A REVIEW OF THE DIFFERENT LIQUID CORE MODELS USED FOR THE COMPUTATION OF THE DYNAMICAL EFFECTS ON NUTATIONS AND EARTH TIDES

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Note: This short review was given during the Symposium at the request of a number of participants to serve as an elementary introduction to the discussion of the results obtained from different Earth models. It was not prepared in advance and is not an original contribution.

1 - Elementary Models for the Core

The first approximation of the figure of a fluid planet is obtained by assuming hydrostatic equilibrium with respect to its gravitational self-attraction and the centrifugal force. When the speed of rotation is not too fast, the equipotential surfaces may be considered to an excellent approximation as ellipsoids of revolution. It is easy to show that these hydrostatic equipotential surfaces are surfaces of equal density (ρ).

Clairaut obtained under these conditions a differential equation of the second order describing the flattening (e) of the equipotential surfaces as a function of their radial distance (r) from the center of mass. In the last century, when no information was available from seismology, great efforts were made to investigate the internal constitution of the Earth on the basis of the Clairaut equation. In particular, Roche showed that the equation was integrable if one chose a law for the distribution of density of the form

$$\rho = \rho_o(1 - a\ r^2) \quad (\rho_o\text{: density at the centre}) \tag{1}$$

which evidently has no experimental basis.

Radau introduced a new parameter [$\eta = (r/e)(de/dr)$] which proved to be useful in the analytical development of the Clairaut equation. One has indeed the relation

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$$1 - \frac{2}{5}\sqrt{1+\eta} = \left(\frac{C-A}{C}\right)^{-1} \left(e - \frac{q}{2}\right) \tag{2}$$

with $q = \omega^2 a^3/GM$.

This parameter varies between two limits:

$\eta = 0$ for a perfectly homogeneous fluid
$\eta = 3$ for an envelope with a dimensionless heavy particle at its centre.

As examples, at the surface of the Earth $\eta = 0.576$, while for Saturn $\eta = 1.711$ (at the surface).

At the time Jeffreys and Vicente (1957a,b) and Molodensky (1961) published their work on the dynamic effects of the earth's liquid core, very little information was available concerning the density of the core. Thus they had no other alternative than to use the ellipticities derived from the Clairaut equation and adopt either

(1) a homogeneous core,
(2) a core with a central particle, or
(3) a core with a Roche law of density.

With some justification, they felt that if these crude but very different models provided similar numerical values for the Love numbers there was a good probability that any more realistic and sophisticated model should not give a much different result. Very recent results based upon much more realistic models (Shen and Mansinha, 1976) seem to prove that this assumption was right.

2 - Models for the Mantle - The Choice of Variables

Jeffreys and Vicente adopted for the mantle a solution obtained by Takeuchi in 1950. This was calculated with a model consisting of numerical values of the density (ρ) and the elastic moduli (λ,μ) given for 12 points in the mantle, the last nine being separated by 300 km in radial distance. For computing the displacements, Takeuchi had to calculate the first derivatives of the quantities ρ, λ, and μ with respect to the radial distance, which was obviously a critical operation. A more suitable choice of variables (Molodensky, 1953; Alterman, Jarosch and Pekeris, 1960) made it possible to avoid this operation, but the Jeffreys and Vicente developments have not been recalculated with this new formulation. Molodensky, however, did use this approach. Note that the difference is wholly one of formulation; the two approaches are mathematically identical.

3 - Adams Williamson Condition

Until very recently there was another limiting characteristic in the models used for the liquid core. It has been noted that when the equations of elastic equilibrium where reduced to the case of a liquid body (by putting the rigidity μ equal to zero), a relation between pressure changes and density was implied for the zero frequency case ($\omega_i = 0$):

$$\frac{1}{\lambda}\frac{dp}{dr} - \frac{1}{\rho}\frac{d\rho}{dr} = 0 \quad . \tag{3}$$

This relation, which had been assumed by Adams and Williamson in 1923, states that the change in density inside the core only depends upon the change of pressure with radial distance. It implies adiabaticity and chemical homogeneity inside the core.

The first distribution of density in the Earth constructed by Bullen in 1936 was based upon the fulfillment of the Adams-Williamson condition in the different regions of the mantle and a Roche law. Pekeris and Accad (1972) have considered cores where

$$\frac{1}{\rho}\frac{d\rho}{dr} - \frac{1}{\lambda}\frac{dp}{dr} = \beta(r)$$

with $\beta(r) < 0$ (stable stratification)
$\beta(r) > 0$ (unstable stratification)

as well as

$\beta(r) = 0$ (neutral equilibrium or Adams-Williamson condition fulfilled).

The cores in the Jeffreys-Vicente and Molodensky models are Adams-Williamson cores. This is not assumed in the Shen and Mansinha models, but their results concerning the nutations are not appreciably different from those of Molodensky.

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