# ON THE ERRORS INVOLVED IN THE DETERMINATION OF COOLING POWERS AND AIR VELOCITIES WITH A KATA-THERMOMETER

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(With 5 Figures in the Text)

During the experimental comparison of different types of hot-water heating systems in a school at Easter 1934 about a thousand kata readings were taken. These were taken mostly in pairs, one immediately after the other, and a third reading was taken only when the first two differed by more than about 10 per cent.

The standard deviation of the difference between the first and second kata cooling times can easily be calculated, and from this it is possible to deduce the errors involved in the measurement of cooling powers and air velocities. The statistical method used involves two assumptions:

(i) That the errors of the first and second cooling times have a chance distribution, when they are taken consecutively with the same instrument. This may be checked by plotting the distribution of the differences between the first and second cooling times and seeing whether they form a normal curve.

(ii) That the standard error of the first reading is equal to that of the second and that the two errors are uncorrelated.

If $x_1$, $x_2$ are two cooling times and $d=x_1-x_2$, $\sigma$ being the standard error of a single reading, then $\sigma_d{}^2=2\sigma^2(1-r)$, where $r$ is the correlation in error between the readings.

If
$$\chi=\frac{x_1+x_2}{2},$$

$$\sigma_\chi{}^2=\frac{\sigma^2}{2}(1+r).$$

Hence, if $r=0$, $$\sigma_\chi{}^2=\sigma_d{}^2/4 \quad \text{or} \quad \sigma_\chi=\tfrac{1}{2}\sigma_d.$$

That is to say, the standard error of the mean of two kata readings is equal to half the standard deviation of the difference between the first and second cooling times.

## The results

### (i) *Ordinary kata*

The percentage differences between the first and second cooling times were calculated for each pair of readings, the percentages being calculated on the mean of the two cooling times. These differences were then grouped according to the size of this difference and according to air velocity calculated from the cooling times. The air velocity groups were (*a*) 0–19, (*b*) 20–49, (*c*) 50–99, and (*d*) 100 ft. per min. upwards. The distribution of the differences for groups (*a*) and (*b*) are shown in Figs. 1 and 2. The (*c*) and (*d*) groups contain too few readings for this, but Fig. 3 shows the distribution of the differences when all the readings are taken together[1].

Taking all the readings together, the second cooling time is on the average 2 per cent. $\pm 0{\cdot}42$ longer than the first one. The standard deviation of the differences between the first and second readings is 7·04 per cent. (s.e. 0·30) when it is calculated about the mean difference. In the low air velocity group (*a*) the standard deviation of the differences is significantly less than in the moderate air velocity group (*b*). The difference is 2·5 per cent. $\pm 0{\cdot}56$.

### (ii) *The silvered kata*

The same procedure was followed with the 160 pairs of silvered kata cooling times, and the distribution curve for all the readings together is shown in Fig. 4[2]. The second cooling time is 0·8 per cent. (s.e. 0·56) longer than the first, but this difference is not significant. The standard deviation of the difference between the first and second readings is 7·05 per cent. (s.e. 0·4) when it is calculated about the mean difference.

## Discussion

The significant difference of 2 per cent. between the first and second cooling times of the ordinary kata may be due to the stem of the instrument being warmer during the second cooling time than during the first. Thus during the first cooling time more heat would be absorbed in warming the stem than during the second cooling time, with a consequent shortening of the first cooling time. But with the silvered kata the mean difference between the first and second cooling times is not significant. This is probably due to the longer cooling time of the silvered kata, owing to the reduction of the radiant heat loss and to the effect of the silver and copper coating which extends up the stem for about 1·5 cm.; this coating would help to warm up

[1] The normality of these curves has been tested by calculating the values of $\sqrt{\beta_1}$ and $\beta_2 - 3$. $\sqrt{\beta_1}$ is insignificant for all these distributions but $\beta_2 - 3$ is $2\frac{1}{2}$ times its standard error for the air velocity group 20–49 feet per minute and four times its standard error when all the readings are taken together. Thus the distribution curves are not significantly skewed but for air velocities above 20 feet per minute the distributions are slightly leptokurtic.

