Part 3
Concepts, Definitions, Models

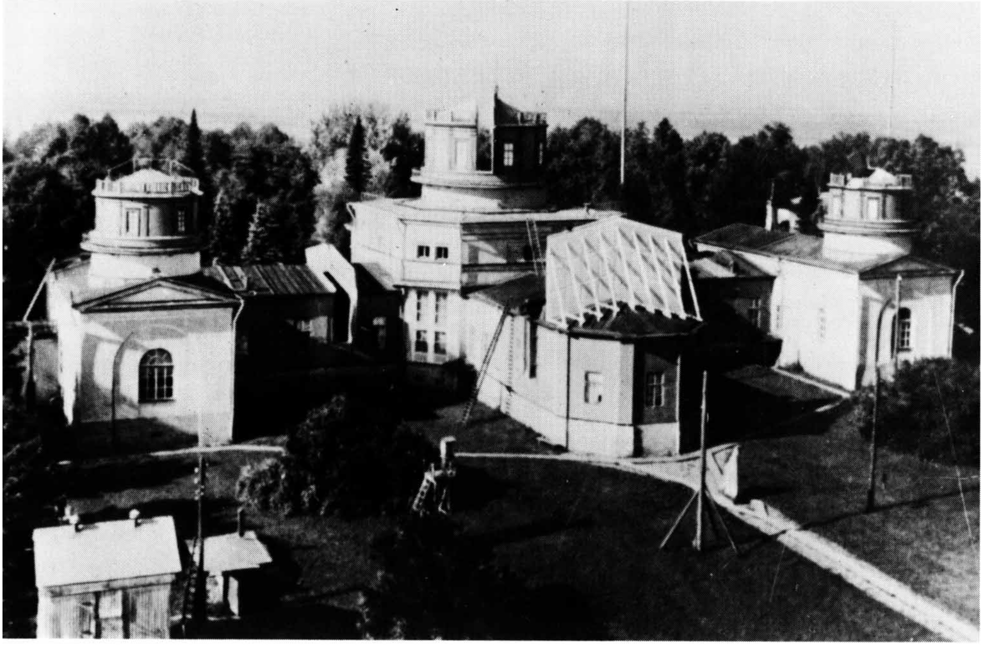

Plate VI. The original Pulkovo Observatory viewed from the southwest. The large screen protects the prime vertical instrument from direct sunlight. Photograph provided by Pulkovo Observatory.

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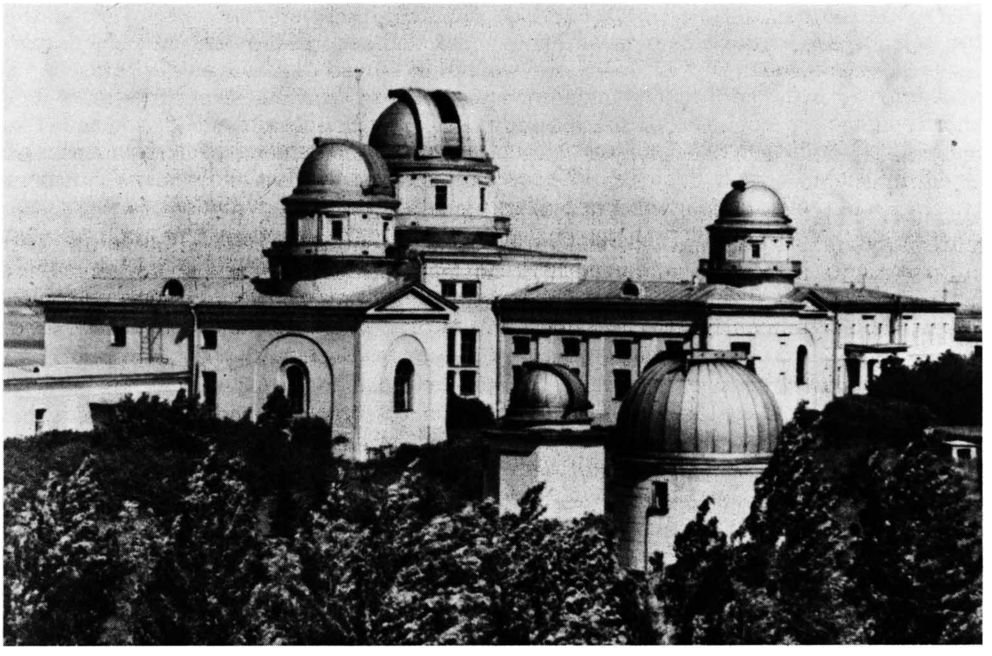

Plate VII. The reconstructed Pulkovo Observatory viewed from the southwest. Photograph provided by Pulkovo Observatory.

