Some Problems of Interstellar Grains

J. MAYO GREENBERG AND A. C. LIND
Rensselaer Polytechnic Institute
Troy, New York

RADIATION FORCES ON GRAINS

THE INTERACTIONS OF GRAINS WITH RADIATION show up either as modifications of the radiation field (scattering, extinction) by grains or as modifications of grains by the radiation. The principal effects on grains are to control their temperature and to exert forces (radiation pressure). This paper presents some radiation pressure calculations which may be pertinent to the problem of propulsion of grains outward from very hot, bright stars or more generally OB complexes. The radiation force is equal to the rate of transfer of momentum to the grain, which is equal, in turn, to the net rate of loss of momentum in the incident beam of electromagnetic radiation. It is readily shown that this force F is given by

$$cF = C_{abs} + (1 - \langle \cos\theta \rangle)\, C_{sc} \qquad (1)$$

where c is the velocity of light, C_{abs} is the absorption cross section, C_{sc} is the total scattering cross section. The average value of the cosine of the scattering angle is given by

$$\langle \cos\theta \rangle = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \frac{C_{sc}(\theta,\phi)}{C_{sc}} \cos\theta \sin\theta \, d\theta \, d\phi \qquad (2)$$

where $C_{sc}(\theta, \phi)$ is the differential scattering cross section.

The scattering cross section C_{sc} is a function of the size and the optical properties of the grain and also of the wavelength of the radiation impinging on the grain. In figure 1 are shown the complex indices which are considered to be representations for a "dirty ice" grain. In figure 2 is shown the radiation pressure (given as force per unit volume) acting on grains of three different sizes in the presence of black-body radiation fields of temperatures from 1° K to 100 000° K. The radiation pressure forces are also given in table I. In order to calculate the force at some

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

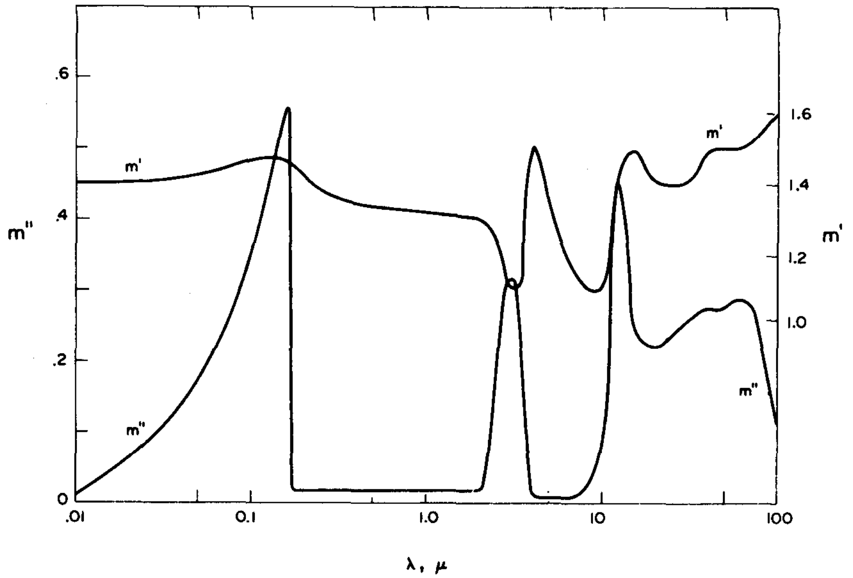

FIGURE 1.—Complex indices chosen as representative of ice grains. "Dirty ice" is arbitrarily given a value of $m''=0.02$ in the "visible" region to account for impurities. For dirty ice. $m=m'-m''i$.

