The Rise of a Magnetic Flux Tube through the Radiative Envelope of a 9 M_{\odot} Star

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Abstract. We explore the possibility that the magnetic field generated by a dynamo at the interface between the convective core and radiative envelope of a massive star can be transported to the surface by buoyancy.

1. Application of solar flux tube theory to massive stars

Among the topics receiving attention at the present meeting has been the possible role of magnetic fields in producing some of the phenomena observed in the atmospheres and envelopes of Be stars. In the case of the Sun, the causal connection between magnetic fields and activity in the solar atmosphere and surface layers is well-established. In this paper, we examine one aspect of the conjecture that hot stars, particularly Be stars, might possess surface magnetic fields whose ultimate source is a hydromagnetic dynamo located deep within the stellar interior. It is widely thought that the magnetism of massive stars is primordial in origin. Here we assume that a magnetic field can be generated by a dynamo that operates in the vicinity of the interface between the convective and radiative portions of the stellar interior, similar to what is believed to occur in the Sun (Parker 1993; MacGregor & Charbonneau 1997; Charbonneau & Mac-Gregor 1997). Our primary concern is how the generated fields are transported over the considerable distance between the site of the dynamo and the surface of the star. Resistive effects alone are incapable of accomplishing this, since simple estimates indicate that for stars more massive than a few M_{\odot} , the diffusion time across the radiative envelope exceeds the main sequence lifetime. We herein briefly explore the alternative possibility that the fields are transported to the surface in fibril form under the action of the buoyant force.

In the Sun, magnetic flux emerges from the photosphere in the form of fibrils or flux tubes. The field is thought to assume this form in or near the dynamo domain at the bottom of the convective envelope. If the formation of flux tubes is a process that is not specific to the fields contained inside the Sun, then the possibility exists that a dynamo generated field at the interface between the convective and radiative zones in a hot star, might be configured in a similar way. In this case, the buoyant force might likewise enable flux from deep in the interior to reach the surface in a time scale that is shorter than the evolutionary time scales.

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There is an important structural distinction between the Sun and massive stars. In the Sun, the outer envelope of the interior is convective, while for massive stars it is radiative. It follows from the convective instability criterion of Schwarzschild that an adiabatically displaced cell in the convection region will continue to rise, for much the same reason as a hot air balloon rises. However in the radiative envelope of a massive star, the temperature structure is subadiabatic, so that a cell displaced adiabatically upward will be colder than its surroundings and sink back down again. Hence, it is not at all obvious whether or not an initially buoyant flux tube will rise from the interface region to the surface of a massive star.

2. The buoyancy of a flux tube

Motivated by the above considerations, we have performed an extensive series of calculations, intended to elucidate the dynamics of magnetic flux tubes in the radiative interiors of early type stars. Our numerical model describes an axisymmetric, toroidal flux distribution of specified initial field strength and cross-sectional radius as is illustrated in Figure 1.

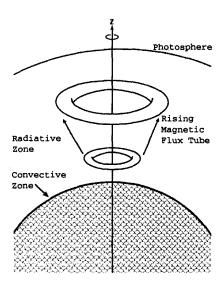


Figure 1. The rise of a thin flux tube through the envelope of a massive star. The field in the toroidal tube is assumed to be produced by interface dynamo activity at the base of the radiative envelope. The rising tube is assumed to maintain the enclosed field as well as the initial mass of the tube.

It is assumed that the slender flux tube remains in total pressure equilibrium with the external, unmagnetized stellar interior at all times. Beginning from an initial location at a prescribed latitude on the periphery of the convective core, the position and velocity of the flux ring at later times is determined by

solving an equation of motion that includes explicit treatment of the buoyant, centrifugal, Coriolis, magnetic tension, and aerodynamic drag forces. At the outset of the calculation, the ring is assumed to be in thermal equilibrium, so that its initial temperature is the same as the gas outside it. If the subscripts i and e refer, respectively, to the values of quantities inside and external to the tube, then from the conditions of mechanical and thermal equilibrium at t=0,

$$\frac{\rho_i k T_i}{\mu m_H} + \frac{B_i^2}{8\pi} = \frac{\rho_e k T_e}{\mu m_H},\tag{1}$$

$$T_i = T_e, (2)$$

it is clear that $\rho_i < \rho_e$ and the tube is initially buoyant

After the tube begins to rise, it does not remain in thermal equilibrium with the subadiabatically stratified gas in which it is immersed. The thermodynamic state of the tube following its release from the core is therefore governed by an energy equation that includes explicit treatment of (i) the heating that arises because of the radiative interaction between the tube and its environment, and (ii), the cooling that results from the expansion of the tube as it travels outward in the stellar interior.

3. The rise and stall of a flux tube

The theoretical framework described above has been used to study the behavior of flux tubes inside a 9 M_{\odot} star with a surface equatorial rotational velocity of 150 km s⁻¹. The rings are free to move in the azimuthal direction, i.e., to rotate relative to the surrounding, rigidly rotating stellar interior through which they rise.

