THE LONG-TERM DYNAMICAL BEHAVIOR OF SMALL BODIES IN THE KUIPER BELT

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1. Introduction

Recent numerical calculations [1,2,3] have shown that Jupiter-family comets, which are on low inclination orbits, cannot originate from the gravitational scatter of long-period comets. Work by Quinn, Tremaine & Duncan [1] shows that objects originally on low-inclination, Neptune crossing orbits will evolve into a population of objects with orbital parameters consistent with those of Jupiter-family comets. However,they point out that the timescale to deplete this initial population of planet-crossing objects is short. Therefore, they conclude there must be a system of objects that are evolving into planet-crossers on the timescale of the age of the solar system. The most likely source of these objects is a region just beyond the orbit of Neptune, the *Kuiper belt.*

In order to complete this theory, it is still necessary to show that objects that formed in the Kuiper belt can become Neptune-crossers and to determine the timescale for this process. For if the length of time to deplete the Kuiper belt is too short then it can no longer be the source for the short-period comets seen today. If the length of time is too long then it will be difficult to reproduce a large enough flux of new comets to explain the number of observed short-period comets. Unfortunately, because the timescales involved must be on the order of the age of the solar system, it is not possible to determine them through the use of direct numerical integrations of orbits with current computer technology. In this paper we calculate the timescales of the evolution of objects in the Kuiper belt using a new technique that treats the evolution of orbits in integral space as a diffusion problem.

2. Technique

The technique employed in this paper is a new member of a class that treats the dynamical evolution of small gravitationally noninteracting objects within the solar system as a diffusion problem in integral space. We divide the two-dimensional integral space of perihelion (q) and aphelion (Q) distance into small bins. For each bin, we numerically integrate the orbits of 100 particles that initially start in that bin for approximately 60 orbital periods. From the behavior of the particles, we can calculate the probability that a particle will cross a bin boundary in a particular length of time represented by Δt. Thus, we have a matrix $\mathbf{P}$, such that the ij^{th} element is the probability of a particle moving from bin i to bin j in time Δt. This matrix is a member of a well studied type known as a *Markov chain.* It is possible

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to calculate the long term behavior of the dynamical system of interest by directly studying this matrix.

In order to calculate the probability matrix, we must have an understanding of the short term evolution of orbits within the bins. To accomplish this, we numerically integrate the orbits of 100 objects in each bin and follow the changes in q and Q for each object. We integrate the orbits of the particles in three dimensions under the gravitational influence of the Sun and the four jovian planets, but the particles themselves are not gravitationally interacting with each other. The initial values of q and Q for the particles are chosen from a uniform distribution and were constrained to lie within a bin.

We can directly calculate the values of P_{ij} from our empirical distribution of $\dot{q}$ and $\dot{Q}$. P_{ij} is the probability that a particle that starts in bin i will enter bin j in a set amount of time, $\Delta t = \frac{1}{2\pi}\, yrs$. We need to add two special bins to act as the boundary condition for the problem. Particles enter these bins when they leave the system we study, and never return. We represent this in our probability matrix by assigning $P_{ii} = 1$ and $P_{i(j \neq i)} = 0$ for these boundary bins. Here, the inner edge is at $q < 30AU$ (the object becomes a Neptune-crosser). We arbitrarily set the outer edge at $a = 100AU$. Reference [4] discuss effects of changing the outer boundary on our results.

The most basic characteristic of the Markov chain, $\mathbf{P}$, is that it allow us to numerically integrate the behavior of the system. If we define a vector $\mathbf{n}$ so that the i^{th} element of $\mathbf{n}$ is the number of particles in bin i, then $\mathbf{n}(t_j) = \mathbf{P}^j\mathbf{n}(0)$. To integrate the system for the age of the solar system then $j = t/\Delta t = 3 \times 10^{10}$. This is not as impossible as it may seem because $j \approx 2^{35}$. Thus, to get the probability matrix that relates that distribution of particles after 4.5×10^9 years to their initial distribution, we square $\mathbf{P}$ then square the square of $\mathbf{P}$ and so on, 35 times.

However, it is possible to determine the lifetimes of particles by directly studying $\mathbf{P}$. Define the matrix $Q_{ij} = P_{ij}$ for all i and j that are not boundary bins. The average number of time steps a particle spends in bin j before it leaves the system if it started in bin i is $\mathbf{M} = (\mathbf{I} - \mathbf{Q})^{-1}$. Thus, if $\mathbf{t} \equiv || \, t_i \, ||$ is the average number of time steps a particle spends in the system before it is absorbed if it started in bin i, then $t_i = \sum_j M_{ij}$.

