^+)^s \leqslant 0. \tag{2.19}$$

Using Lemma 2.7 we obtain that $u_1 - u_2 \leqslant 0$. Interchanging the roles of u_1 and u_2, we complete the proof. □

Next we give an existence theorem for explosive solutions, whose proof follows easily from the estimate given in Lemma 2.5. We keep it in the simplest form, but we believe it may be extended to more general situations.

Proof of Theorem 1.2. We first consider an increasing sequence of smooth functions $\{f_n\} \subset L^N(\Omega)$ such

$$\lim_{n\to\infty} \int_\Omega |f_n - f|^N = 0.$$

Then we find u_n, a solution to the problem

$$\begin{aligned} -F(D^2 u_n) + |u_n|^{s-1}u_n &= f_n \quad \text{in } \Omega, \\ u_n &= n \quad \text{in } \partial\Omega. \end{aligned}$$

Letting $w_n = u_{n+1} - u_n$, we see that w_n satisfies

$$-\mathcal{M}^+(D^2 w_n) + |u_{n+1}|^{s-1}u_{n+1} - |u_n|^{s-1}u_n \geqslant f_{n+1} - f_n.$$

Then we may use the Alexandroff–Bakelman–Pucci inequality to obtain $u_{n+1} \geqslant u_n$ in Ω for all $n \in \mathbb{N}$. By arguments similar to those given in the proof of Theorem 1.1 (existence), using Lemma 2.5, we obtain a subsequence (which we continue to call $\{u_n\}$) such that u_n converges uniformly to a solution u of (1.2 a). Moreover, $u \geqslant u_n$ in Ω for all n, so that $\liminf_{x\to\partial\Omega} u \geqslant n$ for all n, and so u also satisfies (1.2 b). □

Remark 2.8. The argument given above allows to prove the existence of a solution to (1.2 a) assuming that $f \in L^N_{\text{loc}}(\Omega)$. Naturally, in this case we do not know about the behaviour of the solution on the boundary.

3. The radial case

In this section we consider only the particular case when F is a maximal Pucci operator $\mathcal{M}^+$. This operator can be written in a very simple way when we are dealing with radially symmetric functions. Since the eigenvalues of D^2u are u'' of multiplicity 1 and u'/r with multiplicity $N-1$ and defining $\theta(s)=\Lambda$ if $s\geqslant 0$ and $\theta(s)=\lambda$ if $s<0$, then we easily see that, for every u radially symmetric,

$$\mathcal{M}^+(D^2u)(r)=\theta(u''(r))u''(r)+\theta(u'(r))(N-1)\frac{u'(r)}{r}.$$

Then we see that (1.1) in the classical sense becomes

$$-\theta(u''(r))u''-\theta(u'(r))(N-1)\frac{u'}{r}+|u|^{s-1}u=f(r), \tag{3.1}$$

for a radial function f. In order to write this equation in a simpler form, we make some definitions. First we observe that for solutions of (3.1) we have

$$\theta(u''(r))=\theta\biggl\{-\theta(u'(r))(N-1)\frac{u'}{r}+|u|^{s-1}u-f(r)\biggr\},$$

which is more convenient as we see. We define

$$\Theta(r,u(r),u'(r))=\theta\biggl\{-\theta(u'(r))(N-1)\frac{u'}{r}+|u|^{s-1}u-f(r)\biggr\},$$

the 'dimension'

$$N(r,u(r),u'(r))=\frac{\theta(u'(r))}{\Theta(r,u(r),u'(r))}(N-1)+1$$

and the weights

$$\rho(r,u(r),u'(r))=\exp\biggl\{\int_1^r\frac{N(\tau,u(\tau),u'(\tau))-1}{\tau}\,\mathrm{d}\tau\biggr\}$$

