THE PRINCIPLE OF EQUIVALENCE AND THE TROJAN ASTEROIDS. PART II

R.B. ORELLANA and H. VUCETICH
Observatorio Astronómico de La Plata
Paseo del Bosque - (1900)La Plata
Argentina

Abstract. A new value for the Nordtvedt parameter and the mass of Saturn are computed using the ten first Trojan asteroids. From 1262 observations, we find the inverse mass of Saturn 3498.17±0.51 and the Nordtvedt parameter 0.0±0.3.

Key words: relativity - asteroids - gravitation - astronomical constants

1. Introduction

In a previous work (1988), we obtained a limit to possible violation of the Principle of Equivalence (PE) using the photographic observations of the first six Trojan asteroids. The Principle of Equivalence is the basis of general relativity. It states that a freely falling reference frame is locally equivalent to an inertial frame (Einstein, 1911). One of its consequences is the Universality Free Fall (UFF). It is usual to formulate UFF as the equality of inertial and passive gravitational masses. In order to clarify the former statement, let us write the energy of a test body, in the non-relativistic aproximation as (Haugan, 1979).

$$L = -m_R c^2 + \frac{1}{2} m_I \mathbf{v}^2 - m_P \phi(\mathbf{x}) + O\left(\frac{v^4}{c^2}\right). \tag{1}$$

Nordtvedt (1968) proved that theories of gravitation alternatives to general relativity predic a difference between inertial and passive gravitational masses; the so called Nordtvedt effect. The Nordtvedt result can be parametrized in the form

$$\Delta = \frac{m_P - m_I}{m_R} = \eta \frac{\Omega}{m_R c^2}, \tag{2}$$

where η, the Nordtvedt parameter, is a linear function of the PPN parameters (Will, 1981). For fully conservative theories

$$\eta = 4\beta - \gamma - 3. \tag{3}$$

Now, we extend the performed analysis including the next four Trojan asteroids.

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 185–188.

2. Analysis of Asteroids

2.1. SELECTION OF ASTEROIDS

The perturbations induced by Saturn are very large and slowly varying. Hence, an error in the value of the Saturn mass can originate a spurious shift in the Lagrange equilibrium point. For a mass correction, an observation period of 60 years is required. By this reason, only the ten asteroids listed in Table I were admitted.

TABLE I

Observational parameters. The columns show the number and name of the asteroid, accepted number and time span covered by observations.

Asteroid	Span	No. Obs.
(588)Achilles	1906-1990	155
(617)Patroclus	1906–1989	114
(624)Hector	1906–1990	210
(659)Nestor	1908–1990	90
(884)Priamus	1917-1989	93
(911)Agamenon	1919–1984	121
(1143)Odysseus	1930–1990	164
(1172)Aneas	1930–1987	124
(1173)Anchises	1930–1989	80
(1437)Diomedes	1937–1989	111

2.2. OBSERVATIONS

Complete observational sets for each asteroid were collected from several sources before 1950, and from the MPC sheets onwards. All high precision observations were included, and reduced to astrometric position. Observations times were converted to julian dates (JD), and reduced to ephemeris time. The light travel time, the E-terms of aberration, and the parallax correction were applied in computations.

2.3. EQUATIONS OF MOTION

Heliocentric equations of motion, referred to the mean equinox and equator of 1950.0, including the perturbations from the five outer planets, were used for the Trojan asteroid and the outer planets. The variational equations (including the perturbations of Jupiter) included corrections for the initial conditions and perturbations by the Nordtvedt effect and a variation in the mass of Saturn. The inner

https://doi.org/10.1017/S0074180900091130 Published online by Cambridge University Press

planets were treated through the augmented solar mass technique and other relativistic corrections were not included, since they are much smaller than the level of noise.

2.4. Equations of condition

Since we want to find a correction to the mass of Saturn, to the values of Nordtvedt parameter and the initial values of the asteroid our conditional equations included partial derivatives for these parameters. Finally, we must consider the systematics errors in the star catalogues and so we include, three additional parameters in the conditional equations (Shapiro, 1968).

3. Results

The equation of motion of the outer planets and of the asteroid were numerically integrated for the observational period, together with the variational equations for the asteroid and Jupiter. A standard fifth order predictor-corrector routine with a five day step was used. The initial conditions for each asteroid were taken from the "Ephemeris for Minor Planets 1991" for the epoch 2448600.5 JD, and were refined through differential correction to yield a reference orbit. Starting with this orbit, several fits were made for each asteroid including the Nordtvedt parameter and the mass correction. In order to analize the effect of systematic errors on the physical parameters due to star catalogues, simultaneous fits of the observations of all asteroids were performed.

4. Conclusions

In this preliminary research we have found, improved values for the mass of Saturn and the Nordtvedt parameter, from an analysis of ten Trojan asteroids. This value improves the results obtained in (1988), both by increasing the number of asteroids and the span of observations. Table II shows the final results. Since systematics errors my be lurking around however, this result must be considerer a preliminary one.

TABLE II
Final results of the physical parameter.

$\frac{M_\odot}{M_S}$	η
3498.17±0.51	0.0±0.3

https://doi.org/10.1017/S0074180900091130 Published online by Cambridge University Press

Acknowledgements

R.B.O. would like to thank the International Astronomical Union and Dr. S. Ferraz Mello for hospitality and financial help.

References

Einstein, A.: 1911, *Ann. Phys.* **35**, 898
Haugan, M.: 1979, *Ann. Phys.* **118**, 156
Nordtvedt, K.: 1968, *Phys. Rev.* **169**, 1014
Orellana, R.B. and Vucetich, H.: 1988, *Astron. Astrophys.* **200**, 248