PECULIAR RED GIANTS — WHAT KIND OF WHITE DWARFS DO THEY BECOME?

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Abstract. After a brief commentary on the place of "peculiar red giants" in the overall scheme of stellar evolution, an outline is given of the various possibilities for post asymptotic giant branch (AGB) evolution. The behavior of a post-AGB model star is crucially dependent on where in a thermal pulse cycle the mass of the hydrogen-rich envelope is reduced to such an extent that departure from the AGB must follow on a thermal time scale. If departure from the AGB occurs while the model is still burning hydrogen, post-AGB behavior depends on the mass of the helium buffer zone (= zone containing predominantly helium which has been processed through the hydrogen-burning shell following the last thermal pulse on the AGB). If departure occurs at an arbitrary time during the hydrogen-burning phase, then: (1) in ~ 25% of all cases, the post-AGB model will experience a final helium shell flash, and, in consequence of additional mass loss, may become a non-DA white dwarf; (2) in ~ 60% of all cases, the model will cease burning hydrogen when the mass in its hydrogen-rich envelope is reduced to ~ $10^{-4} M_\odot$ and will evolve into a DA white dwarf; and (3) in ~ 15% of all cases, the model will experience a final hydrogen shell flash, but the outcome with regard to spectroscopic type is unclear. If departure from the AGB occurs while the model is burning helium, the result is either the same as in option (3) just described, or mass loss during the post-AGB helium-burning phase may turn the star into a non-DA white dwarf.

1 PRELIMINARY REMARKS

I will begin with a few philosophical comments about "peculiar" red giants (PRGs) and then narrow my remarks to a discussion of their fate "after death". In particular, I will ask the question: what happens to PRGs as they depart from the giant branch, or what happens to their descendants after they have departed from the giant branch, or both, that determines which descendants become DA white dwarfs and which become non-DA white dwarfs. But, first, a few remarks.

That we should call red giants with non-solar distributions of the
heavy elements at their surfaces "peculiar" is ironic: most stars
which are massive enough to evolve off the main sequence in a Hubble
time and to retain their hydrogen-rich envelope until after they have
developed an electron-degenerate core composed of carbon and oxygen and
have entered the helium shell flashing stage develop surface abundance
"peculiarities" in consequence of dredging up material processed
through hydrogen burning and partial helium burning. Thus, all stars
of initial mass between about $1.5 M_\odot$ and $8 M_\odot$ pass through the "peculiar"
phase. The term "peculiar" is a consequence of time scales: the
duration of the shell flashing stage is of the order of only 10^6 yr or
less, and it is this which makes PRGs appear to be such a rare phenome-
non. The helium shell flashing, or thermally pulsing stage, as it is
often called, does not begin until the star has developed a carbon-
oxygen core of mass larger than $\sim 0.5 M_\odot$ and dredge up does not begin
until the star is brighter than at the tip of the "first red giant
branch" through which stars of initial mass less than $\sim 2 M_\odot$ pass before
igniting helium in an electron degenerate core composed of helium. It
was not until studies of the brightest red stars in globular clusters
in the Magellanic Clouds had progressed far enough in this past decade
that a true understanding of the nature of the "asymptotic giant
branch" as distinct from "the first giant branch" became clear in the
context of PRGs. The PRGs that have developed peculiaries of their own
making are on the asymptotic giant branch (AGB). Those which appear to
be on the first red giant branch probably have a white dwarf companion
which was once an AGB star, as Robert McClure and his collaborators
have so elegantly demonstrated (see McClure, this conference).

