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8. DISCUSSION

DR S. A. ZHEVAKIN described his theory of pulsating variables:

This theory [1–7] is based on the idea that stellar pulsations are self-oscillations, continuously maintained from internal sources of energy by means of the valve action of a critical zone of doubly-ionized helium. The theory can be applied to long-period variables of the RR Her type [8], to the β CMa variables, to semi-regular variables like AG Cyg and RS Cnc, and to the long-period o Cet stars. For all these types of stars the theory

1. explains the cause of the oscillations;
2. predicts the correct amplitudes;
3. predicts the correct relations between variations of radial velocity and of brightness;
4. gives the correct asymmetry of the radial-velocity variations;
5. explains why pulsations can occur only in giants and super-giants, and only in those which occupy a definite region in the Hertzsprung-Russell diagram;
6. in the case of the classical cepheids, leads to a $P-L$ relation with zero-point according to Baade and dispersion of about $1^m.0$.

We shall designate as the envelope the outer part of the star; its oscillations are not quasi-adiabatic. The envelope consists of a zone of critical double ionization of helium, which we take as the layer with $\gamma_1 - 1 = (d \ln T/d \ln \rho)_{ad} < 0.26$, and of the atmosphere above that zone. As the parameter of non-adiabaticity of the oscillations of the helium zone we adopt the ratio y_z of the non-adiabatic increase of temperature of this zone under compression to the adiabatic increase of temperature: $y_z = (\delta T/T)_{\mathrm{He\ nad}}/(\delta T/T)_{ad}$. We assume that the non-adiabatic temperature change is derived from the quasi-adiabatic approximation [2].

Computations were made for a series of stellar envelopes, with different values of the effective stellar temperature T_{eff}, of the acceleration of gravity in the envelope g, and with helium content 15–20% by number. These show that while the parameter y_z varies considerably with g and T_{eff} (it increases with g and T_{eff}), the ratio of mass of atmosphere m_a to the mass of the zone of critical ionization m_z remains almost unchanged, m_a/m_z varying from 0.88 to 0.93.

The computations have shown further that, if the opacity of the envelope is a prescribed function of density and temperature (determined by the chemical composition), the character of the non-adiabatic oscillations (i.e. the amount of phase shift between the variations of luminosity and of stellar radius, together with the ratio of the amplitudes of these two variations) is essentially determined by the parameter y_z and the ratio m_a/m_z. It is almost independent of other parameters.

Fig. 12 demonstrates the variation with y_z of the phase shift ψ between the epochs of maximum luminosity and minimum radius (full lines) and of the ratio of the flux leaving the star to the flux entering the ionized zone (dot-dash line). The dashed lines represent parts of the curves for which the full dissipation of the oscillation energy of the star is positive and which, therefore, cannot be realized. The curves were obtained from computations of four-layer spherical models of an envelope containing 85·7 % hydrogen and 14·3 % helium, for the three values of the ratio m_a/m_z, 0·822, 0·894 and 0·930. They are almost independent of details of the model, provided the composition is unchanged.

