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THE STRUCTURE OF CONTINUOUS $\{0, 1\}$-VALUED FUNCTIONS ON A TOPOLOGICAL PRODUCT

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1. Introduction. In this paper we investigate the question of which continuous $\{0, 1\}$-valued functions on a product space $\prod\{X_\alpha : \alpha \in A\}$ admit continuous extensions to $\prod\{\beta X_\alpha : \alpha \in A\}$ where βX_α is the Stone-Čech compactification of X_α and $\{0, 1\}$ denotes the two point discrete space. This problem is clearly equivalent to determining which clopen subsets of $\prod\{X_\alpha : \alpha \in A\}$ have clopen closures in $\prod\{\beta X_\alpha : \alpha \in A\}$.

A space X is called 0-*dimensional* if it has a base of clopen sets. The solution of the above problem in the case of 0-dimensional spaces X_α leads to a characterization of pseudocompactness in 0-dimensional topological products.

All spaces discussed in this paper are assumed to be completely regular and Hausdorff. The notation will be that of [**7**] except for the definition of a 0-dimensional space which is given above.

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2. The main results. It has been known for some time that every 0-dimensional space has a maximal compactification $\beta_0 X$ analagous to βX in the sense that $\beta_0 X$ is 0-dimensional and every function from X to a compact 0-dimensional space admits a continuous extension to $\beta_0 X$. The space $\beta_0 X$ may be more familiar to the reader as the maximal 0-dimensional compactification constructed by Banaschewski in [**1**]. It is clear that if X is 0-dimensional, the closure in $\beta_0 X$ of a clopen subset of X is again clopen.

If X is a compact space, let X^* denote the component space of X, i.e. X^* is the quotient space of X formed by collapsing the connected components of X. Since in a compact space the connected component of a point is equal to its quasi-component (the intersection of all clopen sets containing the point—see Theorem 16.15 of [**7**]), X^* becomes a compact, Hausdorff, 0-dimensional space. Let $q : X \to X^*$ be this quotient map. Suppose $f : X \to Y$ where Y is compact and 0-dimensional. It is easy to see that f must be constant on the connected components of X, and hence there is a map $f^* : X^* \to Y$ such that $f^* \cdot q = f$. Since q is a quotient map, f^* is continuous.

The following lemma is well-known (see [**2**, Part I, p. 110] for a proof).

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2.1 LEMMA. *Let $\{X_\alpha : \alpha \in A\}$ be a family of spaces. Then the connected component of a point $(x_\alpha)_{\alpha \in A} \in \prod\{X_\alpha : \alpha \in A\}$ is equal to $\prod\{C_\alpha : \alpha \in A\}$ where C_α is the connected component of x_α in X_α for each α in A.*

2.2 THEOREM. *Let $\{K_\alpha : \alpha \in A\}$ be a family of compact spaces. Then the map which takes the component of the point $(x_\alpha)_{\alpha \in A}$ in $\prod\{K_\alpha : \alpha \in A\}$ to the product of the components of the x_α's is a homeomorphism from $(\prod\{K_\alpha : \alpha \in A\})^*$ onto $\prod\{K_\alpha^* : \alpha \in A\}$.*

Proof. Let $k : \prod\{K_\alpha : \alpha \in A\} \to (\prod\{K_\alpha : \alpha \in A\})^*$ and $k_\alpha : K_\alpha \to K_\alpha^*$ be the quotient maps. Let h denote the following map: $h((x_\alpha)_{\alpha \in A}) = (k_\alpha(x_\alpha))_{\alpha \in A}$, i.e. h is the product of the k_α's. Thus h is a continuous map from $\prod\{K_\alpha : \alpha \in A\}$ to $\prod\{K_\alpha^* : \alpha \in A\}$. As noted above, there is a map $g : (\prod\{K_\alpha : \alpha \in A\})^* \to \prod\{K_\alpha^* : \alpha \in A\}$ such that $g \cdot k = h$. Clearly g is the map described in the statement of the theorem. Furthermore, g is continuous, as k is a quotient map, g is onto (because h is onto), and g is one-to-one by Lemma 2.1.