and

$$\tilde{\rho}(r,u(r),u'(r))=\frac{\rho(r,u(r),u'(r))}{\Theta(r,u(r),u'(r))}.$$

If we define

$$N_+=\frac{\lambda}{\Lambda}(N-1)+1\quad\text{and}\quad N_-=\frac{\Lambda}{\lambda}(N-1)+1$$

we see that $N_+\leqslant N(r,u(r),u'(r))\leqslant N_-$ and also

$$r^{N_--1}\leqslant\rho(r,u(r),u'(r))\leqslant r^{N_+-1}\quad\text{if } 0\leqslant r\leqslant 1$$

and

$$\frac{\rho}{\Lambda}\leqslant\tilde{\rho}\leqslant\frac{\rho}{\lambda}.$$

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With these definitions we find that (3.1) is equivalent to

$$-(\rho u')' + \tilde{\rho}|u|^{s-1}u = \tilde{\rho}f(r). \tag{3.2}$$

When no confusion arises we omit the arguments in the functions ρ and $\tilde{\rho}$. In particular, when we write $\rho v'$ we mean $\rho(r, v(r), v'(r))v'(r)$ and so on. What is interesting about (3.2) is that it allows us to define a weaker notion of solution which extends the L^N-viscosity sense to more general f. With this new notion we can prove a theorem for the existence of radial solutions of (1.1) with a weaker condition on f than in the non-radial case of § 2 (see Remark 3.7).

We consider the set of test functions defined as

$$H = \{\varphi : [0,\infty) \to \mathbb{R} \mid \exists \phi \in W_0^{1,\infty}(\mathbb{R}^N) \text{ such that } \phi(x) = \varphi(|x|)\},$$

where $W_0^{1,\infty}(\mathbb{R}^N)$ denotes the space of functions in $W^{1,\infty}(\mathbb{R}^N)$ with compact support.

Definition 3.1. We say that $u : [0, R] \to \mathbb{R}$ is a weak solution of (3.2) with Dirichlet boundary condition at $r = R$, if u is absolutely continuous in $(0, R]$, $u(R) = 0$,

$$\int_0^R \rho|u|^s \, \mathrm{d}r < \infty, \qquad \int_0^R \rho|u'| \, \mathrm{d}r < \infty \tag{3.3}$$

and

$$\int_0^R \rho u'\varphi' + \tilde{\rho}|u|^{s-1}u\varphi \, \mathrm{d}r = \int_0^R \tilde{\rho} f \varphi \, \mathrm{d}r \quad \text{for all } \varphi \in H. \tag{3.4}$$

Now we state our theorem (which is a more complete version of Theorem 1.3).

Theorem 3.2. *Assume that $s > 1$ and that f is a radial function satisfying*

$$\int_0^R r^{N_+-1}|f(r)| \, \mathrm{d}r < \infty \quad \textit{for all } R > 0. \tag{3.5}$$

Then (3.2) has a unique weak solution u and if f is non-negative, then u is also non-negative.

Additionally, for any $1 < q < 2s/(s+1)$,

$$\int_0^r \rho|u'|^q \, \mathrm{d}r < \infty \quad \textit{for all } R > 0. \tag{3.6}$$

Moreover, the function $\rho u'$ is differentiable a.e. in $(0,\infty)$ and consequently satisfies

$$\lim_{r\to 0}(\rho u')(r) = 0, \qquad \lim_{r\to 0} \int_0^r \rho|u'| \, \mathrm{d}r = 0. \tag{3.7}$$

In order to prove the above theorem we will perform an approximation procedure as in the general case. Because the problem is radial and has a divergence-form formulation, we can get better estimates and pass to the limit, under weaker assumptions on f.

By regularizing f and using a diagonal procedure, we may find a sequence of radial smooth functions $\{f_n\}$ such that, for all $0 < R$,

$$\lim_{n\to\infty} \int_0^R r^{N_+-1}|f_n(r) - f(r)|\,\mathrm{d}r = 0. \tag{3.8}$$

Moreover, we may assume that there exists a function $g : (0,\infty) \to \mathbb{R}$ such that $|f_n(r)| \leqslant g(r)$ for all $r > 0$ and

$$\int_0^R r^{N_+-1}|g(r)|\,\mathrm{d}r < +\infty \quad \text{for all } R > 0.$$

First we have an existence result for the approximate problems.