THE CELESTIAL REFERENCE SYSTEM IN RELATIVISTIC FRAMEWORK

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ABSTRACT. The concept of reference system, reference frame, coordinate system and celestial sphere in a relativistic framework are given. The problems on the choice of celestial coordinate systems and the definition of the light deflection are discussed. Our suggestions are listed in Sec. 5.

1. Introduction

With the increasing of accuracy of observations, one should pay more and more attention to the relativistic effects in celestial reference systems. Some papers dealing on the definition and realization of reference system have been published (Moritz 1981; Brumberg 1981; Fujimoto *et al.* 1982; Eichhorn 1984; Fukushima *et al.* 1986; Brumberg *et al.* 1989). The monographs titled *Vectorial Astronometry* (Murray 1983), *Relativity in Astrometry, Celestial Mechanics and Geodesy* (Soffel 1989) and *Reference Frames in Astronomy and Geophysics* (Kovalevsky *et al.* 1989) provide the useful tools for construction of the modern astrometric theory in the framework of general relativity.

The concept of the work "reference frame" is used differently in physics and astronomy (Brumberg 1989) and there exist different opinions even among astronomers. According to the suggestion of Kovalevsky and Mueller (1981), "The purpose of a reference *frame* is to provide the means to materialize a reference *system* so that it can be used for the quantitative description of positions and motions on the Earth or of celestial bodies in space." For example, the catalogue of over 1500 star coordinates defines the FK4 frame, materializing the FK4 system. An alternative definition is defined by a set of (physical) points which constitutes a well-defined spatial configuration. To specify the location of a point, which is not an element of the defining set within this frame of reference, one needs a construction that sets numbers to specify completely and uniquely the location of the point with respect to the specific frame of reference. Such a construction is called a coordinate system (Eichhorn 1984). According to this definition the FK4 frame is defined by the barycentre and the FK4 stars themselves rather than their coordinates. It seems that attention must be paid to the distinction between these definitions.

One would also question the conceptual aspects of the barycentric celestial sphere as well as the global equatorial coordinate system, because the equator and the ecliptic could only be defined locally at the Earth when the spacetime is curved.

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Another questionable concept is the light deflection, the most important relativistic effect for optical observations. Because of the space curvature, the two tangent vectors of a light path, at the observer and at the remote source respectively, belong to different tangent spaces, and there is no exact definition to determine the angle made by these two vectors.

In the present paper, we try to make one step to solve these problems. For simplicity our discussions are mainly limited to optical astrometry. The reference system and celestial sphere will be discussed in Section 2, the choice of coordinate systems and the light deflection will be treated in section 3 and 4 respectively. Section 5 lists our conclusions.

2. The Reference System and the Celestial Sphere

In some parts of this section we are much influenced by the textbook of Sachs and Wu (1977), but we use our own words to explain the concepts in order to be understood by astronomers more easily. If there is some misunderstanding we should take the full responsibility.

2.1 Observer

The worldline of any pointlike object is an *observer* (Sachs and Wu 1977, p. 41). Here the word "observer" has a more general meaning than that in astronomy. It could be a real observer or an observational object.

Every observation is made by an instantaneous observer, which could be defined as (**E**,**U**), where **E** represents the event of the observation and the **U** is the instantaneous 4-velocity of the observer that passes through **E** (Sachs and Wu 1977 p. 43).

2.2 Reference frame

Reference frame is a group of selected observers, to which all the observations or motions are referred. The only restriction to these observers is that they can not cross each other, otherwise at one event there could exist two different instantaneous observers that move with respect to each other. The observers, of which the reference frame is comprised, are called stationary observers with respect to the very reference frame.