2.3 *Definition.* Let $\{X_\alpha : \alpha \in A\}$ be a family of spaces, and let $f : \prod\{X_\alpha : \alpha \in A\} \to \{0, 1\}$. Then f is called *finitely decomposable* if there exists a finite subset $F = \{\alpha_1, \ldots, \alpha_n\} \subseteq A$, and finite decompositions (into pairwise disjoint clopen sets) Υ_i of X_{α_i} for $i = 1, \ldots n$ such that f is constant on each set of the form $\prod\{C_i : i = 1, \ldots n\} \times \prod\{X_\alpha : \alpha \neq \alpha_i, i = 1, \ldots, n\}$ where $C_i \in \Upsilon_i$ for $i = 1, \ldots n$.

2.4 THEOREM. *Let $\{X_\alpha : \alpha \in A\}$ be a family of spaces. Suppose $f : \prod\{X_\alpha : \alpha \in A\} \to \{0, 1\}$ is a continuous map. Then f admits a continuous extension to $\prod\{\beta X_\alpha : \alpha \in A\}$ if and only if f is finitely decomposable.*

Proof. Sufficiency. Suppose f is finitely decomposable. Then there is a finite subset $F = \{\alpha_1, \ldots, \alpha_n\} \subseteq A$ and finite decompositions Υ_i of X_{α_i} such that f is constant on each set of the form $\prod\{C_i : i = 1, \ldots n\} \times \prod\{X_\alpha : \alpha \neq \alpha_i, i = 1, \ldots, n\}$, where $C_i \in \Upsilon_i$. Since each C_i is clopen, the set

$$\prod \left\{\mathrm{cl}_{\beta X_{\alpha_i}}(C_i) : i = 1, \ldots, n\right\} \times \prod \left\{\beta X_\alpha : \alpha \neq \alpha_i, i = 1, \ldots, n\right\}$$

is clopen and is the closure in $\prod\{\beta X_\alpha : \alpha \in A\}$ of the set mentioned above. Since Υ_i is finite for $i = 1, \ldots, n$, it follows that the sets of the form

$$\prod \left\{\mathrm{cl}_{\beta X_{\alpha_i}}(C_i) : i = 1, \ldots, n\right\} \times \prod \left\{\beta X_\alpha : \alpha \neq \alpha_i, i = 1, \ldots, n\right\}$$

form a finite decomposition of $\prod\{\beta X_\alpha : \alpha \in A\}$ into clopen sets. If we define $f^* : \prod\{\beta X_\alpha : \alpha \in A\} \to \{0, 1\}$ by

$$f^*\left(\prod \left\{\mathrm{cl}_{\beta X_{\alpha_i}}(C_i) : i = 1, \ldots, n\right\} \times \prod \left\{\beta X_\alpha : \alpha \neq \alpha_i, i = 1, \ldots, n\right\}\right)$$
$$= f\left(\prod \left\{C_i : i = 1, \ldots, n\right\} \times \prod \left\{X_\alpha : \alpha \neq \alpha_i, i = 1, \ldots, n\right\}\right),$$

then f^* is continuous and is an extension of f.

Necessity. Suppose $f^* : \Pi\{\beta X_\alpha : \alpha \in A\} \rightarrow \{0, 1\}$ is a continuous extension of $f : \Pi\{X_\alpha : \alpha \in A\} \rightarrow \{0, 1\}$. Let $k_\alpha : \beta X_\alpha \rightarrow (\beta X_\alpha)^*$ be the quotient maps for all $\alpha \in A$, and let $h : \Pi\{\beta X_\alpha : \alpha \in A\} \rightarrow \Pi\{(\beta X_\alpha)^* : \alpha \in A\}$ be the product of the k_α's. By Theorem 2.2, f^* factors through $\Pi\{(\beta X_\alpha)^* : \alpha \in A\}$, i.e. there is a map $g : \Pi\{(\beta X_\alpha)^* : \alpha \in A\} \rightarrow \{0, 1\}$ such that $g \cdot h = f^*$. Since $\Pi\{(\beta X_\alpha)^* : \alpha \in A\}$ is compact and 0-dimensional, it is easy to see that there is a finite subset of A, $F = \{\alpha_1, \ldots, \alpha_n\}$ such that $g^{\leftarrow}(\{0\})$ and $g^{\leftarrow}(\{1\})$ are sets of the form