Our results indicate that flux rings that start in the equatorial plane stay in that plane. Such rings quickly attain a state in which the outward buoyant force is balanced by the radial components of the Coriolis and magnetic tension forces, both of which are directed inward. When such an equilibrium is established, the outward ascent of the ring ceases, and its radial position remains fixed thereafter. In other words the rise of an equatorial tube "stalls".

Flux rings launched from higher initial latitudes exhibit a greater range of dynamical behaviors. In addition to the strong, radial buoyant acceleration that they initially experience, these rings can undergo oscillations in latitude, driven by the interplay between the Coriolis and magnetic tension forces that act on them. Rings with larger cross-sectional radii (i.e., less drag) and weaker magnetic fields (i.e., less buoyancy) can sustain extended periods of latitudinal oscillations following their release at the outer boundary of the convective core. After this initial phase of dynamical adjustment, the latitudinal oscillations decay, and the rings follow a trajectory that is roughly parallel to the stellar rotation axis, as is shown in Figure 2.

During their ascent, the rings approach adiabaticity, in the sense that the time required for the ring to be heated by the diffusion of radiation from the outside exceeds the characteristic rise time. The thermodynamic conditions that prevail within the ring are such that the mass density and temperature are both just slightly less than the values in the external medium. Once the ring has

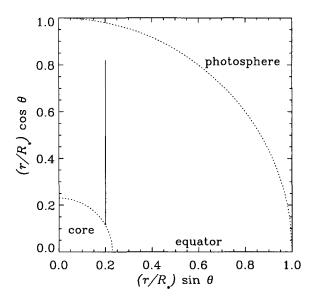


Figure 2. The trajectory of a rising flux tube following release from an initial position $r/R_* = 0.232$ and $\theta = 60^\circ$ on the core-envelope interface. Note that the tube follows a path toward the surface that is approximately parallel to the stellar rotation axis. The rate of rise decreases as the tube approaches the surface, slowing to a speed of a few cm s⁻¹ at 80 % of the stellar radius.

attained this state, the buoyant force acting on it diminishes, and its upward progress slows significantly. In this quasi-stalled situation, a near balance exists between the Coriolis and magnetic tension forces in the radial and meridional directions. In Figure 3, we depict the evolution of some of the physical properties of a tube whose late-time dynamical behavior is governed by a force balance of this kind. Under these conditions, the flux ring rises at a rate that is proportional to the rate at which the ring material is heated by the inflow of radiation from the external medium. Except for the most strongly magnetized rings, this latter rate is sufficiently slow that the rise speed is of order centimeters per second or less. The gradual deceleration of the ring as it tends toward an adiabatic state is illustrated in Figure 4.

4. Discussion: Additional Influences on Flux Transport

The results summarized in the preceding section suggest that the evolving thermodynamic properties of a buoyant flux ring can cause its rate of rise to slow significantly. For example, the time required for the tube of Figures 2-4 to transit from its initial vertical position at $y = (r/R_*) \sin \theta = 0.116$ to y = 0.216 is about $t \approx 20 \ (R_*/u_{A0})$; to travel between $y \approx 0.7$ and 0.8 requires an interval

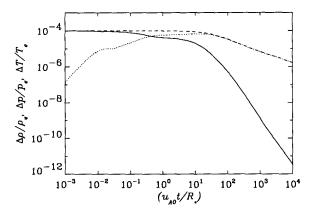


Figure 3. The evolving physical properties of the flux ring depicted in Figure 2. The solid line represents the tube density deficit relative to the external medium, $(\rho_e - \rho_i)/\rho_e$, while the dashed and dotted lines represent the pressure and temperature deficits, $(p_e - p_i)/p_e$ and $(T_e - T_i)/T_e$, respectively. All quantities are shown as functions of time, measured in units of R_*/u_{A0} (= 3.42×10^5 s), where u_{A0} (= 7.49×10^5 cm s⁻¹) is the Alfvén speed in the tube at the start of the simulation. The diminishing density contrast between the tube and its surroundings (and the consequent reduction in the buoyant acceleration) is evident at later times.