For astronomical application a reference frame must be realizable and be carefully chosen (Kovalevsky 1989). The stationary observers could be real celestial bodies such as remote stars or fictitious bodies connected with the real objects such as the barycentre in the solar system.

2.3 Coordinate system

A *coordinate system* draws a 4-dimensional network on the spacetime to make the quantitative measurement of the events and the other physical quantities possible.

After a reference frame has been selected, there still remain infinite choices of possible coordinate systems. On the contrary any coordinate system implies one reference frame, which is sometimes not physically realizable. A family of coordinate systems belong to the same reference frame if the transformation between them is as follows:

$$t = \varphi(t', x_i') ,$$
$$x_i = f_i(x_i') \qquad (1)$$

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where i and i' run from 1 to 3, φ and f are scalar functions (Møller 1973). Eq. (1) tells us that $\partial/\partial t$ and $\partial/\partial t'$ are along the same direction.

A coordinate system is essentially only a mathematical tool for convenience. In traditional astrometry coordinate systems have been usually defined physically. In the language of this paper we could say that the coordinate system defined by the equinox and the equator belongs to the reference frame specified by them. They could be considered as connected with some celestial bodies.

A local tetrad is a local coordinate system, whose reference frame is represented by the instantaneous observer at its centre and some observers in its infinitesimal neighbourhood.

2.4 Reference system

A *reference system* includes three parts:

1) a reference frame:
2) a coordinate system;
3) a recommended set of constants, theories and procedures of data processing.

Parts 1) and 2) construct an idealized reference system, in which the reference frame represents the physical and fundamental part and the coordinate system is its mathematical aspect. Part 3) makes the reference system real. Astronomers make great efforts to improve that in part 3) from time to time and carefully keep the same reference frame and the same coordinate system. It is possible to happen that astronomers want to change the reference frame itself, for example, from the stellar frame to the radio source frame.

2.5 Observer's celestial sphere

Let (**E**,**U**) be an instantaneous observer, then the *local rest space* of (**E**,**U**) is defined as the 3-dimensional subspace of the tangent space at **E**, in which every vector is orthogonal with **U** (Sachs and Wu 1977, p. 45). The light direction observed by (**E**,**U**) is the projection vector of the 4-dimensional tangent vector of the light path at **E** into its local rest space.

It is natural to define the *local celestial sphere* of (**E**,**U**) to be all the unit vectors as a whole in its local rest space. We use the word "unit" here in order to remove one dimension and keep the direction only in the concept of the celestial sphere. Actually the observation is always taken as the event **E** when the photon arrives and only the velocity of the photon, which is different from **U**, is meaningful.

One can also talk about the *celestial sphere of an observer*. It can be defined in the same way as above. It is moving together with the observer. Then one should define the correspondence between the vectors of the local celestial spheres at one instant and another. Different correspondences could represent diferent local reference frames.

There is not much sense to talk about the stellar reference system in the local celestial sphere, in which the star light is changing from time to time. This is not only due to the aberration or parallax but also to the curvature of the spacetime. The invariant direction of the stars can only make sense in the barycentric celestial sphere described in the next subsection.

2.6 Barycentric celestial sphere

The *barycentric celestial sphere* can be defined as the reference frame that takes the remote sources and the barycentre as its stationary observers. As in the previous subsection one dimension representing the distance from the barycentre has been suppressed.

To include the barycentre in this definition is due to the fact that in relativity there would be different space and time separation for different Lorentz reference frames even though in a flat spacetime.

In defining the barycentric celestial sphere, there is no restriction on the barycentric reference frame inside the solar system except the barycentre, but it does put some limitation on the barycentric reference frame to assure that the remote sources are stationary. Here and after we will consider the barycentric reference frame that meets this restriction only.

3. Choice of the Coordinate System

3.1 The problem

In solar system dynamics, the PPN metric is popularly adopted (Will 1981). The general form of a metric is

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu \tag{2}$$

and the coefficients of the PPN metric are estimated to be

$$\begin{aligned} &g_{oo} = -1 + O(2), \qquad && g_{oi} = O(3) \\ &g_{ii} = 1 + O(2), && g_{ij} = O(4),\ i \neq j\,. \end{aligned} \tag{3}$$

Here the Greek letters run from 0 to 3 and the Latin letters from 1 to 3. The PPN metric is quasi-isotropic and its coordinate system is quasi-Cartesian.

