too I think we could benefit from a little more detail in some places. For example, on p. 46 we are presented with

$$\int_{\pi}^{\infty} \frac{\sin x}{x}\, \mathrm{d}x$$

as an example of a conditionally convergent integral; convergence is proved but we are just told that it can be shown that the integral is not absolutely convergent. I feel that the whole point of this example is lost by not at least giving a sketch of the proof. I found several typographical errors, whose corrections are mostly obvious. A few examples refer to probability distributions, but otherwise I see no genuine applications in the text—it is arguable, of course, that those who want to apply the theory already know their applications and their need is to understand the tools they have to use.

I am sure that the authors have produced an interesting and useful account of their topic. The overview presented in the book and the illustrative examples should be of value both to those whose interests are in applications and those who are concerned with the theory. However, I feel that the book can only be a prelude to further study if a proper understanding of the applications is to be secured.

I. TWEDDLE

Narkiewicz, W. *The development of prime number theory from Euclid to Hardy and Littlewood* (Monographs in Mathematics, Springer, 2000), xii+448 pp., 3 540 66289 8 (hardback), £58.50.

This is an extraordinary book both in the breadth of the subject matter addressed and its incredible attention to detail. At times, it has the flavour of a specialized and updated version of Dickson's *History of the Theory of Numbers*: there are twelve proofs of the infinitude of primes within the first ten pages and twenty pages on various prime number formulae. Yet very shortly after, we are browsing through Dirichlet's Theorem on primes in arithmetic progressions (with complete and up to date references and proofs) where one notes the influence of Davenport's *Multiplicative Number Theory.*

After a quick foray into Chebychev's Theorem and its applications, we are presented with a chapter on the Riemann zeta-function and Dirichlet series. Here there is a wealth of historical insight in the form of quotations and letters as well as, of course, concrete mathematical proofs of the more familiar results as well as one or two off the beaten path, such as Stieltjes's representation of the zeta function as a Laurent series around 1 and an interesting exposition on the work of Cahen on general Dirichlet series. We also have a discussion and analysis of Riemann's original memoir, especially the part relevant to analytic prime number theory. What is particularly striking from this part of the book is the extraordinary statement (backed up with a certain amount of evidence) that 'the only correct and fully proved result dealing with the growth of $\pi(x)$ and obtained in the period between Riemann's memoir (1860) and Hadamard's novel approach to the zeta-function, . . . , was a theorem of Phragmen (1891)'!

We are now in Chapter 5, which deals with the new era of analytic prime number theory initiated by Hadamard's first paper on the zeta function (1893) and its far reaching implications on the distribution of primes. Included here are the results of von Mangoldt on the Riemann assertions, both Hadamard's and de la Vallée–Poussin's proofs of the Prime Number Theorem and a short assortment of other proofs of the non-vanishing of the zeta-function and L-functions on the line $s = 1$. There is also a discussion on the size of the error term in the standard asymptotic formulae. It is difficult to overstate the extent of sheer detail of references and the scholarship displayed here and elsewhere throughout the book. It suffices to note the almost 80 pages of references to published articles.

The final chapter, entitled 'The Turn of the Century', essentially considers the years up to the end of the first decade of the 20th century, where the story presented in this book ends. This covers the development and progress in function theory by Landau, Jensen, Lindelöf and other well-known historical figures who have had such a big influence on analytic number theory. The work of Landau, in particular, crops up everywhere: from the classical explicit formula for $\psi(x)$ through the initial applications of tauberian theorems and finally to the zeros of the zeta-function on the critical line. After a brief discussion on the sign changes of $\pi(x) - li(x)$, the chapter (and the book) ends with the celebrated conjectures of Hardy and Littlewood (extended by Bateman-Horn and Schinzel) and a look at how far we are from reaching these.

I can thoroughly recommend this entertaining and very informative book to any active analytic number theorist who has, in addition, more than a passing interest in how the subject has evolved in the way it has. The author is to be congratulated for having dedicated such a considerable effort to providing a really useful addition to the existing literature.

M. NAIR

Breuer, T. *Characters and automorphism groups of compact Riemann surfaces* (LMS Lecture Note Series 280, Cambridge, 2000), xii+199 pp., 0 521 79809 4 (paperback), £24.95 (US$39.95).

In the last 20 years, finite group theory has been revolutionized by the classification of finite simple groups, which allows proofs by inspection, by the development of powerful computational techniques, which are beginning to turn the subject into an experimental science, and by the increased willingness of its practitioners to look outside their subject area for applications. One particularly fruitful area is the relationship between groups and Riemann surfaces, based in part on Schwarz's theorem that a compact Riemann surface X of genus $g \geqslant 2$ has a finite automorphism group G. Since there are induced linear actions of G on various modules associated with X (homology, cohomology, differentials, etc.), there are natural roles for representation theory and character theory in this relationship. These are powerful techniques, now enhanced by rich databases such as the ATLAS, and sophisticated group theory programs such as GAP.

Motivated by these developments, Breuer considers two dual objectives in this book: first, to classify all groups of automorphisms of compact Riemann surfaces X of fixed genus $g \geqslant 2$, up to equivalence of the action on the space $\mathcal{H}^1(X)$ of holomorphic abelian differentials on X; secondly, to classify those characters of a given finite group which can arise from its action on such a module. The first problem is solved, with the aid of GAP, up to genus 48; this is a great advance on previous knowledge, which was complete only for $2 \leqslant g \leqslant 5$. Concerning the second problem, necessary conditions on the character are found, and in many cases these are shown also to be sufficient; these results are illustrated with a detailed study of several classes of groups, such as the linear fractional groups $L_2(q)$ and the Suzuki groups $Sz(q)$, again taking us well beyond what was previously known. These are very difficult problems, and it is unreasonable at this stage to expect complete solutions; by bringing together a number of very effective techniques, and by presenting a mass of specific evidence, Breuer has done the mathematical community a considerable service in directing attention towards some interesting and challenging open problems in this area.

Chapter 1 gives a clear and concise account of the theory of compact Riemann surfaces, often referring to easily obtainable standard textbooks (such as Farkas and Kra) for full details. Chapter 2 does the same for character theory, first in general, and then concentrating on the character afforded by $\mathcal{H}^1(X)$. Chapter 3 gives a detailed examination of how this particular character is related to the fixed points of elements of G on X, for instance through the Eichler trace formula. These three chapters give an excellent quick guide to the general theory of compact Riemann surfaces and their automorphisms and holomorphic differentials, though the reader