### Regarding the Raven Paradox

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### 1. Background

In this paper I take Hempel's raven paradox as the claim that statements of the form ' $\sim$ Ru v Bu', 'u is not a raven or u is black,' confirm the hypothesis h '(x)(Rx -> Bx)', 'All ravens are black.' <sup>1</sup> Although Hempel discusses this using a criterion of confirmation expressed wholly in terms of deductive logic (see 1965, pp. 35-9), it has become more common to articulate criteria of confirmation using concepts of probability and, in particular, to employ the positive relevance criterion of confirmation which says that, given background knowledge k, (i) e confirms h if and only if P(h/e.k) > P(h/k); (ii) e disconfirms h if and only if P(h/e.k) > P(h/k) and (iii) e is irrelevant to h if and only if P(h/e.k) = P(h/k).

The positive relevance criterion of confirmation plays a major role in the quantitative argument (see Swinburne 1973, pp. 156-8) which contends that (i) both 'Ru.Bu' and '~Ru.~Bu' confirm 'All R is B', (ii) 'Ru.Bu' confirms this hypothesis to a much greater degree than does '~Ru.~Bu', and (iii) the degree to which '~Ru.~Bu' confirms 'All R is B' is so slight as to account for our intuition that '~Ru.~Bu' does not confirm this hypothesis at all. The background knowledge k assumed is (i) the ratio of Rs to ~Rs in the world is x:1-x; (ii) the ratio of Bs to ~Bs in the world is y:1-y; and (iii) x<<1-y. The quantitative argument considers the relations between P(e/k) and P(e/k.h) where 'e' is replaced by each of the claims 'Ru.Bu', 'Ru.~Bu', '~Ru.Bu', and '~Ru.~Bu'. Since P(e/k.h)/P(e/k) = P(h/e.k)/P(h/k), considering the relations of P(e/k.h) and P(e/k) enables us to determine whether e confirms, disconfirms, or is irrelevant to h according to the positive relevance criterion. Given that the ratio of Rs to ~Rs is x:l-x, that the ratio of Bs to ~Bs is y:l-y, and this is our only relevant information: (i) P(Ru/k)=x; (ii) P(Bu/k)=y; (iii) P(~Ru/k)=l-x; and (iv) P(~Bu/k)=l-y. Assuming that being a B (or a ~B is probabilistically independent of being a R (or a ~R) given k: (i) P(Ru.Bu/k)=xy; (ii) P(-Ru.Bu/k)=y(1-x); (iii) P(Ru.-Bu/k)=x(1-y); and (iv) P(-Ru.-Bu/k)=(1-x)(1-y). Since h is 'All R is B', it follows that P(Ru.~Bu/k.h)=0. Since P(Bu/k.h.Ru)=1 and P(Ru.Bu/k.h) =P(Bu/k.h.Ru)P(Ru/k.h), P(Ru.Bu/k.h)=P(Ru/k.h.). Assuming that Ru and h are independent given k. the proponents of this argument assert that P(Ru/k,h)=P(Ru/k) and conclude that P(Ru, Bu/k,h) =x. By a parallel series of steps, including the assumption that P(-Bu/k.h) = P(-Bu/k), the proponents of this argument conclude that P(-Ru.-Bu/k.h) = $P(\sim Bu/k) = 1-y$ . Hence,  $P(\sim Ru.Bu/k.h) = y-x$ , because  $P(Ru.Bu/k.h) + P(Ru.\sim Bu/k.h) +$ P(Ru.-Bu/k.h) + P(Ru.Bu/k.h) = 1. Assuming that 0 < x << 1-y, the proponents conclude:

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Conclusions (1) and (4) imply that both 'Ru.Bu' and ' $\sim$ Ru. $\sim$ Bu' confirm 'All R is B'. Using a comparative concept of confirmation according to which e confirms h more than e' confirms h if and only if (P(e/h.k)/P(e/k)) > (P(e'/h.k)/P(e'/k)), (5) implies that 'Ru.Bu' confirms the hypothesis much more than does ' $\sim$ Ru. $\sim$ Bu'. Conclusion (2) implies that 'Ru. $\sim$ Bu' disconfirms 'All R is B'-as expected. However, (3) implies that ' $\sim$ Ru.Bu' slightly disconfirms the hypothesis 'All R is B' and this result conflicts with both Hempel's claim that it is confirmatory (see 1965, p. 15) and with a common intuition that this claim is irrelevant to this hypothesis.

