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UNICITY THEOREMS FOR MEROMORPHIC OR ENTIRE FUNCTIONS III

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This paper studies the unique range set of meromorphic functions and shows that the set $S = \{w \mid w^{13} + w^{11} + 1 = 0\}$ is unique range set of meromorphic functions with 13 elements.

1. INTRODUCTION

By a meromorphic function we shall always mean a meromorphic function in the complex plane. We use the usual notations of Nevanlinna theory of meromorphic functions as explained in [4]. We use E to denote any set of positive real numbers of finite linear measure, not necessarily the same at each occurrence. We denote by $S(r, f)$ any quantity satisfying $S(r, f) = o(T(r, f))$ $(r \to \infty,\ r \notin E)$.

Let f be a nonconstant meromorphic function and let S be a subset of distinct elements in the complex plane. Define

$$E_f(S) = \bigcup_{a \in S} \{z \mid f(z) - a = 0\},$$

where each zero of $f(z) - a$ with multiplicity m is repeated m times in $E_f(S)$ (see [1]).

In 1976, Gross [2] proved that there exist three finite sets S_j $(j = 1, 2, 3)$ such that any two entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2, 3$ must be identical, and asked the following question (see [2, Question 6]):

QUESTION 1. Can one find two (or possible even one) finite sets S_j $(j = 1, 2)$ such that any two nonconstant entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must be identical?

Now it is natrual to ask the following question:

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QUESTION 2. Can one find two (or possible even one) finite sets S_j $(j = 1, 2)$ such that any two nonconstant meromorphic functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$ must be identical?

Recently, the present author proved the following results which provide positive answers to Question 1.

THEOREM A. (See [7, Theorem 3].) *Let $S_1 = \{w \mid w^n - 1 = 0\}$, $S_2 = \{a, b\}$, where $n > 6$ is a positive integer, a and b are constants such that $ab \neq 0$, $a^n \neq b^n$, $a^{2n} \neq 1$, $b^{2n} \neq 1$ and $a^n b^n \neq 1$. Suppose that f and g are nonconstant entire functions satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1, 2$. Then $f \equiv g$.*

THEOREM B. (See [8, Theorem 1].) *Let $S = \{w \mid w^n + aw^{n-m} + b = 0\}$, where n and m are two positive integers such that n and m have no common factors and $n \geqslant 2m + 5$, a and b are two nonzero constants such that the algebraic equation $w^n + aw^{n-m} + b = 0$ has no multiple roots. If f and g are nonconstant entire functions satisfying $E_f(S) = E_g(S)$, then $f \equiv g$.*

Recently, the present author proved the following result which is a partial answer of Question 2.

THEOREM C. (See [8, Theorem 2].) *Let $S = \{w \mid w^n + aw^{n-m} + b = 0\}$, where n and m are two positive integers such that $m \geqslant 2$, $n \geqslant 2m + 7$ with n and m having no common factors, a and b are two nonzero constants such that the algebraic equation $w^n + aw^{n-m} + b = 0$ has no multiple roots. Suppose that f and g are nonconstant meromorphic functions satisfying $E_f(S) = E_g(S)$ and $E_f(\{\infty\}) = E_g(\{\infty\})$. Then $f \equiv g$.*

The set S such that for any two nonconstant meromorphic functions f and g the condition $E_f(S) = E_g(S)$ implies $f \equiv g$ is called a unique range set (URS in brief) of meromorphic functions. A similar definition for entire functions can be given. From Theorem B we immediately obtain the following result.

THEOREM B′. *Let S be defined as in Theorem B. Then S is a URS of entire functions.*

As a special case of Theorem B′, we deduce that the set $S = \{w \mid w^7 + w^6 + 1 = 0\}$ in a URS of entire functions with 7 elements. In this paper, we shall exhibit a URS of meromorphic functions with 13 elements. In fact, we prove more generally the following theorem, which provides a positive answer to Question 2.

THEOREM 1. *Let $S = \{w \mid w^n + aw^{n-m} + b = 0\}$, where n and m are two positive integers such that n and m have no common factors, $m \geqslant 2$ and $n > 2m + 8$, a and b are two nonzero constants such that the algebraic equation $w^n + aw^{n-m} + b = 0$ has no multiple roots. Then S is a URS of meromorphic functions.*

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From Theorem 1 we immediately obtain that the set $S = \{w \mid w^{13} + w^{11} + 1 = 0\}$ provides a URS of meromorphic functions with 13 elements, which provides a positive answer to Question 2.