[2] For this distribution both $\sqrt{\beta_1}$ and $\beta_2 - 3$ are less than their standard errors.

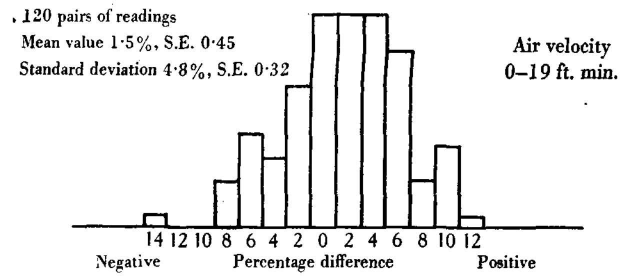


Fig. 1.

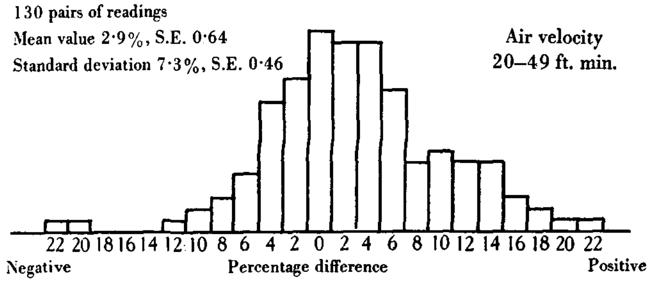


Fig. 2.

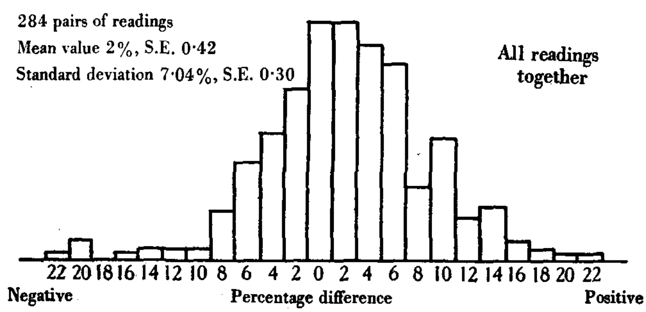


Fig. 3.

Figs. 1–3. Distribution curves for the differences between the first and second cooling times of the ordinary kata. The differences are positive when the second cooling time is longer than the first and standard errors are used throughout.

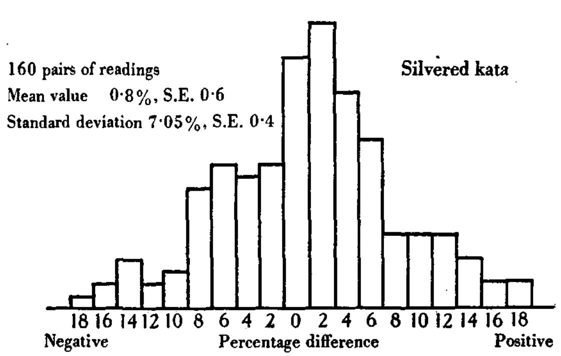


Fig. 4. Distribution curve for the differences between the first and second cooling times of the silvered kata. The differences are positive when the second cooling time is longer than the first and standard errors are used.

the lower part of the stem by conduction from the bulb before the timed cooling period and would thus tend to reduce the heat absorbed by the stem during the first cooling time.

The existence of this difference shows the necessity of neglecting the first cooling time after the kata has been standing for a while, and the desirability of half-filling the top bulb of the kata when warming it up and not merely taking the alcohol up to the bottom of the top bulb.

During this investigation the first cooling time was always neglected when beginning in the morning, but not always when continuing after lunch. This, however, would affect very few out of the total number of readings taken, because about thirty cooling times were taken with each kata during both the morning and afternoon spells.