As has been detailed by David Lambert and others at this conference,
the chemical "peculiarities" are of two main types: C/O ratios greater
than solar (carbon stars show this feature most prominently); and
overabundances of s-process isotopes (stars with technicium show this
most dramatically). Although not as straightforward a theoretical
consequence as we would like, particularly in the case of low-mass AGB
stars (see Lattanzio, this conference), it is generally agreed that the
overabundances of C relative to O are due to dredge up of material
which has experienced partial helium burning (X_{12} = abundance by mass
of $^{12}C \sim 0.25$, X_4 = abundance by mass of $^4 H_e \sim 0.75$). It is further
agreed, both on observational and theoretical grounds, that the source
of neutrons required to produce overabundances of s-process isotopes is
the $^{13}C(\alpha,n)^{16}O$ reaction. However, the manner in which ^{13}C is produced
is subject to controversy. It is still possible that, during a flash,
hydrogen enters the convective shell which is sustained by helium
burning at its base by some sort of "extra-mixing" process (in all
extant models in which radiation pressure is included, formal contact
between the outer edge of the convective shell and hydrogen-containing
material is prevented by an entropy barrier). Should hydrogen find its
way into the convective shell, it would react with the highly abundant
^{12}C there to form ^{13}N, which promptly beta decays into ^{13}C; the ^{13}C
would then be convected down to the center of the convective shell
where it would release neutrons on reacting with α particles.
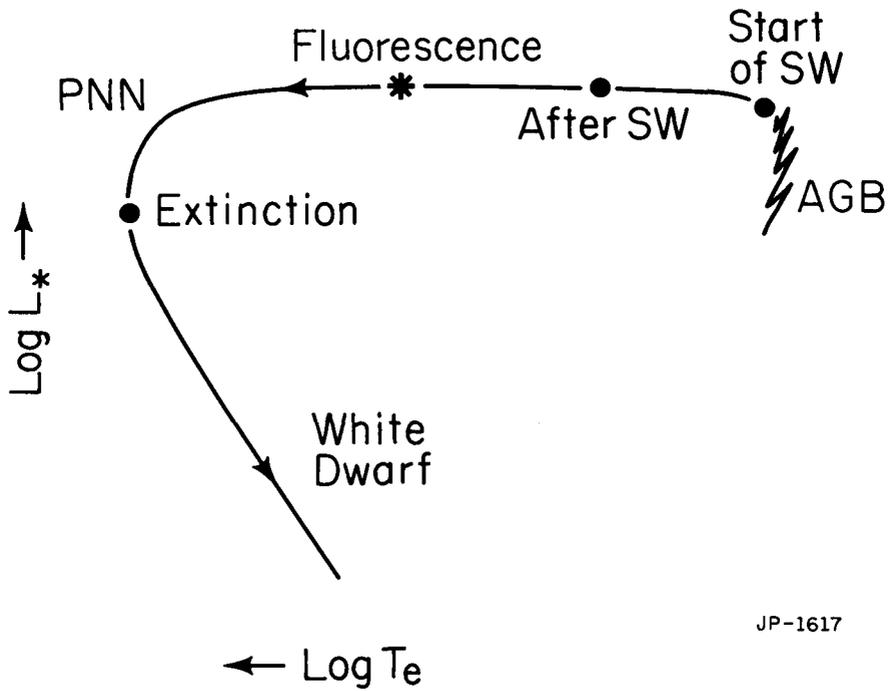
Figure 1. Schematic of evolution in the HR diagram from the asymptotic giant branch (AGB), through the planetary nucleus (PNN) phase, to the white dwarf (WD) phase.

On the other hand, it has been explicitly demonstrated that, following the peak of a thermal pulse in low mass models of low metallicity, semiconvection followed by hydrogen burning produces a layer of ^{13}C near the outer edge of the ^{12}C-rich region left behind by the retreating helium convective shell, and this ^{13}C is swept up by the convective shell which is formed during the next thermal pulse. The rate at which ^{13}C enters the convective shell is determined by the rate at which this shell grows and this, in turn, determines the rate at which neutrons are released near the base of the shell by the $^{13}C(\alpha,n)O^{16}$ reaction. A poster paper at this meeting by David Hollowell and myself (see also Hollowell and Iben 1988) shows how this works. That this process can lead to the production of s-process isotopes in solar system proportions is demonstrated beautifully by Roberto Gallino in his talk at this conference.

2 THE QUESTION OF WHEN A STAR LEAVES THE AGB
2.1 Overview

The standard picture of the evolution of a star from the AGB to the white dwarf stage is illustrated in Figure 1. Because of wind mass loss, the mass M_e of hydrogen-rich material above the burning zones in the AGB star continues to decline, until, when M_e decreases below a critical value (which depends on the mass of the CO core, on which nuclear fuel is burning, and possibly on other things as well), nuclear burning can be sustained only if the outer layers of the star can contract enough to maintain matter in the burning zone at high temperature. The star evolves rapidly toward the blue through the Hertzsprung gap and then evolves more slowly to the blue once the gap has been traversed. Continued nuclear burning in a shell and possibly continued mass loss act to reduce M_e still further, forcing the star to contract steadily and move to the blue.