$$U = \cup\{(\Pi\{U_{ij} : j = 1, \ldots, n\} \\ \times \Pi\{(\beta X_\alpha)^* : \alpha \neq \alpha_t, t = 1, \ldots, n\}) : i = 1, \ldots, k\}$$

where k and n are positive integers, and U_{ij} is a clopen subset of $(\beta X_{\alpha_j})^*$ for $j = 1, \ldots, n$ and $i = 1, \ldots k$ (this is because the canonical clopen sets form a base for the open sets). Let $k_1, k_2 \in N$ and

$$U = g^{\leftarrow}(\{0\}) = \cup\{(\Pi\{U_{ij} : j = 1, \ldots n\} \\ \times \Pi\{(\beta X_\alpha)^* : \alpha \in A - F\}) : i = 1, \ldots, k_1\},$$

$$V = g^{\leftarrow}(\{1\}) = \cup\{(\Pi\{V_{ij} : j = 1, \ldots n\} \\ \times \Pi\{(\beta X_\alpha)^* : \alpha \in A - F\}) : i = 1, \ldots, k_2\}.$$

Fix $j \leqq n$. Let $\Upsilon_j = \{U_{ij} : i = 1, \ldots, k_1\} \cup \{V_{ij} : i = 1, \ldots, k_2\}$. Clearly $\cup \Upsilon_j = (\beta X_{\alpha_j})^*$ for $j = 1, \ldots, n$. Let $1 \leqq j \leqq n$ and suppose $|\Upsilon_j| = n_j \in N$. Then $\Upsilon_j = \{H_s : s = 1, \ldots, n_j\}$ where each H_s is clopen in $(\beta X_{\alpha_j})^*$. Let $H_s^0 = H_s$ and $H_s^1 = (\beta X_{\alpha_j})^* - H_s$ for $s = 1, \ldots, n_j$. For every $r : n_j \rightarrow \{0, 1\}$, let $H_r = \cap\{H_s^{r(s)} : s = 1, \ldots, n_j\}$ and let $\Upsilon_j^* = \{H_r : r : n_j \rightarrow \{0, 1\}\}$. By the construction of Υ_j^* it is clear that if $T \in \Upsilon_j$ and $H_r \cap T \neq \phi$, then $H_r \subseteq T$.

Suppose $C_j \in \Upsilon_j^*$ for $j = 1, \ldots, n$. Then g is constant on $C = \Pi\{C_j : j = 1, \ldots, n\} \times \Pi\{(\beta X_\alpha)^* : \alpha \in A - F\}$. To verify this, suppose $C \cap U \neq \phi$. Then there is an $i_1 \in N$ such that $1 \leqq i_1 \leqq k_1$, and $C \cap (\Pi\{U_{i_1 j} : j = 1, \ldots, n\} \times \Pi\{(\beta X_\alpha)^* : \alpha \in A - F\}) \neq \phi$. Thus, $C_j \cap U_{i_1 j} \neq \phi$ for $j = 1, \ldots, n$. Since for each $j \leqq n$, C_j is an H_r in Υ_j^*, $C_j \subseteq U_{i_1 j}$ for $j = 1, \ldots, n$ and hence $C \subseteq U$. A similar argument holds if $C \cap V \neq \phi$. Thus g is constant on C.

If we define $\theta_j = \{k_{\alpha_j}{}^{\leftarrow}(W) \cap X_{\alpha_j} : W \in \Upsilon_j^*\}$ for $j = 1, \ldots, n$, then it is easy to see that f is finitely decomposable with respect to the decompositions θ_j of X_{α_j} for $j = 1, \ldots, n$.

It is clear that no special property of βX was used in Theorem 2.4 other than the fact that clopen subsets of X have clopen closures in βX. Thus, if γX_α, $\alpha \in A$ are compactifications of X_α such that clopen subsets of X_α have clopen closures in γX_α, then Theorem 2.4 remains valid with βX_α replaced by γX_α (in fact, the necessity remains valid with βX_α replaced by any compactification of X_α).

