next chapter: "our general goal of avoiding metric considerations as far as possible". All 17 groups are presented in terms of generators and relations; the chapter ends with a complete list and 17 diagrams (p. 75) illustrating the actions of the groups.

Chapter five describes the tesselations in Euclidean space E^n of n dimensions and S^n, the n-sphere in E^{n+1}; an inductive process steps up into E^{n+1} from established geometry in E^n. A reader could perhaps best ponder the procedure (pp. 94–97) of how to construct a 24-cell in E^4 by building upon a cube in E^3; less complicated examples precede this and a decidely more involved one follows it. No claim is made that polytopes assembled by these procedures are regular. One is again afforded a tantalizing foretaste (p. 93) of what could happen in hyperbolic space.

Two shorter chapters follow: "incidence geometry of the affine plane" and "projective geometry". There is nothing wrong with these, quite the contrary; one cannot have everything in a single monograph. But to one to whom "groups and geometry" immediately conjures up, say, a long vista of the multitude of results in projective geometry over finite fields these chapters would be a reminder of so much that is not in them.

Immediately following these topics one is launched on the engrossing trilogy alluded to above. Under the group M of inversions in E^2 the aggregate composed of all lines and circles is closed; its subgroup M^+, of index 2, that preserves orientation is isomorphic to the group of linear fractional transformations of a complex variable z and so is sharply triply transitive on the points of E^2: its operations are parabolic or loxodromic according as they have one or two fixed points, and its finite subgroups are isomorphic to the finite groups of rotations of S^2. The whole of M is realized when not only z but also its complex conjugate are used.

M^+ is transitive on the aggregate of lines and circles, so that the stabilisers H of these are conjugate subgroups: any line or circle may be used, two likely choices being (a) the real axis $y=0$, (b) the unit circle $|z|=1$. So use those bilinear transformations with *real* coefficients. These map $y=0$ onto itself, and do not transpose the two half planes $y>0$ and $y<0$ when, as has been prescribed, they preserve orientation (p. 161). So $y>0$ is a model of the hyperbolic plane, its *lines* being those circles orthogonal to and those lines perpendicular to $y=0$. A metric is introduced first (p. 173) on $x=0$ and then extended to the whole hyperbolic plane by using the necessary invariance under H; this leads to formulae for lengths and areas. The other model $|z|<1$ for the hyperbolic plane can also be used (p. 168). The last theorem of this penultimate chapter plays, we are warned, an important role in the final chapter: it concerns those points at equal (hyperbolic) distance from two separate points.

Fuchsian groups G are *discontinuous* subgroups of H; their study is based on their fundamental regions, and more particularly on Dirichlet regions which are not only fundamental but convex. Their treatment is illustrated by the modular group, triangle groups and Schottky groups. The Dirichlet regions provide a tesselation of the hyperbolic plane; the dual to this is a Cayley tesselation which is used to obtain a presentation for G. One can thence derive Poincaré's polygon theorem and, eventually, the classification of all G with compact fundamental region by their generators and relations (p. 205).

The book's value is enhanced by judiciously chosen selections of problems, and pertinent references occur at the ends of chapters. There is a bibliography of 35 entries.

W. L. EDGE

HALLENBECK, D. J. and MACGREGOR, T. H. *Linear problems and convexity techniques in geometric function theory* (Monographs and studies in mathematics, Vol. 22, Pitman, 1984), 182 pp. £26.50.

Let S denote the set of all functions $f(z)=z+\sum_{n=2}^{\infty} a_n z^n$, analytic and univalent in the unit disk $D=\{z:|z|<1\}$. Great interest has been shown for many years in extremal problems for S. But S is not a linear space and results from convexity theory, topological vector spaces and functional analysis are difficult to apply. If $A(D)$ denotes the functions analytic in D with the topology of local uniform convergence then S is a compact subset of $A(D)$. It is reasonable, therefore, to determine *extreme points* of S considered as a subset of $A(D)$, namely those $u \in S$ so that the equation $u=tx+(1-t)y$, $0<t<1$, has as its only solution in S the solution $x=y$, and the *support points* of S, namely those $v \in S$ which maximise the real part of some linear functional on S which is continuous with respect to the topology of $A(D)$.

