ON THE PARTICLE-SIZE DISTRIBUTION FUNCTION OF COMETARY DUST

Z. Sekanina
Harvard-Smithsonian Center for Astrophysics
Cambridge, Massachusetts, U.S.A.

The Finson-Probstein (1968a) method of analysis of the distribution of light intensity in dust tails has so far been applied to only a handful of comets. Yet, the results already suggest a striking diversity in the properties of the particle-size related distribution function $f(\beta)$, one of three parametric functions determined. Here β is the acceleration exerted on the particle by solar radiation pressure, measured in units of solar attraction. For a spherical particle β is a function of its radius a, density ρ, and the integrated efficiency factor for radiation pressure Q_{pr} (i.e., the ratio of the particle's effective cross-section for radiation pressure to its geometric cross-section):

$$\beta = c_0 \frac{Q_{pr}}{\rho a}, \qquad (1)$$

where $c_0 = 0.585 \times 10^{-4}$ g/cm^2.

The distribution function of particle radii g(a) da, i.e., the relative number of ejected particles whose radii lie between a and a + da, is by definition related to $f(\beta)$ according to

$$f(\beta)\, d\beta = \text{const}\; Q_{scat}\, a^2\, g(a)\, da\,, \qquad (2)$$

where Q_{scat} is the wavelength dependent efficiency factor for scattering. Since ρ, Q_{pr}, and Q_{scat} depend generally on particle size, we have

$$g(a)\, da = \text{const}\; (Q_{pr}/\rho Q_{scat})\; f(\beta)\, a^{-4} [1 + \frac{a}{\rho}\frac{\partial\rho}{\partial a} - \frac{a}{Q_{pr}}\frac{\partial Q_{pr}}{\partial a}]\, da\,. \qquad (3)$$

If the same β corresponds to more than one size, formula (3) must still be modified to include the partition function of $f(\beta)$. If ρ, Q_{pr}, and Q_{scat} can be approximated by constants, relation (3) reduces to the expression derived by Finson and Probstein (1968a). The constant in (3) is always determined by the normalization of g(a).

The normalized functions $f(\beta)$, established for the dust tails of Comets Arend-Roland 1957 III (Finson and Probstein 1968b), Bennett 1970 II

I. Halliday and B. A. McIntosh (eds.), Solid Particles in the Solar System, 251–254.

https://doi.org/10.1017/S007418090006678X Published online by Cambridge University Press

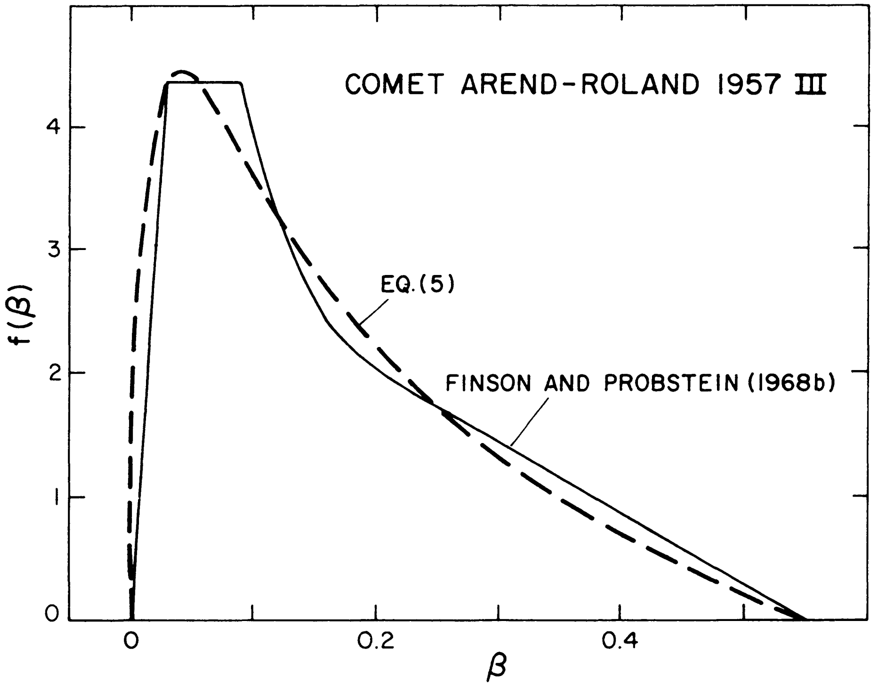

Fig. 1. Comparison of f(β) for Comet Arend-Roland as determined by Finson and Probstein (1968b) with the distribution law (5).