Kata-thermometers have been calibrated several times, and these calibrations have presumably been based on the results of a series of consecutive readings. During the calibration therefore the stem of the instrument would be kept warm and the cooling times may be expected to correspond with the second reading of the pairs of cooling times taken in this investigation. If this is so the first cooling times are 2 per cent. low and the mean of the two cooling times is 1 per cent. low. However, this error is negligible compared with the standard error of $\pm 3{\cdot}4$ per cent. for the value of the mean of two determinations.

*The error of the cooling power determination*

Considering both types of kata-thermometer together and taking the standard deviation of the difference of the first and second cooling times as $\pm 7$ per cent., the standard error of a cooling power determination based on $n$ cooling times is

$$\frac{7}{\sqrt{2n}} \text{ per cent.}$$

Thus when $n$ is 2, the standard error of the cooling power is 3·5 per cent. and the standard error between two different cooling powers is 5 per cent. It is therefore necessary for two such cooling powers to differ by 10 per cent. if the difference is to be considered as significant.

*The error of the air velocity determination*

Taking the formula for the air velocity as

$$\frac{H}{\theta} = c + k\sqrt{v},$$

$$\frac{\delta H}{\theta} = \frac{k\delta v}{2\sqrt{v}},$$

$$\delta v = \frac{2\sqrt{v}\delta H}{k\theta} = 2\left(\frac{H}{\theta} - c\right)\delta H/k^2\theta,$$

or

$$k^2\delta v = 2e\left(\frac{H^2}{\theta^2} - \frac{cH}{\theta}\right), \text{ where } e = \frac{\delta H}{H}.$$

Where $H$ is the kata cooling power,

$\theta$ is (97·7 – the air temperature in ° F.),

$v$ is the air velocity in ft. per min., and

$c$ and $k$ are constants.

For the ordinary kata

$$c = 0{\cdot}1086 \text{ and } k = 0{\cdot}01584,$$

for the silvered kata

$$c = 0{\cdot}0561 \text{ and } k = 0{\cdot}01584.$$

By inserting the values of the constants in the above equation the fol-

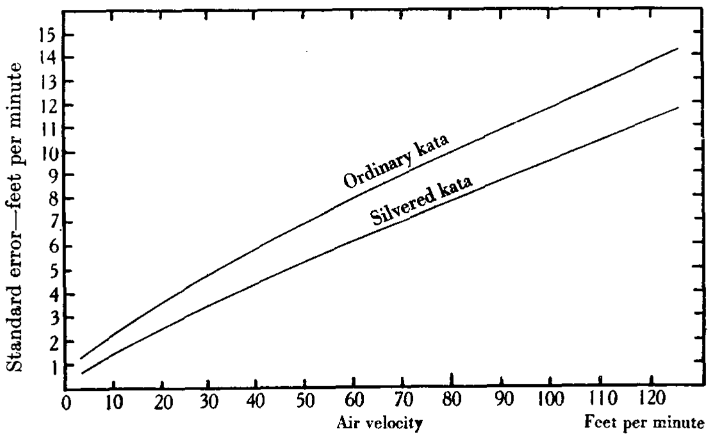


Fig. 5. Curves showing the standard error of the air velocity readings for both the silvered and the ordinary katas. These curves have been calculated for a cooling power with a standard error of 3·5 per cent. i.e., for a cooling power determined from a pair of cooling times.

lowing table is obtained for the standard error of the air velocity readings when $e$ is 3·5 per cent.

| $v$ ft./min. | Ordinary kata ft./min. | Silvered kata ft./min. |
|---|---|---|
| 10 | 2·2 | 1·5 |
| 20 | 3·6 | 2·5 |
| 30 | 4·8 | 3·5 |
| 50 | 6·9 | 5·2 |
| 80 | 9·9 | 7·8 |
| 120 | 13·7 | 11·2 |

These values are shown graphically in Fig. 5. The standard errors for the silvered kata are lower than those for the ordinary kata because of the different values of $c$ in the formulae for the two types of katas.