Lemma 3.3. *For every n there is a solution u_n in $C^2[0,n]$ satisfying $u_n(n) = 0$, (3.3) with $R = n$ and*

$$\int_0^n \rho_n u_n'\varphi' + \tilde{\rho}_n(c_n u_n + |u_n|^{s-1}u_n)\varphi = \int_0^n \rho_n f_n \varphi \quad \text{for all } \varphi \in H, \tag{3.9}$$

where $\rho_n(r) = \rho(r, u_n(r), u_n'(r))$ (similarly for $\tilde{\rho}_n$) and $\{c_n\}$ is a positive sequence converging to zero.

Proof. We may use the same argument of Lemma 2.2 together with the Da Lio and Sirakov symmetry result [**8**]. □

Now we get some estimates following the ideas in [**1**].

Lemma 3.4. *Let $\{u_n\}$ be the sequence of solutions found in Lemma 3.3. Then, for all $0 < R$ and $m \in (0, s-1)$, there is a constant C depending on R, m, s, N, λ and Λ, but not on f or n, such that for all $n \in \mathbb{N}$ we have*

$$\int_0^R \rho_n|u_n|^s\,\mathrm{d}s \leqslant C\biggl(1 + \int_0^{2R} r^{N_+-1}|f|\,\mathrm{d}r\biggr) \tag{3.10}$$

and

$$\int_0^{2R} \frac{\rho_n|u_n'|^2\,\mathrm{d}r}{(1+|u_n|)^{m+1}} \leqslant C\biggl(1 + \int_0^{R} r^{N_+-1}|f|\,\mathrm{d}r\biggr). \tag{3.11}$$

Proof. We consider the function ϕ defined as

$$\phi(t) = \int_0^t \frac{\mathrm{d}t}{(1+s)^{m+1}}, \quad t \geqslant 0,$$

and extended as an odd function to negative t, which is smooth and bounded. We also consider a cut-off function $\theta : [0,\infty) \to \mathbb{R}$ being smooth, with support in $[0, 2R]$, equal to 1 in $[0, R]$, $0 \leqslant \theta \leqslant 1$ and $|\theta'| \leqslant 2/R$.

We define $v = \phi(u)\theta^\alpha$, where $\alpha > 2s/(s-1-m)$. Omitting the index n in what follows, using v as a test function, we obtain

$$
\begin{aligned}
\int_0^{2R} \frac{m\rho|u'|^2\theta^\alpha \,\mathrm{d}r}{(1+|u|)^{1+m}} &+ \int_0^{2R} \tilde{\rho}|u|^{s-1}u\phi(u)\theta^\alpha \,\mathrm{d}r \\
&\leqslant \int_0^{2R} \tilde{\rho} f\phi(u)\theta^\alpha \,\mathrm{d}r - \alpha\int_0^{2R} \rho u'\phi(u)\theta^{\alpha-1}\theta' \,\mathrm{d}r \qquad (3.12\,a) \\
&\leqslant C\biggl(\int_0^{2R} r^{N_+-1}|f| \,\mathrm{d}r + \int_0^{2R} \rho|u'|\theta^{\alpha-1} \,\mathrm{d}r\biggr), \qquad (3.12\,b)
\end{aligned}
$$

where we drop the term with c_n in the first inequality. Using Young's inequality, for some $\varepsilon > 0$, we have

$$
\int_0^{2R} \rho|u'|\theta^{\alpha-1} \,\mathrm{d}r \leqslant \varepsilon \int_0^{2R} \frac{m\rho|u'|^2\theta^\alpha}{(1+|u|)^{1+m}} \,\mathrm{d}r + \frac{1}{4\varepsilon}\int_0^{2R} \rho(1+|u|)^{1+m}\theta^{\alpha-2} \,\mathrm{d}r \qquad (3.13)
$$