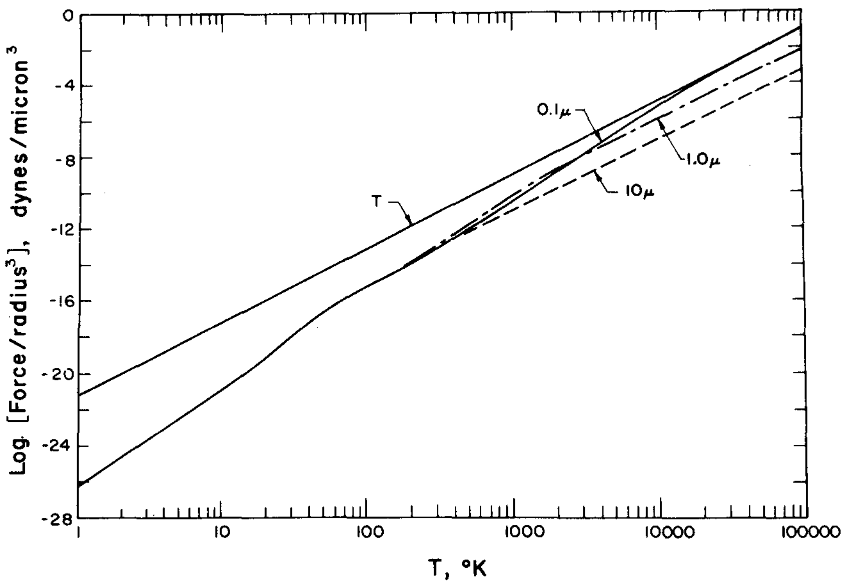

FIGURE 2.—Radiation forces on spherical dirty ice grains in the radiation fields of black bodies of various temperatures.

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

TABLE I.—*Radiation Pressure Forces*

Black-body temperature	Radiation force in dynes for sphere of radius—		
T, °K	0.1 μ	1 μ	10 μ
10	0.1415×10^{-23}	0.1415×10^{-20}	0.1521×10^{-17}
100	$.5197\times10^{-18}$	$.5360\times10^{-15}$	$.6872\times10^{-12}$
500	$.6478\times10^{-15}$	$.1074\times10^{-11}$	$.4834\times10^{-9}$
1000	$.2889\times10^{-13}$	$.4454\times10^{-10}$	$.7325\times10^{-8}$
1600	$.2262\times10^{-12}$	$.4162\times10^{-9}$	$.4545\times10^{-7}$
2500	$.1381\times10^{-11}$	$.2831\times10^{-8}$	$.2597\times10^{-6}$
3500	$.8030\times10^{-11}$	$.1146\times10^{-7}$	$.9696\times10^{-6}$
5000	$.7529\times10^{-10}$	$.4975\times10^{-7}$	$.3913\times10^{-5}$
7000	$.6163\times10^{-9}$	$.1923\times10^{-6}$	$.1462\times10^{-4}$
10 000	$.4587\times10^{-8}$	$.7731\times10^{-6}$	$.5944\times10^{-4}$
16 000	$.4509\times10^{-7}$	$.4651\times10^{-5}$	$.3805\times10^{-3}$
25 000	$.3104\times10^{-6}$	$.2566\times10^{-4}$	$.2236\times10^{-2}$
35 000	$.1226\times10^{-5}$	$.9444\times10^{-4}$	$.8536\times10^{-2}$
50 000	$.5054\times10^{-5}$	$.3806\times10^{-3}$	$.3541\times10^{-1}$
70 000	$.1878\times10^{-4}$	$.1430\times10^{-2}$	.1357
100 000	$.7449\times10^{-4}$	$.5859\times10^{-2}$	.5645

distance R from a star of radius r at temperature T, it is only necessary to multiply the values on the curves by a dilution factor $W=(r/R)^2$. One readily notes that in low temperature fields ($T<200°$) the radiation force per unit volume is almost the same for all the grains in the size range $0.1\ \mu \leqslant a \leqslant 10\ \mu$. For increasing stellar temperatures, the force per unit volume increases for the smaller grains relative to the larger ones until at about 20 000° K the force per unit area is roughly independent of the size so that the force per unit volume is inversely proportional to the size. For low temperatures, then, the ratio of radiation force to gravitational force is independent of the size, whereas in the neighborhood of sufficiently hot stars the ratio of radiation force to gravitational force is inversely proportional to the size.