We now use Theorem 2.4 and Theorem 2.3 of [**3**] to obtain a characterization of pseudocompactness in a 0-dimensional product space. Recall that if X is 0-dimensional, then $\beta_0 X$ can be characterized as the 0-dimensional compactification of X to which every continuous $\{0, 1\}$-valued function on X can be continuously extended. Thus, any clopen subset of X has clopen closure in $\beta_0 X$. Hence, assuming all X_α are 0-dimensional in Theorem 2.4, we may replace βX_α by $\beta_0 X_\alpha$.

2.5 *Definition.* Let $\{X_\alpha : \alpha \in A\}$ be a family of spaces and let $f : \Pi\{X_\alpha : \alpha \in A\} \rightarrow Y$ where Y is any space. Then f is said to *depend on finitely many coordinates* if there exists a finite subset $F \subseteq A$, and a map $g : \Pi\{X_\alpha : \alpha \in F\} \rightarrow Y$ such that $f = g \cdot \pi_F$, where π_F is the projection map from $\Pi\{X_\alpha : \alpha \in A\}$ to $\Pi\{X_\alpha : \alpha \in F\}$.

The following theorem gives various conditions on a 0-dimensional product space which are equivalent to the space being pseudocompact (i.e. every real-valued continuous function is bounded).

2.6 THEOREM. *Let $\{X_\alpha : \alpha \in A\}$ be a family of 0-dimensional spaces such that $\Pi\{X_\alpha : \alpha \in (A - \{\alpha_0\})\}$ is infinite for all $\alpha_0 \in A$. The following are equivalent.*

i) *$\Pi\{X_\alpha : \alpha \in A\}$ is pseudocompact,*

ii) *Every countable subproduct of $\Pi\{X_\alpha : \alpha \in A\}$ is pseudocompact,*

iii) $\beta(\Pi\{X_\alpha : \alpha \in A\}) = \Pi\{\beta X_\alpha : \alpha \in A\}$,

iv) $\beta_0(\Pi\{X_\alpha : \alpha \in A\}) = \Pi\{\beta_0 X_\alpha : \alpha \in A\}$,

v) *Every continuous $\{0, 1\}$-valued function on $\Pi\{X_\alpha : \alpha \in A\}$ is finitely decomposable,*

vi) *Every finite subproduct of $\Pi\{X_\alpha : \alpha \in A\}$ is pseudocompact, and every continuous $\{0, 1\}$-valued function on $\Pi\{X_\alpha : \alpha \in A\}$ depends on finitely many coordinates.*

Proof. Conditions i), ii), and iii) are shown to be equivalent for any family of (not necessarily 0-dimensional) spaces in [**8**] and are included to provide a contrast to iv), v) and vi). Conditions i) and iv) are shown to be equivalent in Theorem 2.3 of [**3**]. Since statement iv) means precisely that every continuous $\{0, 1\}$-valued function on $\Pi\{X_\alpha : \alpha \in A\}$ admits a continuous extension to $\Pi\{\beta_0 X_\alpha : \alpha \in A\}$, by Theorem 2.4 and the remarks following it, we get the equivalence of iv) and v).

i) $\Rightarrow$ vi). Since pseudocompactness is preserved by continuous maps, every subproduct of $\Pi\{X_\alpha : \alpha \in A\}$ is pseudocompact (being the image of $\Pi\{X_\alpha : \alpha \in A\}$ under a projection map) if $\Pi\{X_\alpha : \alpha \in A\}$ is pseudocompact. It is clear that any finitely decomposable map from $\Pi\{X_\alpha : \alpha \in A\}$ to $\{0, 1\}$ depends on finitely many coordinates. Thus, by the equivalence of i) and v) which has already been shown, it follows that every continuous $\{0, 1\}$-valued function on $\Pi\{X_\alpha : \alpha \in A\}$ depends on finitely many coordinates.