This argument raises several difficulties. First, as Swinburne has noted, the assumption that we have background knowledge regarding the ratio of Rs to ~Rs and of Bs to ~Bs in the world is hardly realistic as a general assumption about our epistemic condition ((1973)), p. 159). Second, this solution can apply only to cases where the assumption 0 < x < 1 - y is plausible. Third, on a concept of comparative confirmation saying that e confirms h more than e' confirms h if and only if (P(e/h.k) - P(e/k)) > (P(e/h.k) - P(e/k)), we cannot conclude that 'Ru.Bu' confirms h more than does '~Ru.~Bu' because P(Ru.Bu/k.h) - P(Ru.Bu/k.h) - P(Ru.Bu/k.h) - P(~Ru.~Bu/k.h) - P(~Ru.~Bu/k.h)

Horwich argues that the quantitative argument fails to consider that there is a significant difference between: (i) picking out an object at random and discovering it to be a black raven ('Ru.Bu'); (ii) selecting by method a known raven, examining its color, and observing that it is black ('Ru\*Bu'); and (iii) selecting by method a known black thing and observing that it is a raven ('Bu\*Ru'). Horwich says that these differing methods of observing black ravens generate different items of evidence which do not confirm the hypothesis 'All R is B' equally. Horwich expects that the best evidence is given by 'Ru\*Bu', i.e., where a known raven is examined and found to be black, because in such cases the hypothesis h is subject to the maximum risk of falsification and has passed the most severe test. 'Bu\*Ru', on the other hand, does not test the hypothesis because the method of searching among known Bs to find if there are any that are also Rs will not disclose a counterexample to h (1982, p. 58).

To develop his argument, Horwich proposes a set of probabilities which we formulate as:

SP:	P(Ru.Bu/k)=a	P(Ru.Bu/h.k)=e#
	P(-Ru.Bu/k)=b	$P(\sim Ru.Bu/h.k) = f$
	P(Ru.~Bu/k)=c	$P(Ru.\sim Bu/h.k)=0$
	P(~Ru.~Bu/k)=d	P(~Ru.~Bu/h.k)=g

Horwich describes these as probabilistically coherent, i.e., a+b+c+d=e#+f+g=1. Given SP, he derives  $P(h/Ru.Bu.k) = \{e\#/a\}P(h/k)$  and  $P(h/\sim Ru.\sim Bu.k) = \{g/d\}P(h/k)$ . He claims that as there is no reason to say either(i) e#/a > 1 or (ii) g/d > 1 or (iii) e#/a > g/d, there is no reason to say either that (i) 'Ru.Bu' confirms h or (ii) ' $\sim Ru.\sim Bu'$  confirms h or (iii) 'Ru.Bu' confirms h more than ' $\sim Ru.\sim Bu'$  confirms h (1982, pp. 59-60).

Horwich offers the following equations EQ which are derivable from the probability calculus using SP and appropriate definitions of 'P(Ru\*Bu/h.k)', 'P(Ru\*Bu/k)', etc. I interpret Horwich's use of the '\*'-notation to mean that 'P(Ru\*Bu/k)=P(Bu/Ru.k)', 'P(Ru\*Bu/h.k)=P(Bu/Ru.h.k)', etc. are true by definition.