Because the ratio of radiation force to gravitational force increases with size, the possibility exists for a differential ejection of particles of differing sizes, the larger ones remaining relatively closer to the stars. Compare, for example, the relative situation for ice grains of radii 0.1 μ and 1.0 μ in the neighborhood of a star whose temperature is 70 000° K. An 05 star is assumed whose size and mass are given by $\log\ (r/r_0)=1.3$ and $\log\ (m/m_0)=1.5$, where r_0 and m_0 are the radius and mass of the Sun taken as $r_0=6.96\times10^{10}$ cm and $m_0=1.99\times10^{33}$ g. For grain sizes $a\leqslant1\ \mu$, the force ratio of radiation to gravity is $F_{rad}/F_{grav}\geqslant10^5$, so that

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

the gravitational force may be neglected entirely in calculating the ejection of grains in the neighborhood of such a star. If it is assumed that the grain reaches a terminal speed v_t relatively soon, the terminal speed is then given by $F_{rad} = n_H A m_H v_t^2$, where n_H and m_H are the number density and mass of hydrogen atoms (or ions) and A is the cross-sectional area of the grain. As the grain recedes from the star, the radiation force decreases as the dilution factor $W = \left(\frac{r}{R}\right)^2$. The speed of ejection is then given by

$$v_t^2 = \frac{F_{rad}}{n_H A m_H} W \tag{3}$$

For $A = \pi a^2$, $n_H = 1/\text{cm}^3$, $m_H = 1.6 \times 10^{-24}$ g, the terminal velocity of the 0.1-μ grain is

$$\begin{aligned} v_t(0.1) &= \left[\frac{0.188 \times 10^{-4}}{(1 \times 3.14 \times 10^{-10})(1.6 \times 10^{-24})}\right]^{1/2} \left(\frac{r}{R}\right) \\ &= 1.94 \times 10^{14}\ (r/R)\ \text{cm/sec} \end{aligned} \tag{4}$$

The terminal velocity of a 1.0-μ particle is less by a factor of $(1.43/1.88)^{1/2} = 0.87$. Thus it appears that there is a separation effect. In solving equation (4), we write $v_t = \frac{dR}{dt}$ and obtain for the distance of ejection as a function of time

$$\begin{aligned} \frac{R^2 - r^2}{2r} &= (1.94 \times 10^{14})t && a = 0.1\ \mu \\ &= (0.87 \times 1.94 \times 10^{14})t && a = 1.0\ \mu \end{aligned}$$

The time scale for particle motion to a distance of 10 parsecs (roughly the order of size of the Orion nebulosity) is given by

$$\begin{aligned} t &= 0.25 \times 10^{-14} R^2/r \\ &= 5 \times 10^5 \text{ years} \end{aligned}$$

This is not an unreasonable estimate of the ages of the stars in Orion and it then appears possible for significant changes due to radiation pressure effects to occur in the size distribution of the grains.

EXTINCTION BY CORE MANTLE GRAINS

The suggestion in reference 1 tnat the graphite particles may be an important and perhaps dominant feature in the interstellar grains has led to the further possibility that these grains may form cores upon which dielectric mantles may grow. In figure 3 is shown a series of normalized extinction curves for various size distributions of dirty ice mantles on spherical graphite cores of radius 0.05 μ. The indices of

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

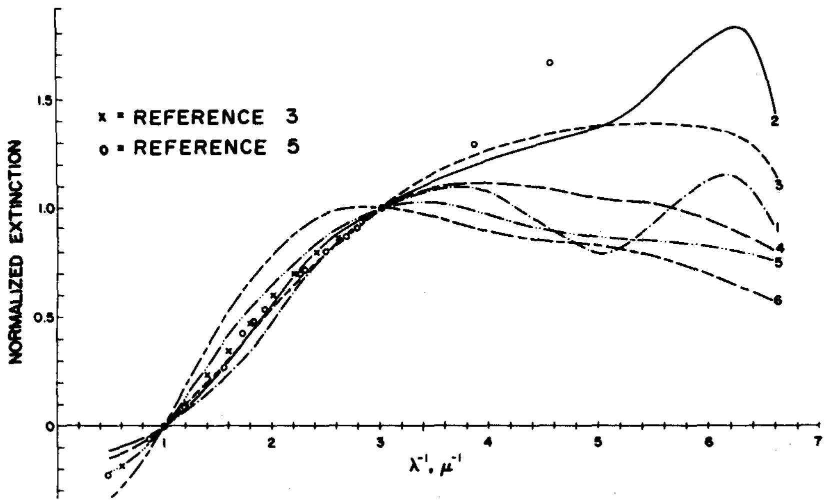

FIGURE 3.—Comparison of theoretical extinction curves for spherical core mantle grains (0.05-μ graphite core plus ice mantle) with observations of interstellar extinction. Mantle size distribution $n(a)=\exp\ [-5(a/a_i)^3]$; $a_i=0.1\ \mu-0.6\ \mu$, curves labeled 1 to 6, respectively.