To summarize, it is possible for us, by using Markov chains, to determine the long term behavior of a dynamical system by simple matrix algebra.

3. Results

The details of our results depend on what region of the solar system is studied. Here we define the Kuiper belt as the region of integral space such that $q \geq 30AU$ and $a \leq 100AU$. Objects tend to diffuse through this region on timescales that are on the order of the age of solar system. These diffusion times imply that it is very unlikely to find an object near to where it formed. This is illustrated in Figure 1, which shows the expected surface density distribution after 4.5×10^9 years for objects that start in circular orbits with $a = 40AU$ ('A') and $a = 70AU$ ('B'). Both distributions fill the entire region being studied. Note that many of the objects that started at $40AU$ moved outward in the solar system.

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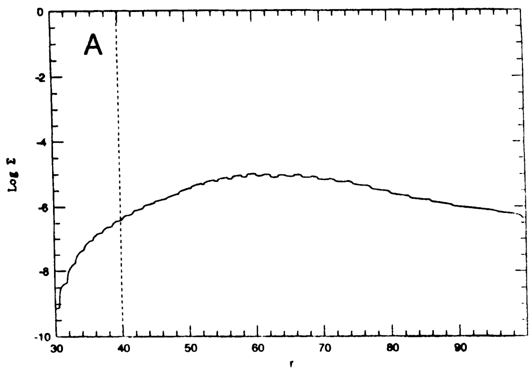

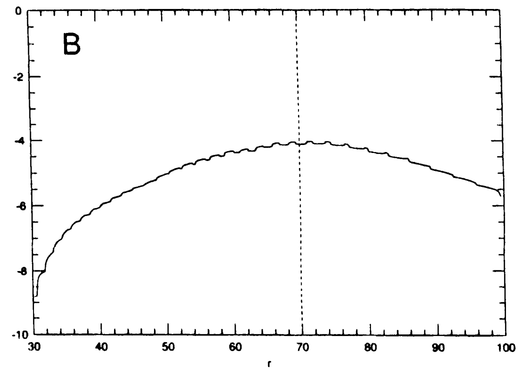

Fig. 1. The surface density as a function of distance from the Sun (r). The solid curve represents the surface density of the Kuiper belt after 4.5×10^9 years if the initial distribution (dashed curve) were a delta function. A) All particles are initially on circular orbits at $40AU$. B) All particles are initially on circular orbits at $70AU$.

A large fraction of integral space is covered with orbits whose lifetimes are on the order of age of the solar system. Here, *lifetime* is defined as the statistically mean length of time it takes for an object to evolve out of the Kuiper belt ($q < 30AU$ or $a > 100AU$). Figure 2 is a contour plot of the expected lifetime of particles as a function of their initial q and Q. The solid contour represents the positions where the expected lifetime is equal to the age of the solar system, 5×10^9 years. The dotted curves are contours of lifetimes less than the age of the solar system and the dashed curve is the contour of 10^{10} years. The longest lifetime in the system is 1.8×10^{10} years. It occurs for a circular orbit at $a = 75AU$. Approximately 30% of these objects become Neptune-crosses and thus provide a source for Jupiter-family comets. The rest are stored in orbits further out in the solar system.

Even objects that form close to the orbit of Neptune have a significant chance to evolve to orbits with $a > 100AU$. For example, objects that formed in circular orbits at $45AU$ have a 50% chance of evolving to orbits with $a > 100AU$. However,