EQ:	P(h/Ru*Bu.k)	=	${(a+c)/a} P(h/k).$
	P(h/Bu*Ru.k)	=	$e#/a$ {(a&b)/(e#+f)}P(h/k)
	$P(h/\sim Bu*\sim Ru.k)$	=	${(d&c)/d}P(h/k)$
	P(h/~Ru*~Bu.k)	=	${g/d}{(d+b)/(g+f)}P(h/k)$
	P(h/Bu*~Ru.k)	=	${f/b}{(a+b)/(e#+f)}P(h/k)$
	P(h/~Ru*Bu.k)	=	${f/b}{(b+d)/(f+g)}P(h/k)$

Horwich argues that since the values a, b, c, d, e#, f, and g are constrained only by the requirement of probabilistic coherence, the equations EQ do not allow us to say either that (i) P(h/Ru\*Bu.k)>P(h/k) or (ii) P(h/~Bu\*~Ru.k)>P(h/k) and so do not allow us to say that 'Ru\*Bu' and '~Bu\*~Ru'\* confirm h. He adds that if we suppose that  $d \subseteq I$  as a part of our background knowledge and thus that a << d, we obtain the results that (i) 'Ru\*Bu' confirms h more than does '~Bu\*~Ru' because  $\{(a+c)/a\}>>\{(d+c)/d\}$  and (ii) '~Bu\*~Ru' only weakly confirms h because  $\{(d+c)/d\} \subseteq I$ . Construing the paradox as the claim that '~Ru.~Bu' confirms h more than does 'Ru.Bu', Horwich claims that this solves the raven paradox ((1982), p. 61).

This purported solution of the raven paradox generates some significant problems. First, Horwich assumes not only that (i)  $d \ge 1$  and (ii) a << d but also (iii) a > 0 and (iv) c > 0. These assumptions cannot be supported by appealing to background knowledge regarding the ratio of non-black non-ravens to black ravens. Second, plausible assumptions regarding the ratio of black ravens to non-black non-ravens do not lead to a general conclusion regarding confirmatory vs non-confirmatory evidence claims. Third, Horwich's subjectivism is too weak to sustain his intuitions regarding confirmation. Since the values of a, b, c, d, e#, f and g are said to be constrained only by the requirement of probabilistic coherence, a rational individual is free to choose values according to which 'Bu\*Ru' confirms h more than 'Ru\*Bu' does, despite his claim that 'Ru\*Bu' is expected to provide better evidence for h than does 'Bu\*Ru'. Fourth, Horwich's subjectivism makes it impossible to say, in general, that any of the evidence claims 'Ru\*Bu', 'Bu\*Ru', '~Ru\*~Bu', '~Bu\*~Ru', 'Bu\*~Ru' and '~Ru\*Bu' are or are not irrelevant or confirmatory or disconfirmatory with respect to h on the basis of the positive relevance criterion. He provides no other criterion of confirmation. Fifth, if, on Horwich's treatment 'Ru.Bu', '~Ru.Bu', '~Ru.~Bu', and 'Ru.~Bu' count as evidence with respect to h and he makes the plausible assumption that c>0, as I have suggested, it is difficult to see how he avoids the raven paradox in the form stated at the beginning of this paper. If c>0, then since P(~Ru v Bu/k)= $P(Ru.Bu/k)+P(\sim Ru.Bu/k)+P(\sim Ru.\sim Bu/k)=1-c$ ,  $P(\sim Ru \vee Bu/h.k)=1$ , and  $P(h/(\sim Ru \vee Bu).k) = 1/(1-c)P(h/k)$ , ' $\sim Ru \vee Bu$ ' confirms h.

## 2. A Modest Proposal

Adopting Horwich's equations SP and EQ with some further interpretation, I attempt to avoid the problems of his subjectivism by moving to an intersubjectivist account of confirmation. This move is not simply *ad hoc* or convenient, but reflects the view that scientific inquiry is intersubjective. Before presenting the intersubjective account of confirmation and a solution to the raven paradox, I propose several plausible assumptions and some conceptual and notational changes.