2. Some lemmas

Lemma 1. (See [5].) *Let f be a nonconstant meromorphic function, and let $P(f)$ be a polynomial in f of the form*

$$P(f) = a_0 f^n + a_1 f^{n-1} + \ldots + a_{n-1} f + a_n,$$

where a_0 $(\neq 0)$, $a_1, \ldots, a_n$ are constants. Then

$$T(r, P(f)) = nT(r, f) + S(r, f).$$

In order to state the second lemma, we introduce the following notation.

Let F be a meromorphic function. We denote by $n_1(r, 1/(F-a))$ the number of simple a-points of F in $|z| \leqslant r$. $N_1(r, 1/(F-a))$ is defined in terms of $n_1(r, 1/(F-a))$ in the usual way (see [6]).

Let F and G be two nonconstant meromorphic functions. If F and G have the same a-points with the same multiplicities, we say F and G share the value a CM (see [3]).

Lemma 2. *Let*

$$H = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right), \tag{1}$$

where F and G are two nonconstant meromorphic functions. If F and G share 1 CM, and $H \not\equiv 0$, then

$$N_1\left(r, \frac{1}{F-1}\right) \leqslant N\left(r, \frac{1}{H}\right).$$

Proof: Suppose that z_0 is a simple 1-point of F. Let

$$F(z) = 1 + a_1(z - z_0) + a_2(z - z_0)^2 + O\Big((z - z_0)^3\Big),$$

$$G(z) = 1 + b_1(z - z_0) + b_2(z - z_0)^2 + O\Big((z - z_0)^3\Big),$$

where $a_1 \neq 0$ and $b_1 \neq 0$. Then an elementary calculation gives that

$$H(z) = O(z - z_0),$$

which proves that z_0 is a zero of H. Thus,

$$N_1\left(r, \frac{1}{F-1}\right) \leqslant N\left(r, \frac{1}{H}\right). \qquad \square$$

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3. Proof of Theorem 1

Suppose that f and g are two nonconstant meromorphic functions satisfying $E_f(S) = E_g(S)$. We proceed to prove $f \equiv g$.

Let

$$F = -\frac{1}{b} f^{n-m}(f^m + a) \quad \text{and} \quad G = -\frac{1}{b} g^{n-m}(g^m + a). \tag{2}$$

From Lemma 1, we have

$$T(r, F) = nT(r, f) + S(r, f) \tag{3}$$

and

$$T(r, G) = nT(r, g) + S(r, g). \tag{4}$$

Let

$$T(r) = \max\{T(r, f), T(r, g)\}$$

and

$$S(r) = o(T(r)) \quad (r \to \infty,\ r \notin E).$$

Noting $S = \{w \mid w^n + aw^{n-m} + b = 0\}$, from $E_f(S) = E_g(S)$ we get that F and G share the value 1 CM.

Let H be given by (1). If $H \not\equiv 0$, from Lemma 2 we have

$$N_1\left(r, \frac{1}{F-1}\right) \leqslant N\left(r, \frac{1}{H}\right) \leqslant T(r, H) + O(1). \tag{5}$$

From (1) we obtain

$$m(r, H) = S(r). \tag{6}$$

From (2) we have

$$F' = -\frac{1}{b} f^{n-m-1}(nf^m + a(n-m))f' \tag{7}$$

and

$$G' = -\frac{1}{b} g^{n-m-1}(ng^m + a(n-m))g'. \tag{8}$$

Since F and G share 1 CM, from (1), (7) and (8),
(9)

$$\begin{aligned} N(r, H) \leqslant \overline{N}(r, f) + \overline{N}(r, g) + \overline{N}\left(r, \frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{nf^m + a(n-m)}\right) + N_0\left(r, \frac{1}{f'}\right) \\ + \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}\left(r, \frac{1}{ng^m + a(n-m)}\right) + N_0\left(r, \frac{1}{g'}\right) \\ \leqslant (m+2)T(r, f) + (m+2)T(r, g) + N_0\left(r, \frac{1}{f'}\right) + N_0\left(r, \frac{1}{g'}\right) + O(1), \end{aligned}$$

where $N_0(r, 1/f')$ denotes the counting function corresponding to the zeros of f' that are not zeros of f and $F-1$, $N_0(r, 1/g')$ denotes the counting function corresponding to the zeros of g' that are not zeros of g and $G-1$. It follows from (5), (6) and (9) that

(10)
$$\begin{aligned} N_1\left(r, \frac{1}{F-1}\right) \leqslant (m+2)T(r, f) + (m+2)T(r, g) \\ + N_0\left(r, \frac{1}{f'}\right) + N_0\left(r, \frac{1}{g'}\right) + S(r). \end{aligned}$$