When the surface temperature of the star becomes high enough (> 30,000K), photons of energy larger than the ionizing potential of hydrogen are emitted at a sufficiently high rate that surrounding material, which was blown off by the precursor both while it was a bonafide AGB star and while it was in the process of leaving the AGB, fluoresces as a planetary nebula. The central star continues to burn nuclear fuel and therefore to contract. Eventually, the central star approaches white dwarf dimensions and, ultimately, the weight of unburned fuel above the burning shell can no longer maintain large enough temperatures for nuclear burning to continue to supply the loss of energy from the surface.

The central star then evolves as a white dwarf to ever smaller luminosities and temperatures, with energy losses being supplied by the thermal energy of the ions in its interior. The emission measure of the expanding nebula drops and the nebula ultimately becomes invisible. As it continues to expand outward, the nebular material becomes incorporated into interstellar clouds, thus enriching the interstellar medium in carbon and s-process elements which were once in the envelope of the precursor PRG.

Note that I have not specified as yet which nuclear fuel is supplying
the energy during the planetary nebula stage. Ever since Shklovski
first proposed this general picture over three decades ago (Shklovski
1956), the general consensus has been that the energy source of most
planetary nebulae is hydrogen. This point of view became solidified
when, almost two decades ago, Paczynski (1970, 1971) constructed
explicit models based on this scenario by stripping hydrogen from the
surface of model AGB stars during the quiescent hydrogen-burning phase
until the models were forced to evolve off the AGB while still burning
hydrogen. About one decade ago, Härm and Schwarzschild (1975) repeated
the Paczynski excercise, but stripped mass from the stellar surface
during the high surface luminosity phase of a thermal pulse cycle when
the AGB model was burning only helium, and showed that one obtains
essentially the same evolutionary track and roughly the same lifetime
for the helium burning central star as for a central star of the same
mass which burns only hydrogen. Schonberner (1979) carried the
exercise one step further by stripping matter from the AGB model star
at a rate which is more or less independent of where in a thermal pulse
cycle the AGB star is, showing that, depending upon precisely where in
this cycle the envelope mass M_e decreases below the critical one (for
the fuel which happens to be burning at the time), a very complicated
post-AGB evolutionary behavior is possible. A frequently occurring
sequence consists of a post-AGB central star which at first burns
hydrogen at high surface temperature, then ignites a final helium shell
flash, returns to the AGB, and finally reverses direction in the HR
diagram once again to reach high surface temperatures during the
remainder of the quiescent helium-burning phase.

Building upon these numerical results, Renzini (1979, 1983) developed a
comprehensive scenario for planetary nebula formation and evolution,
and broadened the scope of the inquiry to postulate a causal connection
between the nature of mass loss on the AGB and the spectral
characteristics of white dwarfs. Iben (1984), Iben and Tutukov (1984),
Iben and MacDonald (1986), Wood and Faulkner (1986), and Schonberner
(1986, and references therein) constructed models useful in extending
Renzini's ideas.

In recent years, there has been a tremendous amplification in our
quantitative understanding of the variation of white dwarf spectral
characteristics with respect to surface temperature and magnitude,
wrought by technological advances and by the energy and dedication of a
large group of observational and theoretical white dwarf "buffs"
including Fontaine, Greenstein, Heber, Holberg, Koester, Kudritzki,
Liebert, Mendes, Wesemael, Schonberner, Shipman, Sion, Wegner,
Weidemann, and Winget, and their minions. One interpretation of these
observations suggests that the picture I am about to paint in the
remainder of this section and in the next cannot possibly be true. In
the final section, I will outline the salient features of the relevant
observations and the currently popular interpretation of the meaning of
these observations, leaving it to the reader to make his own judgement.

