

But, I confess, I shall be very much disappointed if I am suffered to monopolize this department of the *Magazine*.

I am Sir,
Your most obedient servant,

Camden Town,
21st February, 1865.

P. GRAY.

ON THE TABLES OF DEFERRED ANNUITIES PUBLISHED BY
THE NATIONAL DEBT OFFICE.

To the Editor of the Assurance Magazine.

DEAR SIR,—In August, 1861, I drew the attention of your readers to the remarkable discrepancy which exists between the true premiums, as deduced from the Government Tables at 3 per cent, and those charged by the Government on the purchase of those deferred annuities in which the premiums are “returnable,” either on death or at the option of the purchaser at any time prior to the commencement of the annuity, pursuant to 16 & 17 Vict., cap. 45.

I am now induced to revert to the subject, for two especial reasons; the first being, that these premiums are, as I am informed, computed at $3\frac{1}{4}$ per cent., and not at 3 per cent. as assumed in my last letter, whereby the difference is *greater* than I had then stated; the second, because I did not then give the very simple method by which those premiums may be deduced from the materials furnished by the tables themselves—nor, in fact, as far as I am aware, has any method of deducing premiums returnable at the option, as well as on the death, of a purchaser, been hitherto published in any work on life annuities.

The problem then is, to find the single premium for an annuity during the remainder of a life (x) after n years, with the condition that the premium is “returnable,” without interest, on death or at the option of the purchaser at any time prior to the commencement of the annuity.

As the premium (P_x) is repayable at any time during the term (n), but without interest, it must be considered from two points of view; firstly, as a sum held on trust to be ready whenever called for; and, secondly, as a fund yielding an annual income which (not being repayable under any circumstances) is to be applied year by year, during the term, in the purchase of an annuity deferred for n years; but at the expiration of the term of n years, the condition as to the return of P_x having ceased, it must itself be applied to the purchase of an immediate annuity on the life at its then increased age of ($x+n$) years.

Let P_x = single premium “returnable” for an annuity of £1;

i = interest on £1 for a year;

p_{x-n} = the annual premium payable at the end of the year for assuring to x a deferred annuity of £1 after n years;

a_{x+n} = annuity on a life aged ($x+n$);

then $P_x \cdot \frac{i}{p_{x-n}}$ = the amount of deferred annuity which can be assured by the conversion of the annual interest into an annual premium,

and $P_x \cdot \frac{1}{a_{x+n}}$ = the amount of annuity which can be obtained by sinking P_x at the end of the term.

By addition,
$$P_x \left(\frac{i}{p_{x:\overline{1}|}} + \frac{1}{a_{x+n}} \right) = 1,$$

$$\therefore P_x = \frac{1}{\left(\frac{i}{p_{x:\overline{1}|}} + \frac{1}{a_{x+n}} \right)}$$

By an easy transformation to the columnar notation,

$$P_x = \frac{N_{x+n}}{(N_x - N_{x+n})i + D_{x+n}}$$

As in the tables which we are now considering, $p_{x:\overline{1}|}$ (being $= \frac{a_{x:\overline{1}|}}{a_x - a_{x:\overline{1}|}}$)

and a_{x+n} are given, the following table was obtained with great facility, from which it will be seen that in the extreme case of an annuity to 31 after 50 years, the premium charged by the Government is more than *three and a half times* the correct amount.

Deferred Annuities of £30, Males, Single Premiums, returnable without Interest at any time prior to commencement of Annuity, pursuant to 16 & 17 Vict., cap. 45.

Age at Entry.	Term.	Government Premiums.	True Premiums, computed on Data of Tables at 3½ per Cent.	Difference between Government Premiums and true Premiums.	Error per Cent.
21	After 10 years	£ 403·250	£ 394·650	£ 8·600	2·179
"	" 20 "	261·750	242·670	19·080	7·863
"	" 30 "	156·875	133·611	23·264	17·411
"	" 40 "	89·625	62·622	27·003	43·120
"	" 50 "	44·375	21·864	22·511	102·959
31	After 10 years	360·250	352·800	7·450	2·112
"	" 20 "	216·	199·467	16·533	8·289
"	" 30 "	121·375	96·429	24·946	25·869
"	" 40 "	61·125	34·881	26·244	75·238
"	" 50 "	25·250	6·923	18·327	264·726
41	After 10 years	297·375	290·646	6·729	2·315
"	" 20 "	167·125	146·634	20·491	13·974
"	" 30 "	84·125	56·013	28·112	50·188
"	" 40 "	34·750	11·554	23·196	200·761
51	After 10 years	230·	220·467	9·533	4·325
"	" 20 "	115·875	91·128	24·747	27·156
"	" 30 "	47·875	20·550	27·325	132·968

The results given in the foregoing examples are so startling, that I have been induced to prepare a second table, showing what proportion of the whole annuity which is to be entered upon at the end of the term is assured by the yearly application of the interest on the single premium, and what proportion is yielded by sinking the single premium at the end of the term.