Suppose that $w_1, w_2, \ldots, w_n$ are the distinct roots of the equation $w^n + aw^{n-m} + b = 0$. From (2) we have

(11)
$$F - 1 = -\frac{1}{b}(f - w_1)(f - w_2)\ldots(f - w_n)$$

and

(12)
$$G - 1 = -\frac{1}{b}(g - w_1)(g - w_2)\ldots(g - w_n).$$

By the second fundamental theorem, we deduce

(13)
$$\begin{aligned} nT(r, f) < \overline{N}(r, f) + \overline{N}\left(r, \frac{1}{f}\right) + \sum_{j=1}^{n} \overline{N}\left(r, \frac{1}{f - w_j}\right) - N_0\left(r, \frac{1}{f'}\right) + S(r) \\ \leqslant 2T(r, f) + \overline{N}\left(r, \frac{1}{F-1}\right) - N_0\left(r, \frac{1}{f'}\right) + S(r). \end{aligned}$$

In the same manner as above, we have

(14)
$$nT(r, g) < 2T(r, g) + \overline{N}\left(r, \frac{1}{G-1}\right) - N_0\left(r, \frac{1}{g'}\right) + S(r).$$

It is obvious that

$$(15)\qquad \begin{aligned}\overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)&=2\overline{N}\left(r,\frac{1}{F-1}\right)\\&\leqslant N_1\left(r,\frac{1}{F-1}\right)+N\left(r,\frac{1}{F-1}\right)\\&\leqslant N_1\left(r,\frac{1}{F-1}\right)+T(r,F)+O(1)\\&=N_1\left(r,\frac{1}{F-1}\right)+nT(r,f)+S(r)\end{aligned}$$

and

$$(16)\qquad \overline{N}\left(r,\frac{1}{F-1}\right)+\overline{N}\left(r,\frac{1}{G-1}\right)\leqslant N_1\left(r,\frac{1}{F-1}\right)+nT(r,g)+S(r).$$

From (10), (13), (14) and (15) we obtain

$$nT(r,g)\leqslant(m+4)T(r,f)+(m+4)T(r,g)+S(r).$$

From (10), (13), (14) and (16) we obtain

$$nT(r,f)\leqslant(m+4)T(r,f)+(m+4)T(r,g)+S(r).$$

Thus,

$$(17)\qquad \begin{aligned}nT(r)&\leqslant(m+4)T(r,f)+(m+4)T(r,g)+S(r)\\&\leqslant(2m+8)T(r)+S(r).\end{aligned}$$

Since $n>2m+8$, (17) is a contradiction. From this we derive $H\equiv 0$. By integration we have from (1),

$$\frac{1}{G-1}=\frac{A}{F-1}+B,$$

where A $(\neq 0)$ and B are constants. Thus,

$$(18)\qquad G=\frac{(B+1)F+(A-B-1)}{BF+(A-B)}.$$

From (18),

$$T(r,G)=T(r,F)+O(1)$$

and

$$(19)\qquad T(r)=T(r,f)+S(r,f).$$

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From (2) we have

$$\overline{N}(r, F) = \overline{N}(r, f) \leqslant T(r), \tag{20}$$

$$\overline{N}(r, G) = \overline{N}(r, g) \leqslant T(r), \tag{21}$$

$$\overline{N}\left(r, \frac{1}{F}\right) = \overline{N}\left(r, \frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{f^m + a}\right) \leqslant (m+1)T(r) + O(1), \tag{22}$$

$$\overline{N}\left(r, \frac{1}{G}\right) = \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}\left(r, \frac{1}{g^m + a}\right) \leqslant (m+1)T(r) + O(1). \tag{23}$$

We discuss the following three cases.

CASE I. Suppose that $B \neq 0, -1$.

If $A - B - 1 \neq 0$, from (18) we have

$$\overline{N}\left(r, \frac{1}{F + \frac{A-B-1}{B+1}}\right) = \overline{N}\left(r, \frac{1}{G}\right).$$

From this and the second fundamental theorem, we have

$$\begin{aligned} T(r, F) &< \overline{N}(r, F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{F + \frac{A-B-1}{B+1}}\right) + S(r, F) \\ &= \overline{N}(r, F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{G}\right) + S(r, F). \end{aligned}$$

Combining this with (3), (19), (20), (22) and (23), we obtain

$$nT(r) < (2m+3)T(r) + S(r),$$

which contradicts the assumption $n > 2m + 8$. Thus $A - B - 1 = 0$. From (18),

$$G = \frac{(B+1)F}{BF+1}.$$

From this we have

$$\overline{N}\left(r, \frac{1}{F + \frac{1}{B}}\right) = \overline{N}(r, G).$$

Again from the second fundamental theorem, we obtain

$$\begin{aligned} T(r, F) &< \overline{N}(r, F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}\left(r, \frac{1}{F + \frac{1}{B}}\right) + S(r, F) \\ &= \overline{N}(r, F) + \overline{N}\left(r, \frac{1}{F}\right) + \overline{N}(r, G) + S(r, F). \end{aligned}$$

Combining this with (3), (19), (20), (21) and (22), we obtain

$$nT(r) < (m+3)T(r) + S(r),$$

which is impossible.