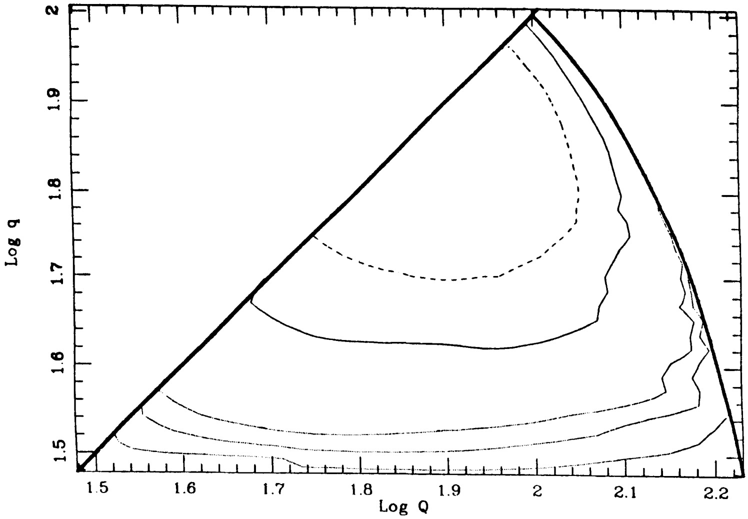

Fig. 2. A contour plot of the mean length of time that a particle spends in the region studied in Run 4 as a function of its initial perihelion and aphelion distances. The thick solid curves represent the boundaries of $q < Q$ and $a < 100AU$. The solid contour is 5×10^9 years. The dashed contour is 10^{10} years. The three doted contours are 10^8, 5×10^8 and 10^9 years.

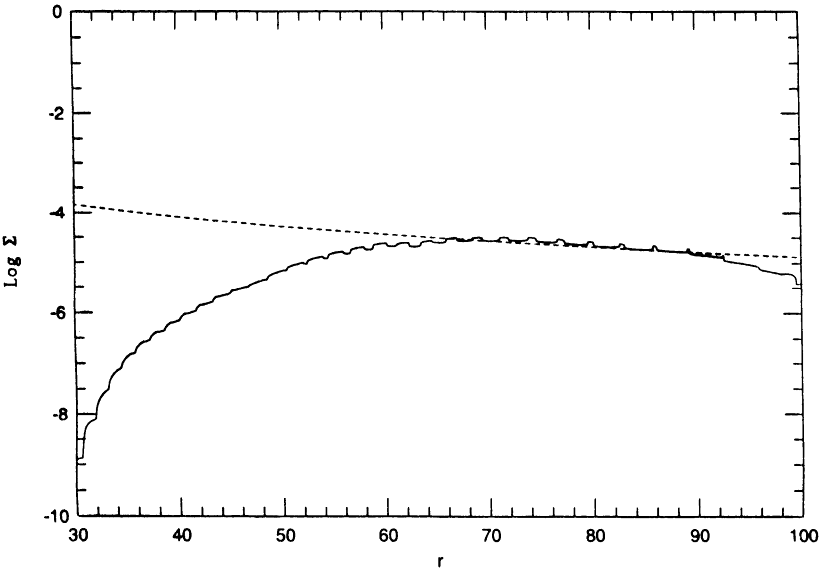

Fig. 3. The surface density as a function of distance from the Sun (r). The solid curve represents the surface density of the Kuiper belt after 4.5×10^9 years if the initial distribution (dashed curve) followed a power law $\Sigma \propto r^{-2}$.

objects stored in this region of the solar system are not precluded from becoming short period comets. It is possible that after being stored for some time they can diffuse back through the system to become Neptune-crossers. Indeed, a significant fraction of objects follow this evolutionary track. They formed near the orbit of Neptune and slowly evolved to orbits with $a > 100AU$. After being stored there for some time (say, approximately 5×10^9 years), they can diffuse back through the Kuiper belt and become Neptune-crossers. Thus, a significant fraction of objects that formed near the orbit of Neptune can currently be evolving into Jupiter-family comets.

If we assume an initial surface density distribution, Σ, we can calculate the current distribution of mass. For example, if initially, $\Sigma \propto r^{-2}$, then Figure 3 shows the current values of Σ. The units are such that the initial total mass in the Kuiper belt was 1. This predicts that the density in the Kuiper belt peaks at about $70AU$. Unfortunately, this result implies that the Kuiper belt will be much harder to detect observationally.

The shape of current surface density distribution is not a strong function of the initial conditions. Compare the current Σ calculated from the power law (Figure 3) to those derived from delta functions (Figures 1A and 1B). Even in the extreme cases the shapes of surface density distributions look similar.

It is also possible to determine the number of comets in the Kuiper belt with this model. We find that

$$N_{kb} = 6 \times 10^9 (\frac{10000\, yrs}{L}),$$

where L is the average lifetime of a Jupiter family comet. If the average mass of a comet is $10^{-11} M_\oplus$ then the mass of Kuiper belt is $0.06(\frac{10000 yrs}{L})M_\oplus$.

References

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