## *Accuracy of thermometers*

The accuracy of the air velocity reading depends partly on the accuracy of $\theta$ and hence on the accuracy of the thermometer used for measuring the air temperature. For a cooling power based on a pair of readings the standard error is 3·5 per cent., so that if the maximum error in the determination of $\theta$

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is $\pm 2{\cdot}5$ per cent. it will have comparatively little effect on the accuracy of the air velocity reading. The following table shows the accuracy of the thermometers required to ensure that the error is kept within these limits.

| Air temp. ° F. | $\theta$ | Maximum permissible error in temp. reading ° F. |
|---|---|---|
| 50 | 47·7 | 1·2 |
| 60 | 37·7 | 0·9 |
| 70 | 27·7 | 0·7 |
| 80 | 17·7 | 0·4 |
| 90 | 7·7 | 0·2 |

A 0·5° F. thermometer will be accurate enough for nearly all work, as it can be relied on to within $\pm 0{\cdot}25°$ F.

### *The error due to the difference between the air temperature and the surround temperature*

The air velocity formula for the ordinary kata assumes that the surrounding surfaces and the air are at the same temperature; but often this is not so, and it is necessary to consider the error introduced by this assumption. The ordinary kata cooling power can be divided into the heat loss due to convection ($Hc$) and the heat loss due to radiation ($Hr$). This radiation loss is given by

$$H_R = 1{\cdot}03\,(T_1^4 - T_0^4)\,10^{-9},[1]$$

where $T_1$ is the surface temperature of the bulb of the kata and $T_0$ is the surround temperature both in degrees Centigrade absolute.

Then, if $T_0$ changes by $\delta T$,

$$\delta H = 1{\cdot}03\,(-4T_0^3\delta T\,.\,10^{-9}) = -4{\cdot}12T_0^3\delta T\,.\,10^{-9}.$$

Hence when $T_0$ is

$$10°\text{ C.}\quad (50°\text{ F.})\quad \delta H = -0{\cdot}094\delta T,$$
$$15{\cdot}5°\text{ C.}\quad (60°\text{ F.})\quad \delta H = -0{\cdot}099\delta T,$$
$$21°\text{ C.}\quad (70°\text{ F.})\quad \delta H = -0{\cdot}105\delta T.$$

Taking 0·099 as the mean value of the constant and converting to the Fahrenheit scale, we get

$$\delta H = 0{\cdot}055\delta T°\text{ F.}$$

The following table has been calculated from this formula when the standard error of the cooling power is 3·5 per cent. and the maximum permissible error due to the surround temperature is $\pm 2{\cdot}5$ per cent.

| $H$ | $\delta H = 0{\cdot}025H$ | Maximum permissible value for $\delta T$ ° F. |
|---|---|---|
| 4 | $\pm 0{\cdot}10$ | $\pm 1{\cdot}8$ |
| 6 | $\pm 0{\cdot}15$ | $\pm 2{\cdot}7$ |
| 8 | $\pm 0{\cdot}20$ | $\pm 3{\cdot}6$ |
| 12 | $\pm 0{\cdot}30$ | $\pm 5{\cdot}5$ |

[1] This formula has been taken from the paper by T. Bedford and C. G. Warner (1933: The influence of radiant heat and air movement on the cooling of the kata-thermometer. *J. Hygiene*, **33**, 330–48), but $H_R$ has been substituted for $H_{RT}$, the emissivity $E$ has been put equal to 0·9, and the corresponding alterations made in the constant.

## CONCLUSION

It has been found possible to calculate the errors involved in the measurement of cooling powers and air velocities with a kata-thermometer when the instrument is used under moderately favourable conditions. The knowledge of the magnitude of these errors will provide a useful check on the validity of deductions made from kata readings.

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