First, if a given hypothesis z is not confirmable, then the raven paradox does not arise in connection with it. According to the positive relevance criterion of confirmation, a form of which I adopt, no confirmable hypothesis has a prior probability (given background knowledge k alone) of either 0 or 1. Hence, for any hypothesis z, if the raven paradox can arise in relation to z, then 0 < P(z/k) < 1. This is not to deny that some hypotheses have a prior probability equal to zero or equal to 1, but to say that the consideration of these hypotheses is irrelevant to the discussion of the raven paradox. Drawing upon the equations SP, we may then say that 0 < c < 1 is a requirement of our discussion, for if c = 1, then P(h/k) = 0 and h is neither confirmable nor disconfirmable on the positive relevance criterion and if c = 0, then P(h/k) = 1 and h is not confirmable on our criterion.

Second, in common mathematical discourse, division by zero is said to be "undefined". If the equations EQ are to be taken as meaningful, we must assume for our discussion that each of a, b, d, e#+f, and g+f has a value > 0. This is not to claim that it is impossible that, e.g., a = 0. Such cases are logically possible but irrelevant to the discussion of the raven paradox. I adopt the symbol 'AV' as an abbreviation for the assumption that each of a, b, c, d, e#+f, and g+f has a value > 0 and also that c < 1.

Third, the literature on the paradoxes of confirmation contains several arguments which may be interpreted as holding that generalizations such as h are not necessarily confirmed by their positive instances. Good (1967, p. 322), Rosenkrantz (1977, pp. 33-35), and Swinburne (1973, pp. 164-6), have shown that, given certain background information k, positive instances such as 'Ru.Bu' may disconfirm h. The role of background knowledge is illustrated by Swinburne's example that the hypothesis 'No monkey is exactly six feet in height' may be disconfirmed by so-called positive instances. Given the similarity of monkeys to other primates, some of which are exactly six feet in height, data which describes instances of monkeys taller than or shorter than six feet may be taken to disconfirm this hypothesis. A parallel point may be made with regard to the contrapositive of the monkey hypothesis and data describing non-monkeys exactly six feet in height.

Swinburne has revised the quantitative argument presented earlier. Taking  $k_0$  as our empirical background evidence, he proposes the probabilities (which I restate using 'h', 'Ru', 'Bu', etc. as above):

$$P(Ru.Bu/k_0) = xy$$
  $P(Ru.Bu/h.k_0) = x_1$   
 $P(Ru.~Bu/k_0) = x - xy$   $P(Ru.~Bu/h.k_0) = 0$   
 $P(~Ru.Bu/k_0) = y - xy$   $P(~Ru.Bu/h.k_0) = y_1 - x_1$   
 $P(~Ru.~Bu/k_0) = (1-x)(1-y)$   $P(~Ru.~Bu/h.k_0) = 1 - y_1$ 

Aside from claiming that  $x > x_1$  and  $y_1 > y$  and recognizing the requirement of probabilistic coherence, Swinburne imposes no further constraints on the permissible values of x, y,  $x_1$ , and  $y_1$ . So, on this revision of the quantitative argument, 'Ru.Bu', '~Ru.Bu', and '~Ru.~Bu' may confirm, disconfirm, or be irrelevant to h. Swinburne claims that these consequences reflect the point-that the instances 'Ru.Bu', '~Ru.Bu', etc. are not

obtained in the pursuit of any method or policy of investigation but are simply chance observations. Rejecting the supposition that we are commonly concerned with the confirming effect of chance observations, Swinburne proposes to change the formulation of confirming instances to reflect the method or policy of investigation q which is employed. Swinburne proposes to write 'P(e/k,q)' rather than 'P(e/k,k)' and 'P(e/h.k,q)' rather than 'P(e/h,k)', where 'q' describes the method or policy of investigation. Swinburne's shift in formulation reflects a shift toward the Popperian notion that confirmation of a hypothesis only occurs in the context of an attempt to falsify the hypothesis. It is worth nothing that thinkers as diverse as Suppes (1966, pp. 199-200) and Watkins (1964, pp. 435-8) have emphasized the role of testing in attempting to confirm laws. Popper has stated this repeatedly and has, at times, required not only that confirmation requires attempted falsification but that the attempted falsification employ the most severe test we can devise (see 1968, p. 36 and p. 240).