The coordinates appear in $g_{\mu\nu}$ only in their differences so that a rigid rotation of the space axes will not change $g_{\mu\nu}$. Traditionally one should choose the direction of the mean equinox of J2000 to be x^1 axis and the mean equator of J2000 to be x^1x^2 plane. The curvature of the spacetime brings in problems to realize this tradition. Actually the equatorial and the ecliptic polices can only be defined locally at the Earth as two vectors and they do not have global definition. To solve this problem we should 1) define a definite connection between the global coordinate system and the local tetrad centered at the Earth, 2) adjust the directions of the space base-vectors of the local tetrad according to the locally defined equinox and the equator, and then 3) make the corresponding adjustment of the space axes of the global coordinate system.

3.2 The connection between the global and the local system

The connection constructed in the following holds not only for the local tetrad at the Earth, but also for that at any observer. It could be considered as the definition of the local tetrad when a global coordinate system is given.

At every event x^α there is an instantaneous observer whose 4-velocity $\mathbf{e}_{(0)}$ is tangential to the t-axis of the global coordinate system and can be expressed as

$$\mathbf{e}_{(0)} = \frac{1}{\sqrt{-g_{00}}}\frac{\partial}{\partial t} \tag{4}$$

The local celestial sphere, *i.e.* the local rest space, of (x^α, $\mathbf{e}_{(0)}$) can be the space expanded by three mutually orthogonal unit base vectors, $\mathbf{e}_{(i)}$. They are also orthogonal to $\mathbf{e}_{(0)}$ according to Section 2. We name this kind of tetrad, $\mathbf{e}_{(\alpha)}$, natural tetrad (NT) or natural frame after Murray (1983). Obviously there are infinitely possible choices of $\mathbf{e}_{(i)}$, which are connected by rigid rotations.

Soffel (1989, p. 77) recommended choosing

$$e_{(m)i} = \left(\hat{g}^{1/2}\right)_{mi} \tag{5}$$

where the 3x3 matrix

$$\hat{\mathbf{g}}^{-1} = (g^{ij})\,. \tag{6}$$

Soffel pointed out that "if the metric $g_{\mu\nu}$ is diagonal the induced tetrad is simply given by

$$\mathbf{e}_{(\alpha)} = |g_{\alpha\alpha}|^{-1/2}\frac{\partial}{\partial x^\alpha}\quad ." \tag{7}$$

In the general case it is impossible to construct the relation between $\mathbf{e}_{(m)}$ and $\partial/\partial x^\alpha$ in a closed form based on (5) and (6). Also $\mathbf{e}_{(1)}$ usually does not agree with $\partial/\partial x^1$ and the plane generated by $\mathbf{e}_{(1)}$ and $\mathbf{e}_{(2)}$ does not coincide with that generated by $\partial/\partial x^1$ and $\partial/\partial x^2$.

We suggest an alternative choice of NT. By Gram-Schmidt orthonomalization (Horn and Johnson 1985) we can construct a mutually orthogonal and normalized tetrad $\mathbf{e}$:

$$\begin{aligned}
\mathbf{e}_{(0)} &= \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^0}\right)^{-1/2}\frac{\partial}{\partial x^0} = (-g_{00})^{-1/2}\frac{\partial}{\partial t}\\
\mathbf{e}_{(\alpha+1)} &= \left(\mathbf{e}'_{(\alpha+1)}, \mathbf{e}'_{(\alpha+1)}\right)\mathbf{e}'_{(\alpha+1)}\\
\mathbf{e}'_{(\alpha+1)} &= \frac{\partial}{\partial x^{\alpha+1}} + \sum_{\beta=0}^{\alpha} a_{\alpha\beta}\,\mathbf{e}_{(\beta)}\\
a_{\alpha\beta} &= -\left(\frac{\partial}{\partial x^{\alpha+1}}, \mathbf{e}_{(\beta)}\right)\left(\mathbf{e}_{(\beta)}, \mathbf{e}_{(\beta)}\right)^{-1}
\end{aligned} \tag{8}$$