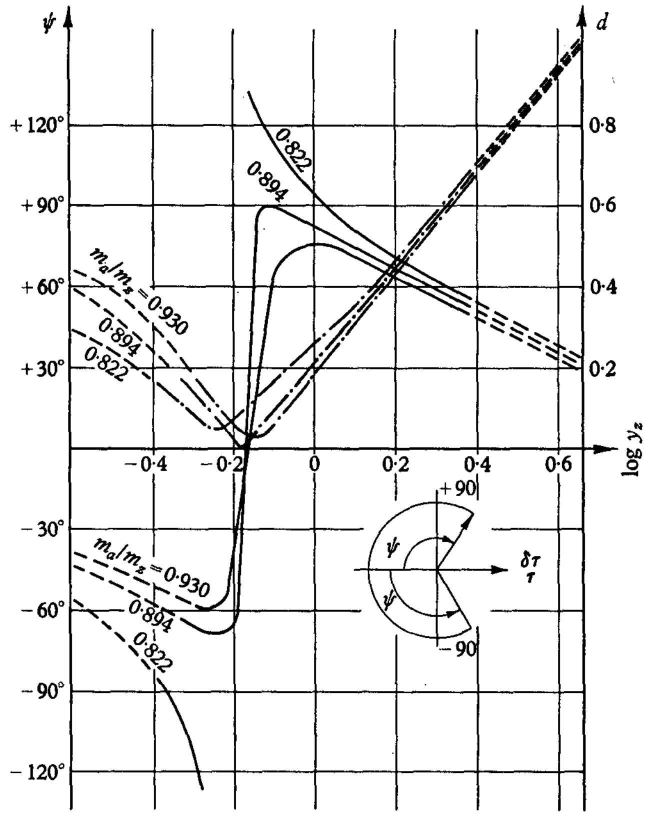

Fig. 12. Variation with the parameter y_z of ψ, the phase shift between epochs of maximum luminosity and minimum radius (full lines), and of d, the ratio of the flux leaving the star to that entering the ionized zone (dot-dash lines).

As we move along the variables of the 'great sequence' [11], from spectral class A to class M, we notice first that the acceleration of gravity in the atmosphere decreases; second, the zone of critical ionization of He II sinks into the denser layers of the star, and finally, the parameter y_z decreases, moving from right to left in Fig. 12. We then obtain the following sequence of stellar variability:

1. $\log y_z \leqslant 0{\cdot}40$—oscillations are absent;

2. $-0{\cdot}1 \leqslant \log y_z \leqslant 0{\cdot}40$—oscillations occur, with the phase shift of $+90°$ characteristic of cepheids and of the RV Tauri stars.

3. $-0{\cdot}25 \leqslant \log y_z \leqslant -0{\cdot}10$—the possibilities now depend on the values of m_a/m_z. For the value 0·894, ψ varies rapidly in this interval, and in consequence minor fluctuations in y_z will produce irregular changes of the phase ψ from $-90°$ to $+90°$, and also irregular three- to four-fold changes in the star's brightness. For values of m_a/m_z differing considerably from 0·894, ψ varies much less rapidly and fluctuations in y_z cannot produce

appreciable changes in ψ and d. These considerations can explain the semi-regular variables, and the RR Her and β CMa variables respectively.

4. $-0{\cdot}35 \leqslant \log y_z \leqslant -0{\cdot}25$—oscillations arise with phase shift $\psi \simeq -90°$, characteristic of the variables of the o Cet type.

5. $\log y_z < -0{\cdot}35$—no oscillations.

It should be emphasized that the above interpretation of semi-regular variability requires an additional, still unjustified, assumption that fluctuations exist in y_z (see however, [2] and [5]). No such assumptions are required to explain the variability of the cepheids or of the β CMa, RR Her or o Cet stars.

Fig. 12 shows that cepheid variability corresponds to a wide range of values of $\log y_z$: $-0{\cdot}1 < \log y_z < 0{\cdot}4$. The middle of this range determines the zero-point of the $P-L$ relation, and its width the dispersion in the relation [10]. If we assume that luminosity L is related to mass $\mathfrak{M}$ by $L \sim \mathfrak{M}^3$, the dispersion in luminosity is 0^m8; if the luminosity is independent of mass, the dispersion is 1^m25. The theory predicts also that high-luminosity cepheids will have smaller variations of brightness and will be bluer than cepheids of the same period and with smaller luminosity. This is in agreement with the results of Sandage for galactic cepheids [12].

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DR L. GRATTON: Work has been going on for some years at the Cordoba Observatory on various selected fields in both Magellanic Clouds. The aim of this work is mainly statistical, so that great attention has been paid to completeness. The results are now ready for one region in the Small Magellanic Cloud, for which twenty-eight pairs of plates were very thoroughly blinked.

About 450 new variables to a limiting magnitude of eighteen have been discovered. This almost duplicates the number of known variables in the region. A method was also derived for obtaining the frequency of periods without actually constructing the light-curves.

The most important result was the finding of a rather large number of stars with a period of less than one day; some of these are eclipsing binaries, but most are true cepheids. Several long-period variables have also been found.