and again

$$
\begin{aligned}
\int_0^{2R} \rho(1+|u|)^{1+m}\theta^{\alpha-2} \,\mathrm{d}r& \\
&\leqslant \varepsilon^2 \int_0^{2R} \rho(1+|u|)^s\theta^\alpha \,\mathrm{d}r + \frac{C}{\varepsilon^2}\int_0^{2R} \rho\theta^{(\alpha(s-m-1)-2s)/(s-m-1)} \,\mathrm{d}r \\
&\leqslant C\biggl(\varepsilon^{-2} + \varepsilon^2 \int_0^{2R} \rho|u|^s\theta^\alpha \,\mathrm{d}r\biggr), \qquad (3.14)
\end{aligned}
$$

where C is a generic constant independent of ε. Here we used our choice of α.

Next we observe that $|t|^s \leqslant |t|^{s-1}t\phi(t)/\phi(1) + 1$ for all $t \in \mathbb{R}$. Using this in (3.14), and then this result and (3.13) in (3.12 b), with the choice of a sufficiently small ε we finally obtain the desired inequalities. □

Corollary 3.5. *For all $q \in (1, 2s/(s+1))$ and for every $0 < R$ there is a constant, as in Lemma 3.4, such that*

$$
\int_0^R \rho_n|u_n'|^q \,\mathrm{d}r \leqslant C\biggl(1 + \int_0^{2R} r^{N_+-1}|f| \,\mathrm{d}r\biggr). \qquad (3.15)
$$

Proof. By the Hölder inequality we find

$$
\int_0^R \rho|u'|^q \,\mathrm{d}r \leqslant \biggl(\int_0^R \frac{\rho|u'|^2 \,\mathrm{d}r}{(1+|u|)^{1+m}}\biggr)^{q/2}\biggl(\int_0^R \rho(1+|u|)^{q(1+m)/(2-q)} \,\mathrm{d}r\biggr)^{(2-q)/2}. \qquad (3.16)
$$

Then by our choice of m in Lemma 3.4 it is possible to choose $q > 1$ such that

$$
\frac{(m+1)q}{2-q} < s
$$

and then we obtain the result from Lemma 3.4. With the adequate choice of m we can cover the range of q. □

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Proof of Theorem 3.2 (existence). We consider the sequence of $\{u_n\}$ of the solution found in Lemma 3.3 satisfying (3.9). In what follows we show that this sequence converges to a weak solution of (3.1).

Now, considering the estimates in Lemma 3.4, we see that the function $\rho_n u_n'$ has weak derivatives in any interval of the form (r_0, R_0) with $0 < r_0 < R_0$. Since the function ρ_n is differentiable a.e., we obtain then that u_n is twice differentiable a.e. and u_n'' is in $L^1(r_0, R_0)$, because of the equation satisfied by u_n and estimates in Lemma 3.4. From here we conclude that u_n' and u_n are uniformly bounded in (r_0, R_0). Using the equation again we then conclude that u_n'' is bounded by an L^1 function in (r_0, R_0), which implies that u_n' is equicontinuous. By the Arzelà–Ascoli Theorem there exists a differentiable function u in the interval (r_0, R_0) such that, up to a subsequence, u_n and u_n' converge uniformly to u and u', respectively, in the interval (r_0, R_0).

We may repeat this argument for any interval (r_0, R_0), so that by a diagonal procedure, we can prove that, up to a subsequence, $\{u_n\}$ and $\{u_n'\}$ converge pointwise to a differentiable function $u : (0, \infty) \to \mathbb{R}$. Notice that $\{\rho_n\}$ converges pointwise to $\rho(r) = \rho(r, u(r), u'(r))$.