Keeping only up to $O(3)$ terms, they are

$$\begin{aligned}
\mathbf{e}_{(0)} &= (-g_{00})^{-1/2}\frac{\partial}{\partial t}\\
\mathbf{e}_{(1)} &= g_{01}\frac{\partial}{\partial t} + g_{11}^{-1/2}\frac{\partial}{\partial x^1}
\end{aligned} \tag{9}$$

$$\mathbf{e}_{(2)} = g_{02}\frac{\partial}{\partial t} - g_{12}\frac{\partial}{\partial x^1} + g_{22}^{-1/2}\frac{\partial}{\partial x^2} \qquad (9, cont.)$$

$$\mathbf{e}_{(3)} = g_{03}\frac{\partial}{\partial t} - g_{13}\frac{\partial}{\partial x^1} - g_{23}\frac{\partial}{\partial x^2} + g_{33}^{-1/2}\frac{\partial}{\partial x^3}$$

For time-orthogonal coordinate system g_{oi} are equal to zero, $\mathbf{e}_{(1)}$ is along the direction $\partial/\partial x^1$ and $\mathbf{e}_{(2)}$ is on the plane generated by $\partial/\partial x^1$ and $\partial/\partial x^2$. Though it is quite complicated, Eq. (8) could be written in a closed form.

Here and after the word "observer" will refer to a real observer. It may be in motion with respect to the background coordinate system. Let τ be its proper time, then its 4-velocity is

$$\tilde{\mathbf{e}}_{(0)} = \frac{\partial}{\partial \tau} = \frac{dx^\alpha}{d\tau}\frac{\partial}{\partial x^\alpha} \qquad (10)$$

One would construct another tetrad, $\tilde{\mathbf{e}}_{(\alpha)}$, called proper tetrad (PT) or proper frame (Murray, 1983), which is centered at the instantaneous observer, (x^α, $\tilde{\mathbf{e}}_{(o)}$), and is related to the above NT by a Lorentz transformation (Soffel, 1989):

$$\tilde{\mathbf{e}}_{(\mu)} = \Lambda^{\nu}_{\tilde{\mu}}\,\mathbf{e}_{(\nu)} \qquad (11)$$

where the Lorentz matrix is

$$\Lambda^0_{\tilde{0}} = \tilde{\gamma}$$
$$\Lambda^0_{\tilde{i}} = \Lambda^i_{\tilde{0}} = \tilde{\gamma}\tilde{\upsilon}^i \qquad (12)$$
$$\Lambda^j_{\tilde{i}} = \delta_{ij} + \frac{(\tilde{\gamma} - 1)}{\tilde{\upsilon}^n\,\tilde{\upsilon}^n}$$

and

$$\tilde{\upsilon}^i = \frac{\tilde{e}_0^{\mu}\,e_{(i)\mu}}{\tilde{e}_0^{\mu}\,e_{(0)\mu}} \qquad (13)$$
$$\tilde{\gamma} = \left(1 - \frac{1}{2}\,\tilde{\upsilon}^n\,\tilde{\upsilon}^n\right)^{-1/2}$$

PT is the comoving tetrad with the observer. It should be considered as the coordinate system in which the astrometrists measure their optical data. Similarly to NT, there are infinitely possible PT. Astronomers have to choose one as conventional when processing their data. Formulae (9) – (13) are our recommendation.

3.3 The adjustment of the space axes

The classical precession and nutation can be considered as the motion of the celestial pole with respect to a tetrad called a Fermi tetrad (FT) or Fermi frame at the Earth. FT is a tetrad that is centered at an observer $x^{(0)}$. Its time axis is $\tilde{\mathbf{e}}_{(0)}$ and space axes could be realized by three gyroscopes that are mutually orthogonal. After correction of the relativistic precession, of which the main term is usually called geodetic precession, one can define the celestial equatorial pole in PT of the Earth.

