Proceedings of the Edinburgh Mathematical Society (2006) **49**, 753–762 ©
Printed in the United Kingdom

BOOK REVIEWS

DOI:10.1017/S0013091506214937

ANDREWS, G. E. AND ERIKSSON, K. *Integer partitions* (Cambridge University Press, 2004), x + 141 pp., 0 521 60090 1 (paperback), £15.99 ($24.99), 0 521 84118 6 (hardback), £40 ($70).

To partition a number n is to express it in the form $n = n_1 + n_2 + \cdots + n_r$, where $n \geqslant n_1 \geqslant \cdots \geqslant n_1 \geqslant 1$. Early work on partitions was carried out by Euler in the mid eighteenth century, one of his earliest results being that the number of partitions of n into odd parts is equal to the number of partitions into distinct parts. Since then many other beautiful and remarkable results have been discovered, one of the most famous being the family of Rogers–Ramanujan identities, the first of which asserts that the number of partitions of n into parts differing by at least two is equal to the number of partitions of n into parts congruent to 1 or 4 mod 5. This identity was first obtained by Rogers in 1894, and rediscovered by Ramanujan in 1913.

Euler used generating functions to prove his results, but many of these results can be obtained by clever alternative approaches. This book starts by discussing proof by bijection—constructing bijections between two sets of partitions which are claimed to be of equal size. Some of these bijections are quite tricky to write down; many are 'dynamic' in the sense that the bijection can be looked on as a process of transforming one type of partition into another. For example, we can start with a partition into distinct parts, split every even part in two, and repeat until a partition into odd parts is obtained. Conversely, we can start with a partition into odd parts, combine equal parts in pairs, and repeat until only distinct parts are left, thereby confirming Euler's theorem. Several examples of ingenious bijections due to Bressoud (around 1980) are given. Another approach is to use Ferrers graphs, where a partition $n = n_1 + n_2 + \cdots + n_r$ is represented by an array with one row of n_1 dots, one of n_2 dots, and so on. Rearranging the dots can lead to proofs which are full of insight.

These ideas are all very clearly explained, and there are examples for the reader to try on the way. Generating functions are dealt with next, and formulae for $p(n, m)$, the number of partitions of n into m parts, are obtained for $m = 2$, 3 and 4. It is also shown that if $p(n)$ denotes the number of partitions of n, then $(p(n))^{1/n} \to 1$ as $n \to \infty$. A study of Gaussian polynomials leads on to a proof of the Rogers–Ramanujan identities, introducing Durfee squares en route. Later chapters discuss contributions of Sylvester, MacMahon, Dyson, Fine, Carlitz, Eriksson and others. Recent work includes the study of random processes, plane partitions and the so-called lecture-hall partitions, which arose 10 years ago out of work done on Coxeter groups.

This is a very readable introduction to the subject, covering a remarkable number of results and ideas in a very accessible way. The publisher claims that an understanding of the book does not require anything more than some familiarity with polynomials and infinite series, but that hidden extra called mathematical maturity would help a lot! Explanations are usually very clear, and there seem to be very few misprints, although a summation sign seems to be missing on p. 59, and the numbering of the solutions and hints in Appendix C goes awry around number 148, where, I suspect, at a late stage of production two extra examples were inserted.

The idea for this book goes back to a meeting of the two authors at a conference in 2000, their view being that it should be possible to study partitions without an advanced knowledge of mathematics. Andrews is well known as a world authority on the subject, having written many papers and books in that area, and Eriksson is a professor of applied mathematics who has recently written several papers on lecture-hall partitions. Together they have produced a book that is very suitable for adoption as a textbook for a course of lectures or for a reading course for honours degree undergraduates; it is very carefully structured to support self-study. I recommend it as an excellent introduction to a fascinating topic.

I. ANDERSON

DOI:10.1017/S0013091506224933

RUSS, S. *The mathematical works of Bernard Bolzano* (Oxford University Press, 2004), xxx + 698 pp., 0 19 853930 4 (hardcover), £125.

Bernard Bolzano (1781–1848) was born in Prague, his father being an Italian immigrant. In 1804 he took holy orders and in 1805 he was appointed to the newly established Chair in Philosophy of Religion at the University of Prague. However, the enlightened views that he put forward did not find favour with the authorities and he was dismissed from this post in 1819 with an interdict on publishing; persecution continued until the mid 1720s. Bolzano is of course familiar to us through his association with the Bolzano–Weierstrass theorem, but probably not a lot more is generally known about him. His mathematical work covered logic, foundations and rigorous analysis; in some aspects he anticipated, and even had priority over, other better-known mathematicians whom we associate with the development of key concepts.

This volume grew out of translations which appeared in Dr Russ's PhD thesis. The works presented are the following (the titles are given as in the translations); each item is provided with a detailed introduction and footnotes are provided to clarify textual and mathematical points.

(i) *Considerations on some objects of elementary geometry (1804).* Here Bolzano is concerned to develop a theory of triangles without assuming the existence of a plane; arguments involving motion are also disqualified.

(ii) *Contributions to a better-grounded presentation of mathematics (1810).* As the title suggests this is concerned with logic and proofs.

(iii) *The binomial theorem and as a consequence from it the polynomial theorem and the series which serve for the calculation of logarithmic and exponential quantities proved more strictly than before (1816).* Ideas of convergence are discussed, the Cauchy criterion is applied, there is an extensive discussion of the binomial series, and series for exponentials and logarithms are also discussed.

(iv) *Purely analytic proof of the theorem that between any two values, which give results of opposite sign, there lies at least one real root of the equation (1817).* The 'intermediate value theorem' is the main topic of this item; again the Cauchy criterion is assumed. The discussion of sups and infs incorporates Bolzano's version of the Bolzano–Weierstrass theorem.

(v) *The three problems of rectification, complanation and cubature, solved without consideration of the infinitely small, without the hypotheses of Archimedes and without any assumption which is not strictly provable (1817).* Taylor's theorem for functions of several variables is applied in a theory of length, area and volume.

(vi) *Pure theory of numbers, seventh section: infinite quantity concepts (posthumous).* Here Bolzano sets up the real numbers (measurable numbers) using a process of bracketing by related rational fractions; the Cauchy criterion is established for his system.