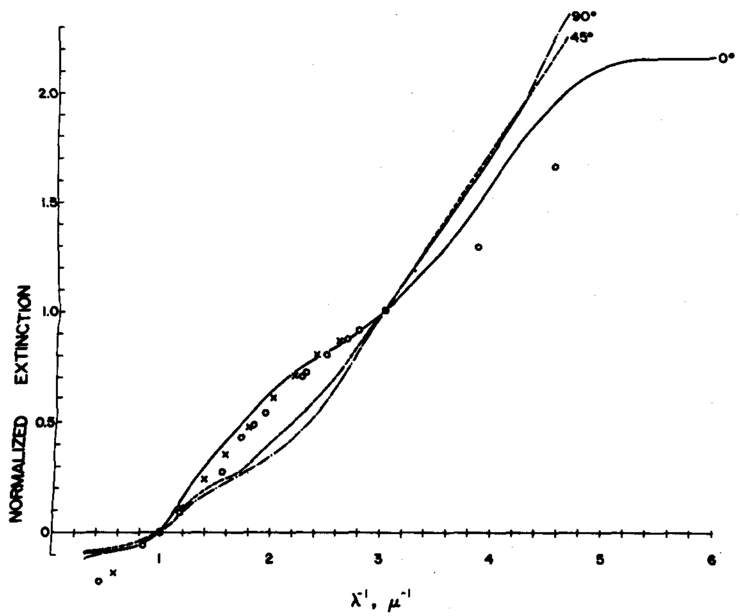

FIGURE 4.—Comparison of theoretical extinction curves for spinning dielectric ($m=1.33$) cylinders of radius $a=0.095\ \mu$. The angles 0°, 45°, 90° denote values of the angle between the plane of rotation of the cylinders and the line of sight.

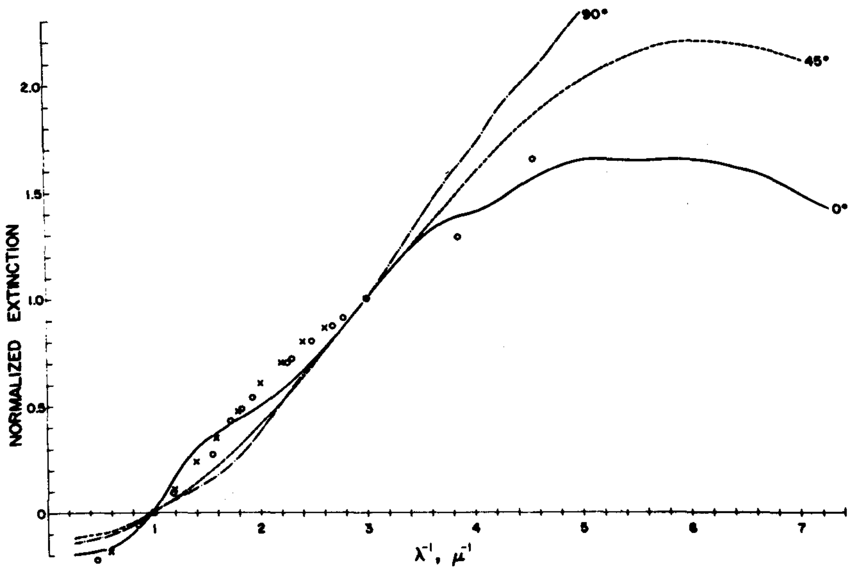

FIGURE 5.—Comparison of theoretical extinction curves for spinning dielectric ($m = 1.33$) cylinders of radius $a = 0.127\,\mu$. The angles 0°, 45°, and 90° denote values of the angle between the plane of rotation of the cylinders and the line of sight.

