# COLLECTIVE PROPERTIES OF A MAGNITUDE-LIMITED SAMPLE OF CATACLYSMIC BINARIES

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ABSTRACT. Results of an analytical discussion of observational selection in a magnitude-limited sample of cataclysmic binaries (CBs) are presented. It is shown that the dependence of the strength of selection in favour of massive white dwarfs, of the mean orbital inclination and the fraction of eclipsing systems in a sample on the limiting magitude $m_V$ reflects the large-scale distribution of CBs in the galaxy and the influence of interstellar absorption. Furthermore it is shown how selection in favour of massive white dwarfs depends on the mass radius relation of white dwarfs.

## 1. INTRODUCTION

Recently, Ritter and Burkert (1985; 1986, paper I) and Ritter and Özkan (1986, paper II) have studied observational selection among cataclysmic binaries (hereafter CBs) in favour of massive white dwarfs. For this purpose they investigated the properties of a V-magnitude-limited sample of those CBs that have a bright quasi-stationary accretion disk, i.e. of dwarf novae during maximum light and of nova-like systems of the UX UMa type. In these computations it was assumed that the accretion disk is the only relevant source of luminosity. The accretion rate that determines the accretion luminosity was thereby obtained from a simple model of the secular evolution of CBs (see e.g. Ritter, 1986a). One major result of these computations was that restriction to a magnitude-limited sample results in an enormous selection effect which is ultimately a consequence of the mass-radius relation of white dwarfs. In fact the selection is strong enough to account fully for the high mean white dwarf mass of $\langle M_{WD} \rangle \approx 1\ M_\odot$ observed among the double-lined spectroscopic CBs (see e.g. Ritter, 1984; 1986c). Furthermore, because of the strong selection effect, the intrinsic mass spectrum of the white dwarfs in CBs need not be significantly different from that of single white dwarfs.

In addition to this it was found that certain properties of a magnitude-limited sample of CBs such as the strength of selection, the mean orbital inclination and the fraction of eclipsing systems depend in an unexpected way on the limiting magnitude $m_V$. In order to inves-

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tigate the cause of this $m_v$-dependence we have solved analytically the equations of the selection problem given in papers I and II. For this we have considered two limiting cases for the spatial distribution of CBs and have treated the influence of interstellar absorption approximately. Specifically we have derived analytical solutions in the following four cases. Case 1: isotropic spatial distribution of CBs with a constant space density and ignoring interstellar absorption; case 2: as case 1 but including interstellar absorption; case 3: isotropic distribution of CBs in a thin disk with constant surface density, interstellar absorption ignored; case 4: as case 3 but including interstellar absorption.

## 2. RESULTS

### 2.1. The Selection Function

The strength of the selection effect in favour of massive white dwarfs is expressed in terms of the selection fucntion $S_1$. $S_1$ is the ratio of the distribution function of the white dwarfs' masses in the magnitude-limited sample $p_o(M_1)$ and the intrinsic white dwarf mass distribution function $p_i(M_1)$, i.e. $S_1 = p_o(M_1)/p_i(M_1)$. Our analytical computations yield the following properties of $S_1$:

- $S_1$ is proportonal to $(M_1/R_1)^\nu$, where $M_1$ and $R_1$ are, respectively, the mass and the radius of the white dwarf. The power $\nu$ depends on the case considered. It is $\nu = 3/2$, $\approx 3/4$, $= 1$ and $\approx 1/2$ in the cases 1,2,3 and 4 respectively. Since $M_1/R_1$ is a steeply increasing function of $M_1$, the factor $(M_1/R_1)^\nu$ completely dominates the selection. In fact it has already been found from numerical experiments in paper I that the selection virtually disappears if $M_1/R_1$ is artificially taken to be constant.
- In each of the four cases studied, $S_1$ does not depend on the limiting magnitude, in apparent contradiction to results obtained in paper I (see Figure 6, in paper I). We shall come back to this point in section 3 where we show why, nevertheless, selection can depend on $m_v$.
- $S_1$ depends relatively weakly on the mass transfer rate $\dot{M}_1$ and thus on the rate of loss of orbital angular momentum $\dot{J}$ that drives the mass transfer. Numerical experiments performed in paper I (see Figure 1) have shown that selection is in fact insensitive to variations in $\dot{J}$.

