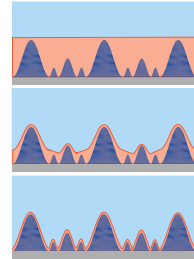# Tuning heat transport via boundary layer topographies

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A primary objective in turbulent thermal convection research is to understand and control the heat transport scaling behaviour. Previous studies have shown that the heat transport can be tuned by manipulating the boundary layer topographies with monoscale roughness elements. Now, Zhu *et al.* (*J. Fluid Mech.*, vol. 869, 2019, R4) have demonstrated that with multiscale wall roughness, the heat transport law with an exponent of 1/2 can be achieved for an extended range of the Rayleigh number, providing a new way to manipulate heat transport by tuning boundary topographies in turbulent flows.

Key words: boundary layer control, Bénard convection, turbulent flows

1. Introduction

Most fluid flows over large scales in nature are thermally driven, and therefore are convective in nature. Examples include planetary and stellar convections, convections in the atmosphere and the oceans, and mantle and core convections of the Earth. Studying thermal convection thus becomes essential in understanding many of the large-scale fluid phenomena occurring in geophysical and astrophysical systems. A paradigmatic model for thermal convection is the so-called Rayleigh–Bénard (RB) system, a fluid layer heated from below and cooled from above (Ahlers, Grossmann & Lohse 2009; Lohse & Xia 2010). A central issue in the study of RB convection is how heat is transported by turbulent flows when the flow itself is thermally driven, which is often framed as a scaling relation, $Nu \sim Ra^{\beta}$, between the Nusselt number Nu (the non-dimensional heat flux) and the Rayleigh number Ra (the non-dimensional driving force). Specifically, one asks what is the value of the exponent β for convective flows at the very large values of Ra that are thought to be relevant to geophysical and astrophysical flows. As current experiments and simulations cannot reach such high values of Ra, extrapolations are needed to bridge the gap. However, this can only be done with some confidence if one knows that the asymptotic state, known as the ultimate state of thermal convection, has been reached. Such a state was

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first suggested by Kraichnan (1962), who argued that when the boundary layers (BLs) become fully turbulent under sufficiently strong thermal driving, heat will be transported ballistically across the fluid layer, resulting in $Nu \sim Ra^{1/2}$ (the existence of a logarithmic correction, however, results in a smaller effective exponent). In contrast, a pure $1/2$ state is now called the asymptotic ultimate state (Grossmann & Lohse 2011). Despite much effort made in the search for the ultimate state, and reports for evidence of it, debates still continue regarding its existence. One way to achieve the ultimate state is to remove the BL in what is called homogeneous convection, which was first demonstrated numerically by Lohse & Toschi (2003), and later experimentally by Gibert *et al.* (2006). In both cases a $Nu \sim Ra^{1/2}$ was observed. A different approach is to retain the BLs, but add roughness to the surface of the top and bottom plates of the convection cell. The idea is that the roughness may help to trigger a transition to a turbulent BL at a lower Ra. Indeed, the value $\beta = 1/2$ has been obtained in experiments (Roche *et al.* 2001; Tisserand *et al.* 2011; Xie & Xia 2017). But direct numerical simulations (DNSs) of RB convection with rough surfaces in two dimensions found that β will fall back from $1/2$ to $1/3$ (the so-called classical scaling) when Ra becomes even higher (Zhu *et al.* 2017). This suggests that the observed $1/2$ scaling is not related to the ultimate state that should survive at arbitrarily high Ra once the system reached that state.

2. Overview

Studying the effect of surface roughness in turbulent thermal convection is not new. The experiment by Shen, Tong & Xia (1996) was perhaps the first such study in the context of RB convection. This is followed by experiments made by Ciliberto & Laroche (1999), Du & Tong (2000), Roche *et al.* (2001) and Qiu, Xia & Tong (2005). In these early studies, a general finding is that heat transport is enhanced once the thermal BL thickness becomes thinner than the roughness height; whether the scaling exponent β changes or not depends on the details of the rough plates (for a brief review on these earlier studies, see, for example, Wei *et al.* (2014)).