As a result the frequency maximum has been shifted towards even shorter periods than was found by Shapley. It must be understood that the cepheids with periods less than one day found here are true cepheids almost two magnitudes brighter than the RR Lyrae stars with the same periods found in globular clusters.

This work is being carried on by Dr Landi-Dessy.

DR S. C. B. GASCOIGNE (comment on Irwin's paper): According to Petrie (*Astr. J.* **63**, 199, 1958) the companion to δ Cep has an absolute magnitude, estimated from the strength of Hγ, of $+0{\cdot}9$. This would give δ Cep the very faint median M_{pg} of $-0{\cdot}65$. There appears to be an anomaly in this system.

MLLE R. CANAVAGGIA (remark on the intrinsic colour of type C cepheids): Dr Irwin suggests that the type C cepheids are redder, intrinsically, than types A and B. It is possible to give some confirmation of this hypothesis from galactic cepheids.

1. α UMi: Six-colour photometry shows that α UMi has the same intrinsic colour as δ Cep at a phase of 0·20. This implies that α UMi is redder than δ Cep, at maximum, by $(B-V) \sim 0^m 11$.

2. I have studied spectro-photometrically δ Cep, η Aql and ζ Gem (method of M. Chalonge). For the three stars, at maximum light, it is possible to find the discontinuities: $D_{max} = +0{\cdot}40$, $+0{\cdot}35$ and $+0{\cdot}17$. Thus the discontinuity of ζ Gem at maximum is somewhat weaker than the cepheids of types A and B. This indicates that the spectral type is later.

(*Note.* According to Sandage (*Ap. J.* **127**, 513, 1958, Fig. 1) the type C cepheids are bluer than the type AB. *Editor.*)

DR O. A. MELNIKOV stressed the need for new methods of determining the zero-point of the $P-L$ relation independent of galactic absorption. He pointed out that if we know:

1. $M-M_\odot$, $R/R_\odot$, $T/T_\odot$;
2. $P(\rho/\rho_\odot)^{\frac{1}{2}} = Q$;
3. $g/g_\odot$, $\mathfrak{M}/\mathfrak{M}_\odot$,

it is possible to obtain in a semi-empirical way the value of Q as a function of ΔM_{pg} for the Shapley 1930 curve. The results are as follows:

Q	0·02	0·04	0·06	0·08	0·10
ΔM_{pg}	$-3^m 5$	$-0^m 5$	$+1^m 2$	$+2^m 5$	$+3^m 5$

A standard model with $Q \simeq 0{\cdot}04$ has a $\Delta M_{pg} \simeq -0^m 5$, agreeing with his own value (*Bull. Abastumani Obs.* **8**, 57, 1945; *Publ. Pulkovo Obs.* **74**, 1950) and supported by Kukarkin (*Variable Stars, Moscow*, **7**, 1949). For a model by Kosirev (*Publ. Crim. astrophys. Obs.* II, 3, 1948; VI. 54, 1951) in which Q is considerably greater, we obtain $\Delta M_{pg} \gg 0$.

DR M. W. FEAST: I should like very briefly to draw your attention to one or two results of recent work at the Radcliffe Observatory which have a bearing on the problem of the luminosity of cepheids.

1. Spectra of four cepheids in the SMC and six in the LMC with periods from 34^d–134^d have been obtained. There appears to be no spectroscopic evidence for their being other than normal classical cepheids. Spectroscopic work on Magellanic Cloud cepheids is being continued.

2. As was reported in one paper at the symposium on the Hertzsprung-Russell diagram, we have deduced from spectra and three-colour work, amongst other things, that the early-type super-giants in both Clouds show appreciable reddening. We find for these stars a total visual absorption of about $0^m 5$. Other evidence, such as Dr Wesselink's failure to discover background extra-galactic nebulae in the centre of the SMC, supports the conclusion that considerable amounts of dust exist in these systems. It will be recalled that the SMC has sometimes been considered more or less dust free, but this is contrary to our findings. Our findings would appear to have an important bearing on the question of the determination of absolute magnitude and colours for the cepheids from Magellanic Cloud studies.