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Case II. Suppose that $B=-1$.

From (18) we have

$$G=\frac{A}{-F+(A+1)}. \tag{24}$$

If $A+1\neq 0$, from (24) we obtain

$$\overline{N}\left(r,\frac{1}{F-(A+1)}\right)=\overline{N}(r,G).$$

Thus, in the same manner as above, we have a contradiction. From this we obtain $A+1=0$. Again from (24) we derive $F\cdot G\equiv 1$. This and (2) yield

$$f^{n-m}(f-a_1)(f-a_2)\dots(f-a_m)g^{n-m}(g^m+a)\equiv b^2, \tag{25}$$

where $a_1, a_2, \dots, a_m$ are the distinct roots of the equation $\omega^m+a=0$.

Suppose that z_0 is a zero of f of order p. From (25) we know that z_0 is a pole of g. Suppose that z_0 is a pole of g of order q. From (25) we obtain

$$(n-m)p=nq. \tag{26}$$

Noting that n and m have no common factors, from (26) we get $n\leqslant p$. Thus,

$$\overline{N}\left(r,\frac{1}{f}\right)\leqslant\frac{1}{n}N\left(r,\frac{1}{f}\right)\leqslant\frac{1}{n}T(r,f)+O(1). \tag{27}$$

Suppose that z_j $(j=1,2,\dots,m)$ is a zero of $f-a_j$ of order p_j. From (25) we know that z_j is a pole of g. Suppose that z_j is a pole of g of order q_j. From (25) we obtain

$$p_j=nq_j.$$

Thus $n\leqslant p_j$ and hence

$$\overline{N}\left(r,\frac{1}{f-a_j}\right)\leqslant\frac{1}{n}N\left(r,\frac{1}{f-a_j}\right)\leqslant\frac{1}{n}T(r,f)+O(1). \tag{28}$$

By the second fundamental theorem, from (27) and (28) we have

$$\begin{aligned}(m-1)T(r,f)&<\overline{N}\left(r,\frac{1}{f}\right)+\sum_{j=1}^{m}\overline{N}\left(r,\frac{1}{f-a_j}\right)+S(r,f)\\&\leqslant\frac{m+1}{n}T(r,f)+S(r,f),\end{aligned}$$

which is impossible.

Case III. Suppose that $B = 0$.

From (18) we have

$$G = \frac{F + (A-1)}{A}. \tag{29}$$

If $A - 1 \neq 0$, from (29) we obtain

$$\overline{N}\left(r, \frac{1}{F + (A-1)}\right) = \overline{N}\left(r, \frac{1}{G}\right).$$

Thus, in the same manner as above, we have a contradiction. From this we obtain $A - 1 = 0$. Again from (29) we derive $F \equiv G$. This and (2) yield

$$f^n - g^n = -a\left(f^{n-m} - g^{n-m}\right). \tag{30}$$

If $f^n \not\equiv g^n$, from (30) we obtain

$$g^m = -\frac{a(h-v)(h-v^2)\dots(h-v^{n-m-1})}{(h-u)(h-u^2)\dots(h-u^{n-1})}, \tag{31}$$

where $h = f/g$, $u = \exp((2\pi i)/n)$ and $v = \exp((2\pi i)/(n-m))$. From (31) we know that h is a nonconstant meromorphic function. Since n and m have no common factors, we have $u^j \neq v^k$ $(j = 1, 2, \dots, n-1;\ k = 1, 2, \dots, n-m-1)$. Suppose that z_j $(j = 1, 2, \dots, n-1)$ is a zero of $h - u^j$ of order p_j. From (31) we have $p_j \geqslant m$. Thus

$$\overline{N}\left(r, \frac{1}{h-u^j}\right) \leqslant \frac{1}{m} N\left(r, \frac{1}{h-u^j}\right) \leqslant \frac{1}{2} T(r, h) + O(1). \tag{32}$$

By the second fundamental theorem, from (32) we obtain

$$\begin{aligned}(n-3)T(r, h) &< \sum_{j=1}^{n-1} \overline{N}\left(r, \frac{1}{h-u^j}\right) + S(r, h)\\ &\leqslant \frac{n-1}{2} T(r, h) + S(r, h),\end{aligned}$$

which is impossible. Thus $f^n \equiv g^n$ and $f^{n-m} \equiv g^{n-m}$. However, since n and m have no common factors, we get $f \equiv g$.