According to the Grossmann–Lohse theory, the kinetic energy and thermal variance dissipation rates can be split into contributions from the bulk and the BLs, and when the dissipation is bulk-dominated $Nu \sim Ra^{1/2}$ asymptotically, whereas when BLs become dominant $Nu \sim Ra^{1/3}$, again asymptotically (Grossmann & Lohse 2000). So, by perturbing the BL with surface roughness, one can reduce the BL contribution in the total dissipation and therefore change the overall heat transport to a more bulk-dominated behaviour. Based on this idea, Xie & Xia (2017) investigated experimentally using a series of convection cells with varying roughness topographies, which would introduce varying levels of BL perturbation. Indeed, they found that the value of β can be tuned by the roughness topography. They further identified two scaling regimes in the heat-transport-enhanced regime, termed as regime II (corresponding to the thermal BL becoming submerged below the roughness height) and regime III (even the viscous BL becoming submerged; in contrast, regime I corresponds to both BLs being above the roughness height). By defining a roughness parameter λ, i.e. the aspect ratio of the roughness elements, they showed that when λ changed from 0.5 to 4, the corresponding value of β in regime II (a transient regime) changed from 0.36 to 0.59, and it changed from 0.3 to 0.5 in regime III (see, for example, figure 1(*a*) for the case with $\lambda = 4.0$). As Ra of the experiment is well below that at which one would expect to see the ultimate state, the observed $Nu \sim Ra^{1/2}$ scaling is clearly not related to the asymptotic state. Rather, this study
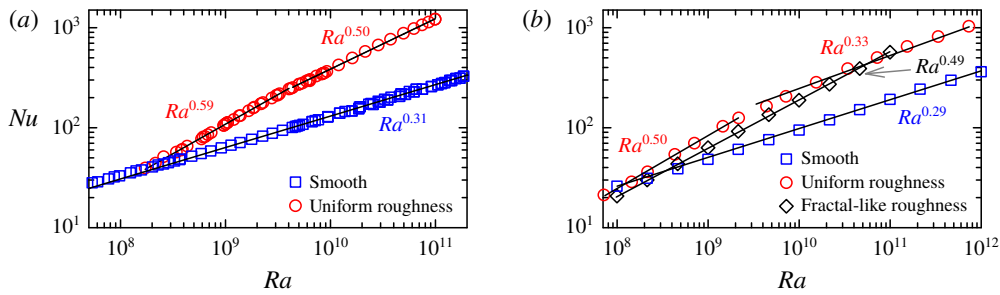
FIGURE 1. Here *Nu* versus *Ra* with smooth and rough boundaries. (*a*) Three-dimensional (3-D) experiments (Xie & Xia 2017) and (*b*) two-dimensional (2-D) DNS (Zhu *et al.* 2019). The solid lines are power-law fits to the respective data sets.

demonstrates that one can tune the roughness topography to obtain a desired value of β. The 1/2 scaling has also be obtained in 2-D DNS studies by Toppaladoddi, Succi & Wettlaufer (2017) and Zhu *et al.* (2017). While in the former the authors believe their results evidence a transition to the ultimate state, in the latter the authors demonstrate, surprisingly, that at even higher *Ra*, the scaling exponent will fall back to the classical 1/3 scaling (figure 1*b*), which implies that the observed 1/2 scaling should not be related to the ultimate state.

By taking a different route than many of the previous studies that use monoscale roughness, Zhu *et al.* (2019) adopted a multiscale roughness topography in their simulation. By doing so, they find a 1/2 scaling over three decades of *Ra* from 1×10^8 to 1×10^{11} (figure 1*b*). They show that the thermal BL is protruded by the three roughness heights sequentially (see the title image): at small *Ra*, all the roughness elements are submerged inside the thermal BL; with increasing *Ra*, part of the roughness elements protrude the thermal BL, with the smaller roughness still being submerged in the thermal BL; at the largest *Ra*, the thermal BL follows the surface roughness, acting as a effective smooth surface, resulting in the classical 1/3 scaling (figure 1*b*). This represents a new approach to perturb the BL in controlling heat transport scaling. In the simulation by Zhu *et al.* (2019), a three-level fractal-like structure was used, but in principle the level of scales is only limited by computational resources. One may then obtain a wider range of a 1/2 scaling with a hierarchy of roughness scales. A similar approach can be taken for other scaling exponents (for example, with different values of λ and multiscale roughness), making it possible to 'design' thermal convection systems with specific heat transport scalings.

3. Future

Although both experimental and DNS studies show convincingly that the heat transport in turbulent convection can be tuned by manipulating the boundary layer topographies, there are differences between the 3-D experiments and 2-D simulations. The falling-back of β from 1/2 to the classical value of 1/3 for $Ra > 2 \times 10^9$ observed by Zhu *et al.* (2017) is not seen in experiments up to $Ra \approx 1 \times 10^{11}$ (Xie & Xia 2017). Therefore, experimental studies with *Ra* larger than 1×10^{11} will be needed to complete the picture. The physical mechanism leading to the observed relation between β and λ is not fully understood. To this end, studies on the spatial distribution of the energy dissipation rate and the thermal dissipation rate

for different roughness configurations should provide some insight, which can only be done with realistic prospects using 3-D DNS. Moreover, the Prandtl number *Pr*, which characterises the relative thickness of the thermal BL to the viscous BL, is found to have a large impact on the heat transport in rough surface cells (Xie & Xia 2017). Studies with a wider range of *Pr* will thus complement our understanding of turbulent convection over rough surfaces. Finally, a crucial step towards real applications is the experimental realisation of the observations by Zhu *et al.* (2019) in turbulent convection with multiscale roughness. With their tuning method, it is possible to design systems with desired heat transport efficiencies, which should find many practical applications.

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