It seems to me that a requirement of testing, i.e., of following a method or policy q the purpose of which is to find counterexamples to h is a sufficient requirement for confirming evidence. A minimum requirement for such a method is that it imply or presuppose that c>0. Severity of testing is discussed below in relation to degrees of confirmation.

Horwich's '\*'-formulae may be used to express Swinburne's point more perspicuously than Swinburne's own formulation which retains the usual data claim forms 'Ru.Bu' etc.. It is clear that in, e.g., 'Ru\*Bu', 'Ru' indicates that a method or policy of investigating known Rs is employed. Since there may be several methods or policies of investigating known Rs, I follow Swinburne and specify the method by the symbol 'q<sub>n</sub>', where  $n \ge 1$ 0. The formula 'P(Ru\*Bu/h.k.q<sub>n</sub>)'and its definitional equivalent 'P(Bu/h.k.Ru.q<sub>n</sub>)'are to be understood as 'the probability that u is a B, given that u is an R obtained by a method of testing q<sub>n</sub> among known Rs'. Since I want to avoid absurd cases where an R is obtained by a method that implies or presupposes that no Rs exist, I exclude such cases as impermissible by requiring that the epistemic condition expressed by the symbol immediately to the left of the '\*' operator be consistent with the method q<sub>n</sub>. In permissible formulas, q<sub>n</sub> is to be understood as a method of investigation among items that satisfy the epistemic condition. Moreover, I exclude as improper data claim forms such as '(Ru v ~Ru)\*(Ru.Bu)', since 'Ru v ~Ru' is universally applicable for formal reasons and following a so-called method or policy of investigating known items that satisfy 'Ru v ~Ru', or any other tautologous condition, amounts to no method at all. I extend this prohibition to cases where the predicate to the left of the '\*' symbol, indicating the method or policy employed, is equivalent syntactically or semantically to a tautologous condition such as 'Ru v ~Ru'. Hence, 'Ru\*Bu', 'Bu\*Ru', "~Ru\*Bu', 'Bu\*~Ru', 'Ru\*~Bu', '~Bu\*Ru', '~Ru\*~Bu', and '~Bu\*~Ru' are permissible forms of data claims.

I propose that, for a policy of investigation  $q_n$  with a value of c>0, understood as part of background knowledge k, and where e is one of the permissible '\*'-forms of data: (i) e N-confirms h if and only if  $P(h/k.e.q_n) > P(h/k.q_n)$  for all choices of values for variables a through g consistent with AV and the requirement PC of probabilistic coherence; (ii) e N-disconfirms h if and only if  $P(h/k.e.q_n) < P(h/k.q_n)$  for all choices of values for a through g consistent with AV and PC; and (iii) e is N-irrelevant to the confirmation of h if and only if e neither N-confirms nor N-disconfirms h. Reinterpreting SP and EQ with 'k' replaced by 'k.q\_n' throughout, 'Ru\*-Bu' and '-Bu\*-Ru' each N-disconfirm h, if  $P(h/k.q_n) > 0$ ; if  $O < P(h/k.q_n) < 1$ , 'Ru\*-Bu' and '-Bu\*-Ru' each N-confirm h; and the remaining '\*'-forms in EQ are N-irrelevant to h. This is an intersubjective criterion of confirmation in that N-confirmation is independent of the particular valuations of individuals, but not of the range of possible valuations.

Popper (1968, p. 391) has proposed two measures of the severity of test e as supporting evidence for h given background knowledge k which, taking  $c = P(\sim h/k)$ , we may express as follows:

S(e,h,k) = (P(h/e,k)+c-1)/(P(h/e,k)+1-c)S'(e,h,k) = P(h/e,k)/P(h/k) = P(h/k,e)/(1-c)

On either measure, the severity of a given test e as supporting evidence for h increases with increasing values of c. Equations EQ imply that, with increasing values of c, data claims of the forms 'Ru\*Bu' and '~Bu\*~Ru' confirm a given h to a greater degree even if a and d are constant.