### 2.2. The Mean Orbital Inclination

The mean inclination $\bar{i}$ considered here is defined in the following way. Let $N(i)$ denote the total number of systems in a magnitude-limited sample of CBs that all have the same orbital inclination $i$. Then $\bar{i}$ is defined via the equation $N(\bar{i}) = N(0^\circ \leq i \leq 90^\circ)$, where $N(0^\circ \leq i \leq 90^\circ)$ is the total number of systems in the i-integrated magnitude-limited sample of CBs with randomly distributed orbital inclinations. Ignoring limb-darkening of the accretion disk the analytical approximations yield $\bar{i}$ = 57°.12, 61°.69, 60° and 63°.61 in the cases 1,2,3 and 4 respectively. Furthermore we find that in each of the four cases $\bar{i}$ does not depend on $m_V$.

## 2.3. The Fraction of Eclipsing Systems

If we denote by $i_{ecl}$ the minimum inclination above which eclipses occur, the fraction of eclipsing systems in a sample is given by $f_{ecl} = N(i>i_{ecl})/N(0° \leq i \leq 90°)$. Since $i_{ecl}$ depends on the parameters of a system, i.e. on the masses of the components (see paper II), the notation $N(i>i_{ecl})$ is symbolic because $i_{ecl}$ is defined for a system but not for a sample. However, in order to make an estimate, we may assume that $i_{ecl}$ is the same for all systems. In this case the analytical solution yields $f_{ecl} = (\cos i_{ecl})^{\mu}$ with $\mu$ = 5/2, 7/4, 2 and 3/2 in the cases 1,2,3 and 4 respectively. Taking $i_{ecl}$ = 65° as an estimate we obtain $f_{ecl}$ = 0.116, 0.222, 0.179 and 0.275 in the four cases 1,2,3 and 4 respectively. Again we find that $f_{ecl}$ does not depend on $m_V$ in any of the four cases studied in apparent contradiction to numerical results (cf. Table 2 in paper I).

# 3. DEPENDENCE ON THE LIMITING MAGNITUDE

In all of the four cases considered, neither the selection function $S_1$ nor the mean inclination $\bar{i}$ and the fraction of eclipsing systems $f_{ecl}$ depend on the limiting magnitude $m_V$ in apparent contradiction to the results obtained in papers I and II. However, we find that the strength of the selection is quite different in each of the four cases studied. Thus selection can depend on $m_V$ if the different cases apply to different ranges of $m_V$. When we consider the approximations made in the four cases we realize that case 1 is relevant to the immediate solar neighbourhood, i.e. to the brightest systems, while case 4 represents the situation for the most distant and faintest systems. Cases 2 and 3 are intermediate. Thus, if we follow selection as a function of increasing $m_V$, the sample is drawn from an increasingly larger part of the galaxy. The relevant approximations as a function of increasing $m_V$ are thus case 1, case 2 or 3 and finally case 4. Whether case 2 or 3 is relevant depends on which effect dominates first, interstellar absorption or the transition to disk geometry. In the limit of very high $m_V$ the reservoir of CBs in the galaxy will be exhausted and the volume-limited sample will be approached. In this way it may be understood as to why the selection function becomes flatter with increasing $m_V$ and why $\bar{i}$ and $f_{ecl}$ increase with $m_V$. It is the influence of interstellar absorption and of the large-scale distribution of CBs in the galaxy that determines how selection depends on $m_V$.

# 4. CONCLUSIONS

The analytical solutions of the selection problem discussed above show that the dependence of the selection effect on the limiting magnitude $m_V$ reflects the large-scale distribution of CBs in the galaxy and the

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influence of interstellar absorption. Furthermore our computations show

- that the dependence of the selection function $S_1$ on $M_1$ is completely dominated by the mass-radius relation of white dwarfs.
- that the mean inclination $\bar{i}$ of a magnitude-limited sample is slowly increasing with $m_V$. However, for all practical purposes it is always close to 60°.
- that the fraction of eclipsing systems $f_{ecl}$ is increasing with $m_V$. For the brightest systems $f_{ecl} \approx 0.1$ and thus significantly smaller than that observed (see e.g. Ritter, 1984; 1986c). This must be due to the fact that the probability of detecting an eclipsing binary is intrinsically higher.

A more detailed account of this work will be given elsewhere (Ritter, 1986b).

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