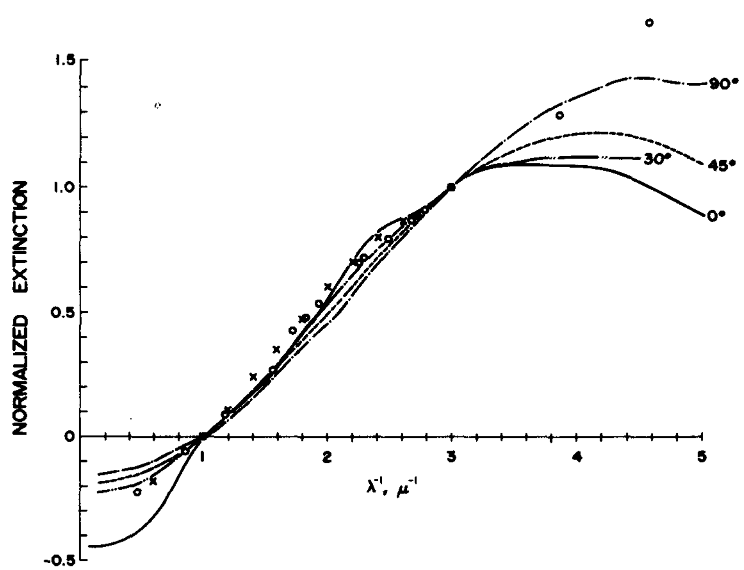

FIGURE 6.—Comparison of theoretical extinction curves for spinning dielectric ($m = 1.33$) cylinders of radius $a = 0.19\,\mu$. The angles 0°, 30°, 45°, and 90° denote values of the angle between the plane of rotation of the cylinders and the line of sight.

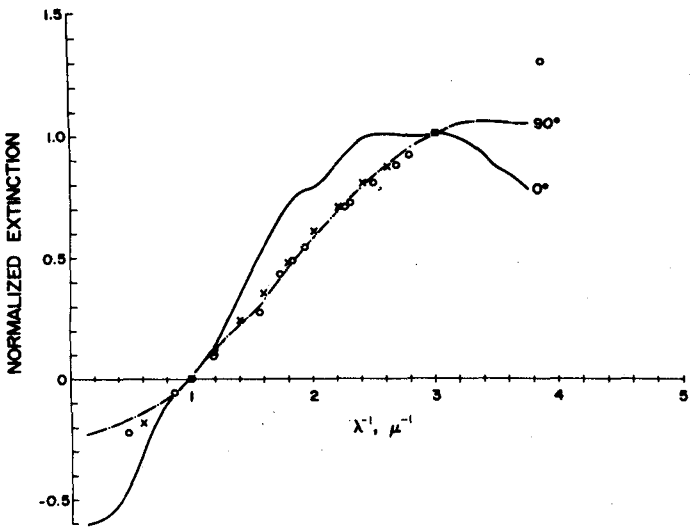

FIGURE 7.—Comparison of theoretical extinction curves for spinning dielectric ($m = 1.33$) cylinders of radius $a = 0.254\ \mu$. The angles 0° and 90° denote values of the angle between The plane of rotation of the cylinders and the line of sight.

refraction of the graphite are taken from reference 2. Comparison is made with the observed extinction for $\lambda^{-1} < 3\ \mu^{-1}$ in references 3 and 4 and for $\lambda^{-1} > 3\ \mu^{-1}$ in reference 5. The curve labeled as number 1 corresponds to a mantle size distribution which is almost negligible and therefore is essentially indistinguishable from the extinction which one obtains for the graphite core alone. It should be noted that only for a small (but not too small) mantle size can a reasonable match to the observed extinction in the ultraviolet as well as the visible be obtained. This effect was previously noted in reference 6 in calculations using a somewhat different form for the graphite refractive indices.