Next we use the estimate (3.15) to prove that the sequence $\{\rho_n u_n'\}$ is equi-integrable in $[0, R]$ and then it converges in $L^1[0, R]$ to $\rho u'$ for all $R > 0$. Then it is only left to prove that $\{\tilde{\rho}_n |u_n|^s\}$ converges in $L^1[0, R]$. For this purpose we introduce, as in [**1**], a new function ϕ in $\mathbb{R}$ defined as $\phi(\nu) = \min\{\nu - t, 1\}$ if $\nu \geqslant 0$ and extended as an odd function to all $\mathbb{R}$, for a parameter $t > 0$. Then we consider inequality (3.12 a) with the cut-off function $\phi(u_n)\theta$ to get

$$\int_{E_n^{t+1} \cap (0,R)} \tilde{\rho}_n |u_n|^s \, \mathrm{d}r \leqslant \int_{E_n^t \cap (0,2R)} \tilde{\rho}_n |f_n| \, \mathrm{d}r + C \int_{E_n^t \cap (0,2R)} \rho_n |u_n'| \, \mathrm{d}r,$$

where $E_n^t = \{r > 0 \mid |u_n(r)| > t\}$. From (3.10) and (3.15) it follows that the second integral approaches zero if $t \to \infty$. From here the equi-integrability of $\rho_n |u_n|^s$ follows and we conclude.

Finally, (3.7) is a consequence of the integrability properties just proved for u_n that also hold for u. This finishes the proof. □

Now we prove the remaining part of Theorem 3.2, that is uniqueness and non-negativity of weak solutions. For this purpose it would be natural to use comparison arguments. However, these are a little delicate in this case. In fact, we may naturally define the notion of weak subsolutions (supersolutions) by writing '$\leqslant$' ('$\geqslant$') and using only non-negative (non-positive) test functions in (3.4). It so happens that, if u is a weak subsolution and v is a weak supersolution, we cannot be sure that $w = u - v$ is a weak subsolution, since we do not have good control of $\rho w'$ at the origin.

We first consider the non-negativity of solutions of when f is non-negative. For this purpose we need to find appropriate test functions.

Lemma 3.6. *If u is a solution of (3.1) in the weak sense and $f \leqslant 0$ a.e. in $[0, \infty)$, then $u \leqslant 0$ for all $r > 0$.*

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Proof. As in the general case, we consider the function U given in the proof of Lemma 2.7, which satisfies (2.18) in B_R. On the other hand, by the regularity of u given above, we have that $u(x) = u(r)$ satisfies (1.1) a.e. We may subtract the equations for U and u and get

$$-\mathcal{M}^+(D^2(u-U)) + |u|^{s-1}u - U^s \leqslant 0 \quad \text{a.e. in } B_R.$$

If we set $w = u - U$, we see that

$$-(\rho w')' + \tilde{\rho}(|u|^{s-1}u - U^s) \leqslant 0 \quad \text{a.e. in } (0, R). \tag{3.17}$$

Here the function ρ and $\tilde{\rho}$ are defined in the natural way with $\theta(r) = \theta(w'(r))$ and Θ given by

$$\Theta(r) = \theta(w''(r)) \quad \text{a.e. in } (0, R).$$

We see that the function w is negative near R. If there exists $0 < r_1 < r_2 < R$ such that $w > 0$ in (r_1, r_2) and $w(r_1) = w(r_2) = 0$, then we may choose the function φ, defined as $\varphi = w$ in (r_1, r_2) and $\varphi \equiv 0$ elsewhere, as a test function in (3.17) to get

$$\int_{r_0}^{R} \rho|w'|^2 + \tilde{\rho}(|u|^{s-1}u - U^s)w \, \mathrm{d}r \leqslant 0.$$

But each term in the left-hand side is positive; thus, $w = 0$ in (r_1, r_2).