3. In view of Dr Weaver's reference to a value for the Oort constant of 14 km/sec/kpc, it is necessary to point out that, in a recent analysis of radial velocities of distant B type stars by Dr Thackeray and myself, which incorporates much new observational material, we have deduced a value of 17·5 for this constant.

DR H. J. SMITH: Attention has been called to the non-homogeneous character of the RR Lyrae stars. The most striking example of this appears in the galactic intrinsic variables which have periods less than $0^d 2$. It can be shown that practically all of these objects have characteristics in common which suggest that another classification besides that of RR Lyrae may be more appropriate.

Specifically, these extreme short-period objects occupy a discrete, separated place in

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the period-colour and period-spectrum planes; they show a period-amplitude relation decreasing from about 1^m0 at 0^d5 to about 0^m2 at 0^d2. They have positions and motions in the Galaxy more characteristic of a disk than a halo; and they appear to have a very pronounced period-luminosity relation extending from about $M = +4$ for the shortest-period objects to $M = 0$ at 0^d2. Such low absolute magnitudes are consistent with the short periods, and accordingly the probable high density of the variables.

An abstract of this work has appeared (*Astr. J.* **60**, 179, 1955); more complete publication will follow.

DR G. M. IDLIS: From dynamic considerations, it is possible to deduce a relative distribution of the density of a typical spherical sub-system of the Galaxy along the *z* co-ordinate in the solar neighbourhood. The corresponding observed distribution of short-period cepheids is in good accord with this if their absolute magnitudes are, as indicated by Kukarkin, $M = +0^m5$.

z kpc	0·0	1·5	2·6	4·2	6·0	9·0
Theoretical	1·0	0·61	0·30	0·11	0·04	0·01
Observed	1·0	0·50	0·30	0·10	0·05	0·01

This would confirm the estimate $M = +0^m5$.

DRS K. BAHNER AND L. MAVRIDIS: Before photometric observations of cepheids can safely be used for the determination of individual distances, the problem of the supposed sub-classification must be solved. For this purpose precise light- and colour-curves are needed for as many galactic cepheids as possible. As a step in this direction, two-colour photo-electric observations have been made of the following stars with the 28-inch reflector at Heidelberg:

RT, RX, SY Aur; RW Cam; SU Cas; VZ, CD Cyg; V, X, Y, Z, RR, BG Lac; RS Ori; SV, AW Per; U Vul (data rather incomplete); TU Cas (many observations, light-curve variable).

About thirty-five observations have been made of each star. Reductions are preliminary, but the following conclusions may be drawn:

1. There are no differences between individual cycles for $P < 9^d5$ and only a few suspected cases at longer periods.

2. The light-curves of Y Lac, VZ Cyg, BG Lac, RR Lac, AW Per, RS Ori very closely resemble the mean light-curves as published for demonstration of the Hertzsprung relation. SU Cas and X Lac show symmetric light-curves and small amplitudes; RT Aur and V Lac asymmetric light-curves of large amplitude. The secondary maxima at the rising branch of RW Cam and CD Cyg ($P = 16^d4$ and 17^d1 respectively) are quite sharp rather than flat.

3. Cepheids with $P \approx 10^d$ may have sharp and narrow maxima (Z Lac, SV Per). Therefore: (*a*) Data from incompletely observed light-curves in this period range must be considered with caution; (*b*) a revision of Eggen's criterion ($M - m \lesssim 0^p37$) for the distinction between the AB and C cepheids seems necessary (compare, e.g. Z Lac and ζ Gem, which are both classified C by Eggen).

4. Since we observed no hump in the light-curve of BG Lac ($P = 5^d3$), the question arises whether the sub-division of the group AB in Eggen's groups A and B is simply a matter of the periods; no hump up to $P \approx 6^d$, hump for $P > 6^d$. Serious contradictions to this interpretation are only (from the data of Eggen *et al.*) in the light-curves of SS Sct and T Cru (which might be C, however).

5. Some differences up to 0^m15 exist between our measurements and those of Oosterhoff *et al.* and Eggen *et al.* for the stars in common.