EXTINCTION BY INFINITE DIELECTRIC CYLINDERS

The calculations presented in the paper of this compilation by Greenberg and Shah are shown here in figures 4 to 7 as normalized extinction curves for a range of selected single sizes. As before, particles in perfect Davis-Greenstein spinning orientation about magnetic field directions making different angles with the normal to the line of sight are considered. Thus, 0° and 90° correspond to directions of the magnetic field B perpendicular to and along the line of sight, respectively. Comparison is made with the same observational points that are shown in figure 3. It appears that elongated particles in a size range between $a = 0.095\ \mu$

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

and $a = 0.127\ \mu$ may provide a good overall fit to the average extinction in the ultraviolet as well as in the visible region.

REFERENCES

1. BOGGESS, A., III; and BORGMAN, J.: Interstellar Extinction in the Middle Ultraviolet. Astrophys. J., vol. 140, 1964, p. 1636.

2. GREENBERG, J. M.: The Optics of Interstellar Grains. Proceedings of the I.A.U. Symposium No. 24: Spectral Classification and Multicolor Photometry. (Saltsjobäden, Sweden), 1964.

3. HOYLE, F.; and WICKRAMASINGHE, N. C.: On Graphite Particles as Interstellar Grains. Roy. Astron. Soc., Monthly Notices, vol. 124, 1962, p. 417.

4. JOHNSON, H. L.; and BORGMAN, J.: The Law of Interstellar Extinction. Bull. Astron. Inst. Netherlands, vol. 17, 1963, p. 115.

5. WHITFORD, A. E.: The Law of Interstellar Reddening. Astron. J., vol. 63, 1958, p. 201.

6. WICKRAMASINGHE, N. C.; and GUILLAUME, C.: Interstellar Extinction by Graphite Grains. Nature, vol. 207, 1965, p. 366.

DISCUSSION

Wickramasinghe: Dr. Greenberg, I would like to refer to the curves you got for the graphite-core–ice-mantle case where you said you could not get a fit with the extinction curve by using the new graphite data. I have showed that one could get a very good fit by using a realistic size distribution; namely, an exponential size distribution of mantles around graphite cores of radii less than 0.6 μ. The detailed fit depends upon the equilibrium size distribution.

Greenberg: The important difference between the mantles you used and those which we used lies in the choice of index of refraction. Your mantles have a pure real index of refraction, whereas our mantles have complex indices, as does ice in the ultraviolet. The use of complex indices has the effect of reducing the extinction in the ultraviolet and therefore, if the interstellar grain mantles are indeed icelike (have ultraviolet absorption like most dielectric substances), our calculations are more realistic. Furthermore, we have calculated extinction by various linear combinations of mantle size distributions ($n = \sum n_i e^{-5(a/a_i)^3}$) where size a_i defines a curve as shown in figure 3. These size distributions bear little resemblance to the Oort-van de Hulst distribution and, therefore, the theoretical extinction curves which we obtain for these distributions are quite general and they involve as many as seven parameters. In any case, none of these linear combinations can produce an extinction in the ultraviolet relative to the visible larger than that given by curve (2) in figure 3.

Wickramasinghe: For the mantle distribution obtained from my theory, I get perfect fit with the Cygnus curve all the way down from the infrared measurements of Johnson, through the curve of Nandy in the visible including the knee, to ultraviolet points of Boggess and Borgman.

I would also like to question the fact that you can get a good fit with your ice cylinders. For a curve of extinction efficiency Q_{ext} as a function of $2\pi a$ times inverse wavelength for a single ice sphere, I could make the curve go up wherever I wanted by choosing a sufficiently small particle size. The only problem here is that if one arranged to match the *UV* points he would get a curve bowed down in the visible. If you demand a close fit with the λ^{-1} curve in the visible, you are essentially using a very small region in the curve of Q_{ext} as a function of $2\pi a/\lambda$. Once you pin this down in the visible by choosing a, you are forcing the extinction curve to turn down beyond $\lambda^{-1} \sim 3\mu^{-1}$.

Greenberg: I think the fits I got for the various individual sizes were not very good. But the qualitative fits were not as bad as you say. Actually the relatively small infinite cylinders did not fit that badly. It should be noted that the extinction by the smallest cylinders (see fig. 4) was not necessarily bowed down in the visible as would have been the case for spheres whose diameters were equal to the cylinder diameter.