To define the celestial ecliptic pole is more difficult. During the construction of DE ephemerides as described by Standish (1980), a vector $\mathbf{r} \times d\mathbf{r}/dt$ is defined as the direction of the normal of the instantaneous orbit plane of the Earth, where the components of $\mathbf{r}$ and $d\mathbf{r}/dt$ are x^i and dx^i/dt of the Earth respectively in a simplified PPN coordinate system. In the relativistic framework dx^i/dt could be replaced by the 4-velocity $dx^\alpha/d\tau$ that is coordinate system independent. But $\mathbf{r}$ or x^i is coordinate system dependent. This fact shows the difficulty to define the ecliptic in a coordinate system independent way. We have not solved this problem.

Now let us assume that the celestial equatorial and the ecliptic pole of epoch have been defined in the NT at $(x^\alpha, \mathbf{e}_{(0)})$. One could determine the dynamic equinox and the equator of epoch in the very NT, then adjust $\mathbf{e}_{(1)}$ along the direction of the equinox of epoch and $\mathbf{e}_{(2)}$ on the equator of epoch. This adjustment is a rotation in the local rest space (the local celestial sphere) of $(x^\alpha, \mathbf{e}_{(0)})$. Let P be the orthogonal matrix representing the rotation for the adjustment of NT. In order to keep the relation between the local NT and the background coordinate system as described by Eq.(9), one has to rotate the background space axes. Let M represent the rotation matrix of the background system. It is necessary to find the relation between M and P. Within post-newtonian accuracy we found that

$$M = P - P\,H + H'\,P\,, \tag{14}$$

and

$$P = M - H'\,M + M\,H \tag{15}$$

where

$$\begin{aligned} &H + H^T = G - I\,, \\ &H' + H'^T = G' - I\,, \\ &G = (g_{ij}) \qquad G' = (g'_{ij}) = M\,G\,M^T \\ &I = (\delta_{ij})\,, \end{aligned} \tag{16}$$

and H^T is the transposed matrix of H. In other words

$$H = \begin{vmatrix} \frac{1}{2}h_{11} & 0 & 0 \\ h_{12} & \frac{1}{2}h_{22} & 0 \\ h_{13} & h_{23} & \frac{1}{2}h_{33} \end{vmatrix} \tag{17}$$

$$h_{ij} = g_{ij} - \delta_{ij}$$

H′ is defined by a similar formula.

The elements of H and H′ are all in the magnitude of $O(2)$. Since the adjustment from the catalog equinox and equator to the dynamic equinox and equator is a very small quantity, which is usually less than 0".1, one can neglect the difference between M and P.

Eqs. (14) and (15) can also be used when the transformation of the coordinate system between different epochs is necessary. One important nature of Eqs. (14) and (15) is that the relation between M and P is location dependent. Factually, M – P depends on the metric coefficients, $g_{\mu\nu}$, which are functions of the location of the observer. Here we would like to point out that M is equal to P exactly if $g_{oi} = 0$ and $g_{ij} = 0$ $(i \neq j)$, which implies that the background coordinate system is isotropic. The real metric in the solar system does not meet this demand but could satisfy it within a certain accuracy. We call the metric that nearly meets this condition quasi-isotropic metric and it is preferable.

4. The Light Deflection

4.1 The problem

To complete the construction of the celestial reference system a conventional procedure for data processing has to be founded. For optical observations the most important relativistic effect is the light deflection. Here we limit ourself to the discussion of this effect.

Many works on the light deflection have been published. The following lists some of the recent publications. Murray (1981) derived his formulae within the post-newtonian accuracy. Epstein and Shapiro (1980), Fischbach and Freeman (1980) worked up to post-post-newtonian terms and later Xu *et al.* (1984) extended their results to the case that treats a source and an observer that are both inside the solar system. All of them dealt with a spherically symmetric metric only. Richter and Matzner (1982) demonstrated that knowledge about light propagation in the solar system to any given order requires knowledge of every term in the metric to that same order. They extended the PPN metric to a metric called parametrized post-lineal (PPL) metric by including the second-order relativistic contributions from the Sun into g_{ij}. But they calculated the gravitational deflection of a photon in the equatorial plane of the Sun only.