Following Swinburne (see 1973, p. 170)), I add the requirement to adopt a policy of investigation  $q_n$  for which c has the highest value compatible with 0 < c < 1, if there is a choice of policies with differing values of c. Let us call this the requirement of the maximal value of c. Adopting a policy with some lower value of c would mean that our best evidence would confirm the hypothesis under investigation less strongly than it could under some available alternative policy. This result is particularly undesirable in the context of the comparison of competing hypotheses with respect to their relative degree of confirmation on a given body of evidence e.

The N-confirmation criterion explains not only why a non-black non-raven may constitute confirming evidence for 'All ravens are black' but also why a black non-raven fails to confirm this hypothesis. Moreover, on the plausible assumption that there are many more non-black non-ravens than there are black ravens, i.e., d >> a, this criterion accounts for our common sense notion that ravens found to be black confirm the hypothesis more than do non-black things found to be non-ravens.

The N-confirmation criterion applies not only to extensional accidental generalizations such as 'All ravens are black', but also to non-extensional, subjunctive conditionals used to express lawlike sentences such as LS 'For all x and all t, if x were gold at time t, then x would be malleable at time t'. Using the symbol '->-' for the subjunctive fork developed by Fetzer and Nute, we symbolize LS as '(x)(t)(Gxt ->- Mxt)'. According to Fetzer, LS implies both the extensional generalization AG '(x)(t)(Gxt -> Mxt)' and DG '(x)(t)(~Gxt v Mxt)' (see 1981, pp. 152-7).

Demonstrating that the fork operator '->-' is not subject to transposition, Fetzer seems to think that the non-transposibility of lawlike subjunctive generalizations such as LS allows him to avoid the raven paradox (1981, pp. 192-4). Although Fetzer does not develop a theory of confirmation for accidental generalizations, he does say that such generalizations are confirmed by their instances (1981, p. 190 and p. 256). The reasoning employed in this confirmation seems, on Fetzer's view, to be a form of argument to the best explanation which he describes as a likelihood inference. He proposes a converse consequence condition with regard to such likelihood inferences, saying that any evidence which confirms h by a likelihood inference also confirms any hypothesis that entails h (1981, pp. 268-9). Since '~Gyu v Myu' confirms DG and LS entails DG, it appears that Fetzer's view is open to the raven paradox.

The N-confirmation criterion blocks this route to the raven paradox by denying that claims of the form '~Gyu v Myu' are proper evidence claims for DG, AG or LS. The N-confirmation view also provides a positive account of evidence for LS, viz., the same account offered for AG or DG. That is, (i) 'Gyu\*~Myu' and '~Myu\*Gyu' each N-disconfirm LS, if  $P(LS/k.q_n)>0$ ; (ii) 'Gyu\*Myu' and '~Myu\*~Gyu' each N-confirm LS, if  $0<P(LS/k.q_n)<1$ ; and (iii) the other permissible '\*'-forms whose counterparts are listed in EQ are N-irrelevant to LS.

The N-confirmation criterion is limited in at least two significant ways because it is a view of the confirmation of accidental generalizations and of lawlike sentences by individual cases. First, the N-confirmation criterion provides no account of the confirmation of

laws by other laws or by accidental generalizations. Second, it does not offer a complete treatment of the confirmation of statistical hypotheses of the form 'r% of A is M', where 0<r<100. If one construes such statistical hypotheses in terms of generalizations attributing single case propensities of strength r, i.e., as 'All As have an M-propensity of strength r', then these hypotheses have falsifying cases and the N-confirmation criterion is applicable if it is known that individuals have or lack the propensity. However, the determination that a given (member of) A has or lacks an M-propensity of strength r would seem to require the consideration not of single cases but of sets or runs of cases, except in the trivial condition that the strength r is universal or null.

#### Note

<sup>1</sup>In this paper, (i) the symbol '-> stands for the truth-functional 'if-then', (ii) 'Ru.Bu', 'Ru.~Bu' etc. are used to stand for any instances of the forms 'Ru.Bu', 'Ru.~Bu' etc., and (iii) 'Ru.Bu' is to be understood as implying and as implied by 'Some R is B'.

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