Wickramasinghe: My claim is that you get the same thing for single spherical isotropic ice spheres. By choosing the particle size small enough, one could make the curve rise at 2200 Å and go through the *UV* point, but the curve no longer fits the observations in the visible.

Greenberg: That is not correct.

Wickramasinghe: What I am trying to say is that the ice cylinders do not give a situation essentially different from the case of the spherical particle. The moment you fit your λ^{-1} curve in the visible (the best determined part of the interstellar extinction curve) accurately, your curve immediately goes down in the ultraviolet.

Greenberg: That is incorrect. There is a noticeable difference between long and spherical particles of diameter $2d = 0.2\,\mu$. The extinction by the spherical particles of diameters of the order of $2a = 0.2\,\mu$ will not go up as high in the ultraviolet and still be reasonably matched to observations in the infrared.

Wickramasinghe: I don't agree.

Greenberg: The size distribution of the nonspherical particles, which appear to fit the extinction, has radii which are somewhat smaller than I would have anticipated on the basis of spheres. The major difference in the ultraviolet extinction would be a function of both the size as well as the orientation.

Nandy: If the variation in the ratio of the slope of the ultraviolet relative to the blue is not dependent on the orientation properties of the particles, is it conclusive that the particles are not dielectric?

Greenberg: I think it might be true. However, since the graphite is anisotropic in its scattering characteristics, it seems to me that the

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

graphite should also exhibit some differences in extinction. These differences are probably smaller because most of the extinction is characterized by the index of refraction; i.e., the absorption in the ultraviolet. For the dielectric particles, the extinction is characterized more by the particle shape. As I said before I would not expect the effect to extend as far into the ultraviolet for graphite as for dielectric particles.

Nandy: But there is yet another parameter: the coating of ice.

Wickramasinghe: For graphite, or graphite-core–ice-mantle grains, both the knee that Nandy gets and the ultraviolet observations emerge from physical properties of the graphite.

Greenberg: This is correct.

Wickramasinghe: I believe Stecher has obtained the computer output of the experimental data on the refractive index of graphite and it is quite noticeable that a change in the absorptive index takes place pretty close to $\lambda^{-1}=2.2\,\mu^{-1}$. This is the wavelength that Nandy gets for his kink.

Greenberg: I think that this deserves considerable attention. No theory that I know of on dielectric particles predicts any kink at $2.2\,\mu^{-1}$.

Field: One effect which I don't think has been accounted for in these radiation pressure calculations is the fact that these particles would tend to be coupled with the magnetic field. I have just calculated the gyrofrequency of the particles in the magnetic field for a charge of 1 electron. The value I obtained was 10^4 years. I also made some rough estimates of the charge that might be expected, and it would be on the order of 100 electrons, I believe, in an H II region. The result would be then that they gyrate every 100 years. The effect of radiation pressure, I think, would be to induce a drift perpendicular to B and to the radius vector, leading to a circulation of the particles rather than an outward expansion.

Greenberg: What fields do you use in the H II regions?

Field: I used 1 gamma as an example.

Greenberg: Is such a magnetic field in a H II region expected?

Field: I think you can argue that the magnetic field may be pushed out by expansion of the H II region if it is an old region; but young H II regions have not expanded and should contain whatever magnetic field was present before the ionization took place.

Greenberg: I would like a clarification on this. I had always thought that the H II regions were essentially free of even modest magnetic fields.

Hall: The stars in the directions of H II fields don't show any differences in polarization from that found for other stars.

Greenberg: That would seem to indicate the possibility of the same size magnetic fields.

Field: I think that the Oort-van de Hulst theory of clouds when they consider that the grains are essentially independent of the cloud during

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press

the collision and then colliding with each other depends very critically on this question. If the grains are coupled to the magnetic field, and the field is coupled to the ions and atoms of the cloud, the grains would not be able to leave the cloud. Hence, the region of interpenetration of grains from various clouds would be much reduced and the rate of destruction might be reduced by a factor of 10 or 100.

Wickramasinghe: The sputtering could come in here, couldn't it?

Greenberg: We can always make and destroy grains. However, we certainly know they exist.

https://doi.org/10.1017/S0252921100118482 Published online by Cambridge University Press