Being different from physicists, astronomers are not so interested in the total deflection of a light from a remote source to a remote observer. The most accurate observations are made on or near the Earth. This fact poses a problem. The two tangent vectors of the light path that are at the source and at the observer belong to different tangent spaces at different locations. There is no known definition of the angle between the two tangent vectors. Atkinson (1963) has pointed out that the relativistic effect in a stationary gravitational field with spherical symmetry can be calculated by pure Euclidean geometry. The real metric in the solar system is much more complicated. It is neither stationary nor spherically symmetric. When astronomers have to pursue higher precisions in their data processing, it is necessary to give a definition of the light deflection that can be followed in calculation for any case.

4.2 The definition of the light deflection

Let $\mathbf{P}_s$ and $\mathbf{P}_o$ be the two 4-velocities of the light path from a source to an observer and let them be in the corresponding tangent spaces at two events, $\mathbf{E}_s$ and $\mathbf{E}_o$, respectively. Here $\mathbf{E}_s$ is the event that the source launched the photon; $\mathbf{E}_o$ is the event that the observer received the same photon. It is evident that $\mathbf{P}_s$ and $\mathbf{P}_o$ are independent of coordinate systems and also of reference frames.

Assume $\mathcal{F}$ to be a family of barycentric coordinate systems that imply the same barycentric celestial sphere as introduced in Section 2. Also, without loss of generality, we assume that they are all quasi-Cartesian coordinate so that the corresponding metric at infinity is Minkowsky metric and the base vectors, $\partial/\partial t$ and $\partial/\partial x^i$, are all the same at infinity among this family. The coordinate resolution of $\mathbf{P}_s$ and $\mathbf{P}_o$ in one member of $\mathcal{F}$ are

$$\mathbf{P}_s = P_s^o \frac{\partial}{\partial t} + P_s^i \frac{\partial}{\partial x^i}$$
$$\mathbf{P}_o = P_o^o \frac{\partial}{\partial t} + P_o^i \frac{\partial}{\partial x^i} \tag{18}$$

The 3-dimensional vector $P_s^i\partial/\partial x^i$ can be defined as the position of the source in the barycentric celestial sphere and it is coordinate system independent among $\mathcal{F}$. And $P_o^i\partial/\partial x^i$ represents the observed direction by an instantaneous observer ($\mathbf{E}_o$, $\mathbf{U}'_o$) where $\mathbf{U}'_o$ is along the direction of $\partial/\partial t$. It is evident that $P_o^i\partial/\partial x^i$ is generally coordinate system dependent among $\mathcal{F}$, but it is coordinate system independent among a subset of $\mathcal{F}$, between which the transformation takes the form of Eq. (1).

Several authors (Soffel, 1989; Brumberg, 1989) calculated $P_s^i - P_o^i$. Obviously it is coordinate system dependent among $\mathcal{F}$ and even among its subset mentioned above because the spatial base, $\partial/\partial x^i$, at $\mathbf{E}_o$ is generally different from one coordinate system to another.

Astronomers possibly would not like to define $P_s^i - P_o^i$. as the light deflection. The problem is that the spacial base, $\partial/\partial x^i$, is neither orthogonal nor normal and $P_o^i P_o^i$. is generally not equal to 1. On this kind of base one can not introduce the spherical equatorial coordinates, the right ascension and the declination. A more natural and practical definition we suggest here is the following. Firstly, construct NT at every event over the spacetime, which is related to the local base, $\partial/\partial x^\alpha$, by a set of defined formulae. Here and after we use $\mathbf{e}_{(\alpha)}(\mathbf{E})$ to denote the NT at event $\mathbf{E}$. Resolving the vectors $\mathbf{P}_s$ and $\mathbf{P}_o$ with respect to $\mathbf{e}_{(\alpha)}(\mathbf{E}_s)$ and $\mathbf{e}_{(\alpha)}(\mathbf{E}_o)$ respectively, we have

$$\mathbf{P}_s = n_s^o \mathbf{e}_{(o)}(\mathbf{E}_s) - n_s^i \mathbf{e}_{(i)}(\mathbf{E}_s)$$
$$\mathbf{P}_o = n_o^o \mathbf{e}_{(o)}(\mathbf{E}_o) - n_o^i \mathbf{e}_{(i)}(\mathbf{E}_o) \tag{19}$$

and $n_s^i n_s^i = n_o^i n_o^i = 1$ because the measured light speed in a local inertial frame is equal to 1. Then we define the light deflection as $\mathbf{n}_s^i - \mathbf{n}_o^i$ and the angle between the two directions as

$$\Delta\theta = |\mathbf{n}_s \times \mathbf{n}_s|$$
$$\mathbf{n}_s = [\, n_s^1, n_s^2, n_s^3 \,]^T, \qquad \mathbf{n}_o = [\, n_o^1, n_o^2, n_o^3 \,]^T. \tag{20}$$