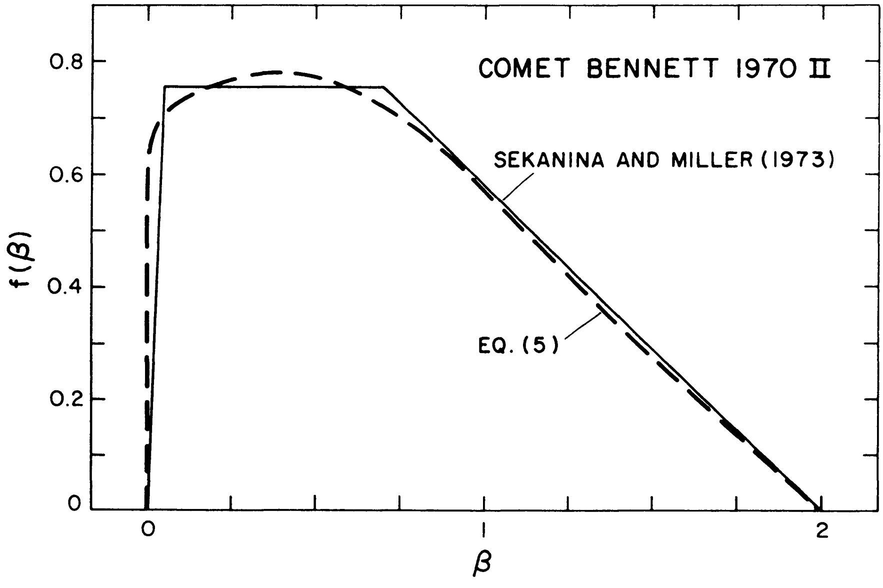

Fig. 2. Comparison of f(β) for Comet Bennett as determined by Sekanina and Miller (1973) with the distribution law (5).

https://doi.org/10.1017/S007418090006678X Published online by Cambridge University Press

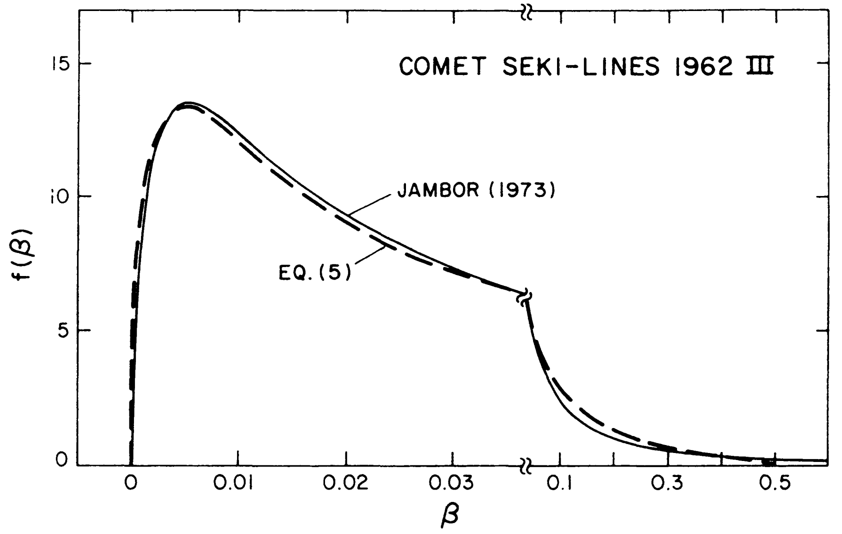

Fig. 3. Comparison of f(β) for Comet Seki-Lines as determined by Jambor (1973) with the distribution law (5).

(Sekanina and Miller 1973), and Seki-Lines 1962 III (Jambor 1973), are plotted in Figs. 1 to 3. Although the variety of behavior is clearly demonstrated, the three distributions do have certain features in common, one of these being the existence of a peak or plateau. Unfortunately, the rate of decrease toward $\beta = 0$ (i.e., in the domain of large particles) is not well determined, because such particles contribute to the intensity in an ordinary dust tail only marginally. Hence, solutions based on the Finson-Probstein approach are not sensitive to the character of f(β) to the left of the peak or plateau, and the adopted approximations reflect largely mathematical convenience (Sekanina 1980).