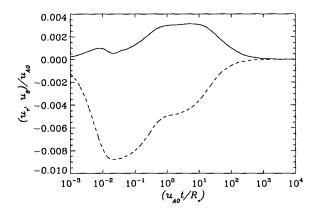


Figure 4. The evolution of the velocity components u_r (solid line) and u_{θ} (dashed line) for the flux ring shown in Figures 2 and 3.

of more than 1.5×10^4 (R_*/u_{A0}). If τ_S is a measure of the time needed to heat the tube by radiative diffusion, the radial rise speed of a quasi-adiabatic tube can be shown to vary as $u_r \sim h_e/\tau_S$, where h_e is the pressure scale height in

the external medium; for the tube considered herein, $u_r = 1.2$ cm s⁻¹ when $t = 3 \times 10^4 \ (R_*/u_A)$, the last point on the trajectory depicted in Figure 2. Although this ascent rate is still sufficient to enable the ring in question to traverse the radiative envelope in a time of order several $\times 10^3$ years, for initial tube properties other than those of the present example (i.e., weaker initial fields), even slower rates of rise can be obtained. Consequently, it is of interest to ascertain whether there are additional processes that might contribute to the transport of flux from the core to the stellar surface. Among the numerous effects that have been omitted from the present model, two mechanisms in particular might enhance the transport rate above that provided by buoyancy alone.

- 1) The first is an obvious one for those of us who work primarily on stellar winds. Although the final position depicted for the tube of Figure 2 is still $\approx 0.2~R_{\star}$ below the stellar surface, that layer contains less than 1 % of the stellar mass. A wind with mass loss rate of $10^{-8}~M_{\odot}~\rm yr^{-1}$ can remove the outer 0.09 solar masses in 9 million years, a time much smaller than the main sequence lifetime of about $10^8~\rm years$.
- 2) The balance between all the forces we have accounted for is a delicate one near the time that the rise begins to slow, so that any additional force, even a small one, could dominate the motion of the tube. In rapidly rotating Be stars, additional outward impetus might be supplied by rotationally induced, internal circulation currents. Using Eddington-Sweet theory, as employed in the paper by Maheswaran and Cassinelli (1988, 1992) to put limits on fields at the base of winds, Maheswaran estimates that a circulation speed of about 0.1 cm s⁻¹ \times (v_{rot}/v_{crit}) could be obtained in the envelope of our 9 M_{\odot} model. The drag force in this flow would advect the tube to the surface in less than 10⁵ years. Circulation currents may also explain why the non-emission line B stars do not show effects expected from the presence of magnetic fields, while the Be stars do.

A major problem is posed by evidence suggesting that the disks around Be stars are Keplerian. This discovery is surprising since the material in the disks must come from a star that is rotating at a sub-Keplerian rate. Surface magnetic fields appear to be the crucial link needed to explain the transfer of angular momentum from the star to the disk. If the fields of Be stars vary with time, they might sometimes provide support for the disks and at other times not. Since disks are not always present around Be stars, perhaps their mere presence can be considered as a mild analogue to solar activity.

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Discussion

- **Ph.** Stee: You used a 9 M_{\odot} star. What happens if you change the mass of the star? Is it a crucial parameter for the radius of the rise of the flux tube?
- J. Cassinelli: Early B stars have masses of about $9 M_{\odot}$, so it is a good value for our interest here in Be stars. For stars with a somewhat smaller mass the convective core would be smaller fraction of the stellar mass and radius, so a flux tube in such a star would have farther to rise. For stars with masses larger than $9 M_{\odot}$, the interface is closer to the surface, but also the mass loss rates are larger so the mass above the stalling radius would be removed more quickly. However the more massive stars also have complications associated with radiation being an important contributor to the ambient pressure.
- J. Fabregat: Do you think the rise of the magnetic flux tubes can apport matter from the stellar interior to the atmosphere, and hence modify the light element abundances? I wonder if your proposed mechanism could explain the helium abundance enhancement observed in O- stars and early B- stars.
- J. Cassinelli: The flux tubes are assumed to conserve the mass and composition with which they formed. Hence what you are suggesting could occur in principle. However the helium rich material also has a higher mean mass per particle μ , and this would reduce the buoyancy and the radius out to which a flux tube rises. The net helium enrichment of the atmosphere, if any, would depend strongly on the model parameters and assumptions.
- M. Smith: What is the timescale for the flux tube rising to or near the surface? Are there any other rotation-dependent factors needed other than the Eddington Sweet currents for the tubes to rise quickly?
- J. Cassinelli: The rate of rise of the flux tube is initially rather fast, with rise time scales of order 10^4 years. However, near the top of the flux tube rise the time scale becomes very large, longer than the time scale associated with circulation currents. The effects of mass loss and circulation currents are the only auxiliary effects we have considered thus far. However, in the outmost layers the OPAL opacities can increase over a thin zone of radius (in which T is near 3×10^5 K). This increases the temperature gradient, and could accelerate the rate of rise of the any flux tube that reaches that zone.

Rinehart: Given that the initial rise times are faster than the rise owing to the circulation currents and associated drag force, could there be a stack up of flux tubes at the 80% R_* level?

J. Cassinelli: There is a transition from a situation in which the tubes are rising faster than the circulation current to one in which the circulation is faster than the tube rise. So there could occur a stack up or traffic jam with the increased in the number of tubes as you suggest. However, our models have not accounted for the interactions or the dissipation of a flux tube, so it is not yet clear yet what should happen.