This completes the proof of Theorem 1. □

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4. Supplement of Theorem 1

It is reasonable to ask: What can be said if $m = 1$ in Theorem 1? In this section, we prove the following theorem, which is a supplement of Theorem 1.

Theorem 2. *Let $S = \{w \mid w^n + aw^{n-1} + b = 0\}$, where $n > 10$ is a positive integer, a and b are two nonzero constants such that the algebraic equation $w^n + aw^{n-1} + b = 0$ has no multiple roots. If f and g are two distinct nonconstant meromorphic functions satisfying $E_f(S) = E_g(S)$, then*

$$f = -\frac{ah(h^{n-1}-1)}{h^n - 1} \quad \text{and} \quad g = -\frac{a(h^{n-1}-1)}{h^n - 1},$$

where h is a nonconstant meromorphic function.

Proof: Let

$$F = -\frac{1}{b}f^{n-1}(f+a) \quad \text{and} \quad G = -\frac{1}{b}g^{n-1}(g+a). \tag{33}$$

Proceeding as in the proof of Theorem 1, we have $F \cdot G \equiv 1$ or $F \equiv G$. We distinguish the following two cases.

Case I. Assume $F \cdot G \equiv 1$.

From (33) we have

$$f^{n-1}(f+a)g^{n-1}(g+a) \equiv b^2. \tag{34}$$

Suppose that z_0 is a zero of f of order p. From (34) we know that z_0 is a pole of g. Suppose that z_0 is a pole of g of order q. From (34) we obtain $(n-1)p = nq$. From this we get $n \leqslant p$. Thus

$$\overline{N}\left(r, \frac{1}{f}\right) \leqslant \frac{1}{n}N\left(r, \frac{1}{f}\right) \leqslant \frac{1}{n}T(r, f) + O(1). \tag{35}$$

Suppose that z_1 is a zero of $f + a$ of order p_1. From (34) we know that z_1 is a pole of g. Suppose that z_1 is a pole of g of order q_1. From (34) we obtain $p_1 = nq_1$. Thus $n \leqslant p_1$ and hence

$$\overline{N}\left(r, \frac{1}{f+a}\right) \leqslant \frac{1}{n}N\left(r, \frac{1}{f+a}\right) \leqslant \frac{1}{n}T(r, f) + O(1). \tag{36}$$

In the same manner as above, we have

$$\overline{N}\left(r, \frac{1}{g}\right) \leqslant \frac{1}{n}T(r, g) + O(1), \tag{37}$$

$$\overline{N}\left(r, \frac{1}{g+a}\right) \leqslant \frac{1}{n}T(r, g) + O(1). \tag{38}$$

From (34) one sees easily that the poles of f can only be from the zeros of g and $g+a$. Consequently,

$$\overline{N}(r, f) \leqslant \overline{N}\left(r, \frac{1}{g}\right) + \overline{N}\left(r, \frac{1}{g+a}\right).$$

From this, (37) and (38) we obtain

$$\overline{N}(r, f) \leqslant \frac{2}{n}T(r, g) + O(1). \tag{39}$$

By the first fundamental theorem and Lemma 1, from (34) we have

$$T(r, g) = T(r, f) + S(r, f).$$

From this and (39) we obtain

$$\overline{N}(r, f) \leqslant \frac{2}{n}T(r, f) + S(r, f). \tag{40}$$

By the second fundamental theorem, from (35), (36) and (40) we get

$$\begin{aligned} T(r, f) &< \overline{N}\left(r, \frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{f+a}\right) + \overline{N}(r, f) + S(r, f) \\ &\leqslant \frac{4}{n}T(r, f) + S(r, f), \end{aligned}$$

which is impossible.

Case II. Assume $F \equiv G$.

From (33) we have

$$f^n - g^n \equiv -a\left(f^{n-1} - g^{n-1}\right). \tag{41}$$

Noting $f \not\equiv g$, from (41) we obtain

$$g = -\frac{a\left(h^{n-1} - 1\right)}{h^n - 1}, \tag{42}$$

where $h = f/g$. From (42) we know that h is a nonconstant meromorphic function. Thus, from (42) we have

$$f = -\frac{ah\left(h^{n-1} - 1\right)}{h^n - 1}.$$

This completes the proof of Theorem 2. □

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