Meaningful information on f(β) at very small β can fortunately be obtained from photometry of anomalous tails (e.g., Sekanina 1980). Such studies indicate that a reasonable approximation in this range of β is

$$f(\beta)\, d\beta = \text{const}\ \beta^{z}\, d\beta\ , \tag{4}$$

where the exponent z varies from comet to comet, but seems to be generally confined to $0 \lesssim z \lesssim 0.5$. Other f(β) properties shared by the comets are the existence of a sharp cutoff at β_0, or the tendency thereto, so that $f(\beta) = 0$ at $\beta > \beta_0$; and a uniform decrease of f(β) as β approaches β_0.

The existence of common features is of course a prerequisite for a formulation of an *a priori* f(β) distribution law, with which one can approximate all the diversity of behavior (that may reasonably be expected) by changing a few key parameters. The availability of such a law should benefit future applications of the Finson-Probstein method and assist in modeling dust output from comets selected for space exploration. After

TABLE I. Parameters of the Proposed $f(\beta)$ Law for Three Comets ($\beta_\infty = 0$)

Comet	z	β_0	χ	C	β_{peak}
Arend-Roland 1957 III	0.4	0.55	0.14	15.3	0.041
Bennett 1970 II	0.05	2.0	1.2	0.849	0.40
Seki-Lines 1962 III	0.3	0.5	0.021	60.5	0.0050

some experimentation, I now propose the following law

$$f(\beta)\,d\beta = C\,(\beta/\beta_0)^z\{1-\exp[\chi(1-\beta_0/\beta)]\}\,d\beta \quad \text{for } \beta_\infty \leqq \beta \leqq \beta_0\,, \qquad \qquad (5)$$
$$= 0 \quad \text{for } \beta < \beta_\infty \text{ and } \beta > \beta_0\,,$$

where $\beta_\infty \geqq 0$, $\beta_0 > \beta_\infty$, $\chi > 0$, and z are the free parameters, C is a normalizing factor. The position of the peak functional value follows from

$$\exp[-\chi(1-\beta_0/\beta_{peak})] = 1 + (\chi/z)(\beta_0/\beta_{peak}) \qquad (6)$$

and the rate of decrease near β_0 from

$$[df(\beta)/d\beta]_{\beta\to\beta_0} = -C\chi/\beta_0\,. \qquad (7)$$

When $f(\beta)$ is truncated at $\beta_\infty > 0$ (a necessity for large-particle dominated distributions and a convenient option otherwise), C is given by

$$C = (\zeta/\beta_0)\{1-(\beta_\infty/\beta_0)^\zeta - \zeta\,\chi^\zeta e^\chi[\Gamma(-\zeta,\chi) - \Gamma(-\zeta,\chi\beta_0/\beta_\infty)]\}^{-1}, \qquad (8)$$

where $\zeta = z+1$ and $\Gamma(-\zeta,y)$ is the incomplete gamma function with integration limits from y to $+\infty$. For $\beta_\infty = 0$ this expression reduces to

$$C = z(z+1)(\chi\beta_0)^{-1}[1-\chi^z e^\chi\,\Gamma(1-z,\chi)]^{-1}. \qquad (9)$$

The law (5), which is readily seen to converge to (4) for $\beta \ll \beta_0$, has been applied to approximate $f(\beta)$ for the three comets. The choice of parameters used is listed in Table I and very satisfactory agreement between law (5) and the empirical distributions is seen in Figs. 1 to 3. Little effort was expended to optimize the fits.

This work was supported by Grant NGR 09-015-159 from the Planetary Atmospheres Program of the National Aeronautics and Space Administration.

REFERENCES

Finson, M. L. and Probstein, R. F.: 1968a, *Astrophys. J.* 154, pp. 327-352.
Finson, M. L. and Probstein, R. F.: 1968b, *Astrophys. J.* 154, pp. 353-380.
Jambor, B. J.: 1973, *Astrophys. J.* 185, pp. 727-734.
Sekanina, Z.: 1980, review paper in this volume.
Sekanina, Z. and Miller, F. D.: 1973, *Science* 179, pp. 565-567.