The authors give an excellent and carefully thought-out presentation of the theory of extreme points and support points, not just for S but for various subclasses of S such as typically, real functions, starlike functions and close-to-convex functions. They explain clearly the relation between these problems and problems about subordination and H^p-spaces. This is a book which everyone interested in extremal problems in geometric function theory will wish to have.

Unfortunately, just as the book appeared, de Branges announced his solution of the celebrated Bieberbach conjecture stating that $|a_n| \leqq n$ for $f \in S$. This has raised enormous interest not just for the intrinsic importance of the problem, but also for the elegance and unexpected nature of the proof. Hamilton's proof that there exist support points which are not extreme points also came too late for inclusion. I fear that both of these developments will lead to a quickly diminishing interest in extremal problems for S. This would be a pity, since as the authors have so capably shown, the subject is still able to raise deep and interesting questions about univalent functions.

J. M. ANDERSON

HANCHE-OLSEN, H. and STØRMER, E. *Jordan operator algebras* (Monographs and Studies in Mathematics, Vol. 21, Pitman, 1984), 183 pp. £27.50.

Jordan algebras are a class of non-associative algebras first introduced by Jordan, von Neumann and Wigner about fifty years ago in connection with their studies on axiomatic quantum mechanics. For the next thirty years mainly the algebraic aspects of Jordan algebra theory was studied although some interesting links with other branches of mathematics were discovered and investigated. It is relatively recently that Jordan algebras arising in functional analysis have been studied. Initial work by E. Størmer and D. Topping was further developed by E. Alfsen, F. Shultz and E. Størmer to give the theory of classes of Jordan algebras which closely resemble that of C*-algebras and von Neumann algebras. These classes of algebras called JB-algebras and JBW-algebras include the original algebras of Jordan, von Neumann and Wigner and all C*-algebras. As this theory has now achieved a certain degree of completeness it seems an opportune time for the appearance of *Jordan Operator Algebras* which presents the theory of JB-algebras for the first time in monograph form.

The authors aim to present a complete self-contained account of the theory of JB-algebras, presupposing only basic results in functional analysis (concerning Hilbert spaces and Banach spaces) but no familiarity with Jordan algebras. The first chapter contains preliminaries in functional analysis and the second develops the algebraic theory of Jordan algebras required in the sequel. This chapter is in fact the longest in the book, partly because the reader is assumed not to have much knowledge on this topic and partly because some of the proofs are quite technical. All the results in this chapter have previously been published in, for example, *Structure and Representations of Jordan Algebras* by N. Jacobson, but their omission from this monograph would have given the reader quite a lot of difficulty in extracting the relevant material. The more introductory parts of the theory of JB-algebras and JBW-algebras are given in Chapters 3 and 4, for example, ideals, the centre, spectral theory, states and projections while more detailed analysis of specific algebras is given in Chapters 5 and 6, for example, equivalence of projections, and analysis of type I factors. The final chapter contains the main representation theorem together with one or two applications.

The book is clearly written, for example, not only are the technical proofs in Chapter 2 given in detail, but the main steps are separately indicated for the reader who does not wish to get bogged down in these details. In general it succeeds in being self-contained although at least some prior acquaintance with the theory of C*-algebras and von Neumann algebras would be an advantage. On the other hand, although the authors briefly indicate in the preface some of the applications of JB-algebras, this is not further expanded upon later in the book and so there is little indication given of future developments in the area other than as a generalisation of C*-algebra theory. These however are minor points and overall this monograph provides a clear concise introduction to the theory of JB-algebras.

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