Age at Entry.	Term.	Annual Income from Interest on Single Premium.	Deferred Annuity which Annual Interest will assure.	Annuity which Single Premium will purchase at the end of the Term.	Sum of last two Columns, being Total Benefit secured.
(x).	n.	$P_x \cdot i.$	$P_x \cdot \frac{i}{p_x \cdot n}$	$P_x \cdot \frac{1}{a_{x+n}}$	
21	10 years	12·826	8·676	21·324	£30
"	20 "	7·886	15·327	14·673	"
"	30 "	4·342	20·209	9·791	"
"	40 "	2·035	24·073	5·927	"
"	50 "	·710	27·012	2·988	"
31	10 years	11·466	8·666	21·334	£30
"	20 "	6·482	15·385	14·618	"
"	30 "	3·134	20·867	9·133	"
"	40 "	1·133	25·233	4·767	"
"	50 "	·225	28·337	1·663	"
41	10 years	9·446	8·701	21·299	£30
"	20 "	4·765	16·080	13·920	"
"	30 "	1·820	22·344	7·656	"
"	40 "	·375	27·224	2·776	"
51	10 years	7·165	9·119	20·881	£30
"	20 "	2·961	17·545	12·455	"
"	30 "	·667	25·063	4·937	"

If not trespassing too much on your valuable space, I should like to give one practical example in illustration of the previous observations.

Let us then suppose the case of a man aged 25, possessed of a capital of £47. 18s. 4d., which yields him an annual income (at $3\frac{1}{4}$ per cent.) of £1. 11s. 2d.; either, or both, of which he is desirous of devoting to a provision for old age.

On turning to Table No. 2 (money returnable), he will find that his capital is exactly sufficient to provide an annuity of £20 after 41 years, with the option of having his money returned at any time prior to the commencement of the annuity.

But on turning to Table No. 3 (money not returnable), he will see that, by devoting the yearly income of £1. 11s. 2d., he can assure an annuity of £24. 18s. 8d., instead of £20, and not part with his capital at all; and he will further discover, on looking to the table of immediate annuities, that if he wish to part with his money at the end of the term he can purchase a further annuity of £5. 7s. 9d., making together £30. 6s. 5d., instead of £20—a result which will be found to be fully confirmed by my previous deductions.

I have now practically shown that a person can secure a deferred annuity under Table No. 3, greater than he can assure under Table No. 2 by 51·6 per cent., on precisely the same security and at precisely the same expenditure in both instances, with the additional advantage that in the former case he can at the end of the term exercise an option against the Government, should his then state of health render such a course desirable; but even this does not fully describe all the disadvantages which he suffers under Table No. 2, as circumstances, other than his state of health, may render it necessary for him to divert his capital into some other channel at the end of the term; should this be so, he will under Table No. 2 have secured

nothing in the shape of an annuity, whilst under Table No. 3 he will receive £24. 18s. 8d. per annum for the remainder of his life. In this illustration I have disregarded the provision of the Act, that no person can assure an annuity of more than £30, and also any rule which may exist as to insuring fractions of a pound, as these can have no bearing on the question before us; but it may, perhaps, be as well to point out that, as in the Government tables, the premiums for deferred annuities are, of course, payable at the beginning, whilst in the formula which I have given they are payable at the end of the year, the assurance under Table No. 3 must be made at the age of $x + 1$ for $n - 1$ years.

In conclusion, allow me to draw your attention to the table for deferred annuities (money not returnable), which has this week been laid before Parliament. In it are given tables of immediate annuities and tables of deferred annuities; with these materials, the values of annuities for terms of years are immediately obtained, with the following results:—

Age.	MALES—VALUE OF AN ANNUITY OF £1 FOR THE TERM OF																							
	10 Years.		12 Years.		14 Years.		16 Years.		18 Years.		20 Years.		22 Years.		24 Years.									
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.						
20 to 21	7	18	9	2	9	10	4	8	11	4	8	12	3	0	12	19	9	13	15	0	14	8	11	
30 „ 31	7	19	4	9	3	5	10	5	5	11	5	5	12	3	9	13	0	5	13	15	6	14	9	2

in all of which cases it will be seen that the value of a life at 31 is *greater* than that of a life at 21; and similarly it may be shown that the value of an annuity for 10 years is precisely the same at the ages of 21 and 39.

I hope, on a future occasion, to be allowed to prove that the annual premiums payable under the “money returnable” scale are open to the same objections, although not to the same degree, as those which I have urged against the single premiums under a like condition.

I am, dear Sir,

Yours truly,

J. W. STEPHENSON.

London, 25th February, 1865.

MR. SPRAGUE'S LETTER IN THE LAST NUMBER OF THE JOURNAL.

To the Editor of the Assurance Magazine.

SIR,—It was my intention to have asked you to be good enough to insert a reply, which I have written at some length, to the letter of Mr. Sprague which appeared in the last Number of the *Magazine*. But some of my friends, on whose judgment I place great reliance, who have seen the manuscript, urge me to withdraw it, on the ground that the affairs of particular Societies ought not, under any circumstances, to be discussed in the *Journal of the Institute*. For this reason, and for this reason only, I abstain from entering into further controversy on the subject.

I have the honour to be,

Sir,

Your obedient servant,

ARTHUR H. BAILEY.

10th March, 1865.