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A discrete analytic theory of geometric difference functions

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Discrete-analytic function theory is concerned with a study of functions defined only at certain lattice points in the complex plane. Since continuity no longer has any meaning in this theory, the classical concepts of differentiability and analytic function theory can not be applied.

Using difference operators instead of derivatives, Isaacs [3] in 1941 defined the notion of discrete-analyticity and established a theory for functions defined on the set of Gaussian integers. The subject was given impetus by the work of Duffin [2] in 1956 and since then it has been extensively developed by numerous writers. Making use of discrete analogues for contour integration and power series, the resulting discrete-analytic function theory parallels many aspects of classical analysis and of course has a variety of distinguishing features.

Geometric or q-difference functions are a generalization of ordinary differences and constitute an important branch of finite-difference theory. As yet an extension into the realm of discrete-analytic functions has not been made and in this thesis such a theory is established.

The concepts of q-difference and q-integration operators, defined and developed by Jackson [4] and others, are extended into the complex plane comprising lattice points of the form $\left(\pm q^m x, \pm q^n y\right)$ where q, x, y are real and m, n are integers. A class of functions (q-analytic) defined on this set is examined and a discrete-analytic function theory is

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constructed.

Using the resulting discrete contour integral, analogues are obtained for Cauchy integral theorems and the integral formula. A discrete-analytic continuation operator C is devised which enables functions defined on the real axis to be continued into the complex plane as q-analytic functions. This process (in fact an analogue of Taylor's Theorem) is of fundamental importance to the development of the subsequent theory.

A central problem in discrete-analytic function theory is the construction of a suitable analogue for multiplication. The continuation operator C is used to define a multiplicative operator $*$ which has certain advantages over existing operators in the ordinary-difference theory. The function z^n is found to have a convenient analogue in q-analytic function theory which appears to be more amenable to applications in power series than its ordinary-difference counterpart.

An unsolved problem in discrete-analytic theory (see Deeter [1]) has been to find a suitable analogue for the function z^α where α is arbitrary. Such a function is found in the q-analytic theory. New results are also obtained for discrete-analytic polynomials, discrete trigonometric series, q-difference equations and conformal mapping.

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