Thus, either $w(r) \leqslant 0$ in $(0, R)$ or there is $r_0 \in (0, R)$ such that $w > 0$ in $(0, r_0)$ and $w(r_0) = 0$. To see that the second case is impossible we just need to prove that

$$\int_0^{r_0} \rho(w)|w'| \, \mathrm{d}r < \infty \quad \text{and} \quad \lim_{r \to 0} (\rho(w)w')(r) = 0, \tag{3.18}$$

since in this case we may use the function φ, defined as $\varphi = w$ in $(\bar{r}, r_0)$ and $\varphi \equiv w(\bar{r})$ in $(0, \bar{r})$, as a test function in (3.17) and get a contradiction.

Assuming (3.18) for the moment, we see that $u \leqslant U$ in $[0, R]$ and this is true for all $R > 0$. Taking the limit as $R \to \infty$, keeping r fixed, we conclude that $u \leqslant 0$ in $[0, \infty)$.

To complete the proof we prove (3.18). To see this, we first observe that there is a $\bar{r} \in (0, r_0)$ such that $w'(\bar{r}) < 0$, and then from inequality (3.17) we find that $w''(r) > 0$ a.e. and $w'(r) < 0$ in $r \in (0, \bar{r})$. *A posteriori* we see that $w''(r) > 0$ a.e. and $w'(r) < 0$ in $r \in (0, r_0)$ and consequently $\rho(w) = r^{N_+ - 1}$ there. Next we assume that u' is negative at some point in $(0, r_0)$, because otherwise the functions u and u' would be bounded and then w and w' are bounded, yielding (3.18). Since $u'' > U'' > 0$ in $(0, r_0)$, we see then that $u' < 0$ near the origin and consequently $\rho(u) = r^{N_+ - 1}$. Since (3.7) holds we see that (3.18) holds. □

Proof of Theorem 1.1 (uniqueness). Let u_1 and u_2 be two solutions of (3.1) in the weak sense. Then they satisfy (1.1) a.e. in $\mathbb{R}^N$, with abuse of notation $u_i(x) = u_i(|x|)$, $i = 1, 2$. Then we define $w = u_1 - u_2$ and proceed as in the proof of Theorem 1.1 to obtain that w satisfies (2.19) a.e. in $\mathbb{R}^N$. Now we follow the proof of Lemma 3.6. □

Remark 3.7. Let us consider a continuous function f in $\mathbb{R}^N \setminus \{x_i \mid i = 1, \dots, k\}$, such that near each singularity

$$f(x) \sim \frac{c_i}{|x - x_i|^{\alpha_i}}, \quad x \sim x_i,\ i = 1, \dots, k.$$

In order to apply Theorem 1.1 we need $\alpha_i < 1$ for all $i = 1, \dots, k$. In contrast, assuming that f is radially symmetric with a singularity at the origin of the form

$$f(r) \sim \frac{c}{r^\alpha}, \quad r \sim 0,\ r > 0,$$

in order to apply Theorem 3.2, we only need $\alpha < N_+$. We observe that if $\lambda/\Lambda \to 0$, then $N_+ \to 1$, while if $\lambda/\Lambda = 1$, then $N_+ = N$.

When we have a radial function f being in $L^p_{\mathrm{loc}}(\mathbb{R}^N)$ with $p > N/N_+$, f satisfies our hypothesis (3.5) and we may apply Theorem 3.2. This is particularly interesting if N and N_+ are close to each other.

Remark 3.8. Let f be a function in $\mathbb{R}^N$ and define

$$g(r) = \max\{|f(x)| \mid |x| = r\}$$

and assume that g satisfies (3.5). This will be the case if f has a singularity of the form $r^{-\alpha}$ with $\alpha < N_+$.

Then we may construct a solution of (3.2). This solution is a 'candidate' for a supersolution for (1.1) with f as a right-hand side. However, since the two notions of solutions are not compatible, this is not possible.

Remark 3.9. In this section we have considered only the case of the Pucci operator $\mathcal{M}^+$. However, these results can be adapted for the operator $\mathcal{M}^-$ as well.

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