This is a pure Euclidean definition when the two NT are fictitiously coincided with each other. Actually this definition is a new version of that described by Murray (1981, 1986) but in a more general case and in a more clear and definite way.

It is evident that $n_s^i - n_o^i$ and $\Delta\theta$ both are coordinate system dependent. They depend on the choice of NT and the choice of the barycentric coordinate system. We should choose NT to assure that $\mathbf{e}_{(\alpha)}(\mathbf{E}_s)$ coincides with the base vectors at $\mathbf{E}_s$: $\partial/\partial x^\alpha$. With this limitation the natural tetrads are restricted in a family called $\mathcal{N}$ here and after. The natural tetrads recommended by Soffel (1989) or by this paper in Eq. (9) both meet this demand. To be consistent with Section 3, we recommend that defined in Eq. (9).

It has to be mentioned that the angle between two light directions from two sources is coordinate system independent whether at infinity or at the observer. Therefore the correction of the light deflection of this angle is also coordinate system independent. In astronomical practice one needs to do the correction of the observational data one by one so that the discussion in this section is necessary.

The above definition has been applied to the Schwarzschild metric. We found that the formulae for the light deflection in isotropic coordinates is much simpler than that in the standard coordinates.

In Murray's paper (1981) the formulae for the light deflection in these two coordinates are the same. This is due to the fact that his definition of the natural frame brings in the same tetrads for these two coordinates and is different from ours, but it would not be the same in the general case. We do not think that one definition is superior to another solely so far as the light deflection is concerned. The most important thing is that the position of the source in the barycentric celestial sphere after the correction of the light deflection and other system effects is unique and invariant. To this end any selection of the barycentric coordinate system among the family $\mathcal{F}$ and any definition of the natural tetrad among the family $\mathcal{N}$ is allowed.

5. Conclusions

Our main conclusions are

1) Astronomers should have an agreement on the implication of "reference system", "reference frame" and "coordinate system". We suggest that the terminology of astronomers should be close to that of physicists if possible. It is proper to distinguish the physical aspect "reference frame" from the mathematical aspect "coordinate system". We suggest that a "reference system" include these two components and a recommended procedure of data processing, a set of constants and theories that are involved.

2) Astronomers should have an agreement on the relation between the barycentric coordinate system and the local natural tetrad in order to keep a consistent procedure of data processing. We recommend that defined in Eqs. (8) and (9).

3) The barycentric quasi-isotropic coordinate is preferable because the adjustment rotation of the space axes in the global and the local system will be almost the same just as in the classical case and the calculation of the light deflection is simpler as well.

4) We suggest that the light deflection be defined as follows. When a barycentric coordinate system is chosen, the 4-velocity of a photon at an event can be projected into the local space in the local NT to get a 3-vector. The light deflection is the difference between the direction cosines of the two 3-vectors, at the source and at the observer, in their corresponding NT. It also could be described as that the angle between them when the two corresponding NT are fictitiously moved to coincide with each other.

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Discussion

TURYSHEV: The author tells us only about his concept of the description. I would like to ask him about the calculation in a real situation, such as for the solar system.

XU: Please read our full paper which will be published in the proceedings.

KOPEJKIN: It is very difficult to use the local tetrad approach for a description of the gravitational field of the solar system. For example, in the construction of the geocentric coordinate system, we must take into account the gravitational field of the Earth. Therefore, in my opinion, it is better to use coordinate systems which are not local in the tetrad sense.

XU: For description of the motion of the celestial bodies we cannot use the tetrad approach and you are correct. But for the description of the observational data, which are the events at the station, the local (tetrad) approach is adequate and enough.