vi) $\Rightarrow$ iv). Let $f : \Pi\{X_\alpha : \alpha \in A\} \rightarrow \{0, 1\}$ be a continuous map. Then there is a finite set $F \subseteq A$ and a continuous map $g : \Pi\{X_\alpha : \alpha \in F\} \rightarrow \{0, 1\}$ such that $f = g \cdot \pi_F$. But, $\prod\{X_\alpha : \alpha \in F\}$ is pseudocompact by hypothesis. Thus, by the equivalence of i) and iv), g admits a continuous extension g^* to $\prod\{\beta_0 X_\alpha : \alpha \in F\}$ (we do not need the hypothesis that $\prod\{X_\alpha : \alpha \in F - \{\alpha_0\}\}$ is infinite for every $\alpha_0 \in F$, for if one such product is finite then g obviously admits an extension g^*). Let $\bar{\pi}_F : \prod\{\beta_0 X_\alpha : \alpha \in A\} \to \prod\{\beta_0 X_\alpha : \alpha \in F\}$ be the projection map. Then, if we let $f^* = g^* \cdot \bar{\pi}_F$, $f^* : \prod\{\beta_0 X_\alpha : \alpha \in A\} \to \{0, 1\}$ and is an extension of f. Hence $\beta_0(\prod\{X_\alpha : \alpha \in A\}) = \prod\{\beta_0 X_\alpha : \alpha \in A\}$.

If we consider any finite family of 0-dimensional spaces X_i, $i = 1, \ldots, n$ then every continuous function on $\prod\{X_i : i = 1, \ldots, n\}$ depends on finitely many coordinates. However, if X_i is a non-pseudocompact space for each $i = 1, \ldots, n$, then $\prod\{X_i : i = 1, \ldots, n\}$ satisfies the second part of condition vi) but not the first. Furthermore, in [**4**] an example is given of a 0-dimensional, non-pseudocompact product space, all of whose finite subproducts are pseudocompact. Evidently, this space satisfies the first part of condition vi) but not the second. Hence, neither of the two parts of condition vi) may be removed.

3. The product of two spaces.

3.1 *Definition.* A space X is said to be $\{0, 1\}$*-embedded* in a space Y if every continuous $\{0, 1\}$-valued function on X admits a continuous extension to Y. A pair of spaces (X, Y) is called a $\{0, 1\}$*-pair* if $X \times Y$ is $\{0, 1\}$-embedded in both $\beta X \times Y$ and $X \times \beta Y$. If (X, Y) is a $\{0, 1\}$-pair such that $X \times Y$ is not $\{0, 1\}$-embedded in $\beta X \times \beta Y$, then (X, Y) is called a *proper* $\{0, 1\}$*-pair*. A map $f : X \to Y$ is called a *clopen map* if the image under f of a clopen subset of X is clopen in Y.

The following theorem is the "0-dimensional analogue" of Theorem 2.1 of [**6**]. The proof requires only minor modifications of the original proof, to which the reader is referred.

3.2. THEOREM. *Let X and Y be 0-dimensional spaces. If $X \times Y$ is $\{0, 1\}$-embedded in $\beta X \times Y$, then either X is pseudocompact, or Y is a P-space (a P-space is a space in which every G_δ is open).*

An investigation of the proof of Theorem 2.2 of [**3**] shows that for 0-dimensional spaces X and Y, $X \times Y$ is $\{0, 1\}$-embedded in $\beta X \times Y$ if and only if $X \times Y$ is $\{0, 1\}$-embedded in $\beta_0 X \times Y$. Thus, "$\beta X \times Y$" may be replaced by "$\beta_0 X \times Y$" in Theorem 3.2. Theorem 3.2 yields the following analogue to Theorem 2.2 of [**6**].

3.3 THEOREM. *If (X, Y) is a proper $\{0, 1\}$-pair and X and Y are 0-dimensional spaces, then both X and Y are P-spaces.*

Proof. By hypothesis $X \times Y$ is $\{0, 1\}$-embedded in $\beta X \times Y$, hence also in $\beta_0 X \times Y$ by the remarks above. If Y is pseudocompact, then $\beta_0 X \times Y$ is pseudocompact, and then, by Theorem 2.2 of [**3**] $\beta_0(\beta_0 X \times Y) = \beta_0 X \times \beta_0 Y$, i.e. $\beta_0(X \times Y) = \beta_0 X \times \beta_0 Y$ (as $X \times Y$ would then be $\{0, 1\}$-embedded in $\beta_0 X \times \beta_0 Y$) which is contrary to the hypothesis that (X, Y) is a proper $\{0, 1\}$-pair. Thus, Y is not pseudocompact. Hence, by Theorem 3.2, X is a P-space. Similarly, Y is a P-space.

Theorems 2.1 and 2.2 of [**6**] now follow from Theorems 3.2 and 3.3, in the case where X and Y are 0-dimensional spaces. The following are parallels to Theorems 3.1 of [**6**] and 4.3 of [**5**].

3.4 THEOREM. *The following conditions on a product space $X \times Y$ are equivalent.*

i) *The projection map $\pi_Y : X \times Y \to Y$ is a clopen map,*

ii) *$X \times Y$ is $\{0, 1\}$-embedded in $\beta X \times Y$.*

Proof. i) $\Rightarrow$ ii). Trivial modifications of the proof of Theorem 3.1 of [**6**] yield this implication.

ii) $\Rightarrow$ i). Let U be a clopen subset of $X \times Y$. By hypothesis, there is a clopen set $V \subseteq \beta X \times Y$ such that $V \cap (X \times Y) = U$. Let π_Y denote the projection map from $X \times Y$ to Y and $\bar{\pi}_Y$ the projection map from $\beta X \times Y$ to Y. Since βX is compact, $\bar{\pi}_Y$ is a closed map, hence (as all projection maps are open maps) is a clopen map. Thus, $\bar{\pi}_Y(V)$ is clopen in Y. Clearly $\pi_Y(U) \subseteq \bar{\pi}_Y(V)$. If $y \notin \pi_Y(U)$ then $(X \times \{y\}) \cap U = \phi$, hence $(\beta X \times \{y\}) \cap V = \phi$ as $X \times \{y\}$ is dense in $\beta X \times \{y\}$. Thus $y \notin \bar{\pi}_Y(V)$. Therefore $\pi_Y(U) = \bar{\pi}_Y(V)$ and hence π_Y is a clopen map.

3.5 THEOREM. *Let X and Y be 0-dimensional spaces. The following are equivalent.*

i) *$X \times Y$ is pseudocompact,*

ii) *X and Y are both pseudocompact spaces and π_X and (or) π_Y are clopen maps.*

Proof. i) $\Rightarrow$ ii). Since $X \times Y$ is pseudocompact, $\beta(X \times Y) = \beta X \times \beta Y$ by Theorem 1 of [**8**], and hence $X \times Y$ is a $\{0, 1\}$-pair. Thus by Theorem 3.4, both π_X and π_Y are clopen maps. Since both X and Y are continuous images of $X \times Y$, both X and Y are pseudocompact (note that this implication does not make use of the 0-dimensionality of X and Y).

ii) $\Rightarrow$ i). Since π_X is clopen, $X \times Y$ is $\{0, 1\}$-embedded in $X \times \beta Y$ by Theorem 3.4. Thus, by the remarks preceding 3.3, $X \times Y$ is $\{0, 1\}$-embedded in $X \times \beta_0 Y$. Since X is pseudocompact and $\beta_0 Y$ is compact, $X \times \beta_0 Y$ is pseudocompact. Thus, by Theorem 2.2 of [**3**], $\beta_0(X \times \beta_0 Y) = \beta_0 X \times \beta_0 Y$. But $X \times Y$ is $\{0, 1\}$-embedded in $X \times \beta_0 Y$, hence $X \times Y$ is $\{0, 1\}$-embedded in $\beta_0 X \times \beta_0 Y$, i.e. $\beta_0(X \times Y) = \beta_0 X \times \beta_0 Y$. By Theorem 2.2 of [**3**], $X \times Y$ is pseudocompact. Thus, the proof does not require the hypothesis that π_Y be clopen, although this must be the case by Theorem 3.4.

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