Top

Extent of Processing	Layer Content	Typical $M/M_\odot$
Pristine and Dredge-Up Products	H, He, CNO C s-process	3×10^{-4}
Partial H-Burning	^{1}H, ^{4}He, ^{14}N	10^{-4}
Complete H-Burning	^{4}He, ^{14}N Buffer Zone	$\gtrsim 10^{-2}$
Partial He-Burning	^{4}He, ^{12}C, ^{16}O, ^{22}Ne	10^{-2}
Complete He-Burning	^{12}C, ^{16}O, ^{22}Ne	0.6

Bottom   JP-1618

Figure 2.   The compositional structure of an AGB star.   The strata of different compositions are shown schematically, not to scale.
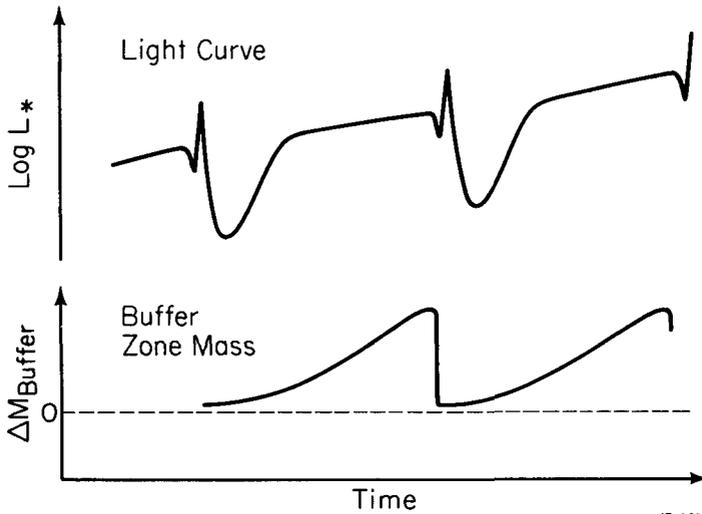
Figure 3.   The variation with time of the surface luminosity and of the mass of the helium buffer zone.   Schematic only.

2.2 Mass Loss and The Composition of a Post-AGB Star
The distribution of the most abundant elements in an AGB model star of core mass ~ $0.6M_\odot$ is shown schematically in Figure 2. As time progresses during a quiescent hydrogen-burning phase, the hydrogen-burning shell, of thickness ~ $10^{-4}M_\odot$ adds ^{4}He and ^{14}N to an underlying "buffer" zone. When the buffer zone grows in mass to about $10^{-2}M_\odot$, temperatures at its base become large enough to ignite helium and a helium shell flash takes place. The mass M_{buf} of the buffer layer is correlated with the surface luminosity of the AGB model in the fashion illustrated schematically in Figure 3. The behavior of a model post-AGB star depends strongly on the mass of the buffer zone when departure from the AGB is assumed to occur. What this mass should be is not known from first principles. At this point one can only guess. In time, with enough observational facts to explain, the answer may be forced upon us, if, as discussed in the section IV, this forcing has not already occurred.

The dependence on time of critical masses is shown schematically in Figure 4. The location of the hydrogen-burning shell is given by the curve labled M_H, the boundary of the convective shell formed during a helium shell flash is given by the outline of the shaded region, and two of many possibilities for the location of the surface are sketched as the curves labled M_*. In placing these latter two curves, it has been assumed that the mass of the hydrogen-rich surface layer, given by $M_e = M_* - M_H$, decreases below the critical value of $M_{e,crit}$ during the quiescent hydrogen-burning phase. $M_{e,crit}$ is about one-tenth of the mass ΔM_H through which the hydrogen-burning shell moves during the quiescent hydrogen-burning phase between pulses. For a core mass of $0.6M_\odot$, $\Delta M_H \sim 0.01M_\odot$, so that $M_{e,crit} \sim 0.001M_\odot$. The maximum buffer mass is, of course, equal to ΔM_H.

For the typical core mass of $0.6M_\odot$, the time between thermal pulses is ~ 200,000yr. If, between pulses, the mass loss rate is of the order of 10^{-5}-$10^{-4}M_\odot yr^{-1}$ (a so-called fast or "superwind" rate), it seems likely that the envelope mass will be reduced below the critical value at some arbitrary point during the interpulse phase, and so it makes sense to consider the post-AGB evolution of model stars with initial $M_e <$ $M_{e,crit}$ and $M_{buf} \sim (0-1)\Delta M_H$. Since the duration of the quiescent helium-burning phase is roughly 10% of the duration of the interpulse phase, approximately 10% of all central stars of planetary nebulae should be powered from the start by helium burning. Further, the critical mass $M_{e,crit,a}$ necessary for a helium-powered AGB star to depart from the AGB is smaller than $M_{e,crit}$ by a factor of 3-10, depending on how far into the helium-burning phase it has progressed. The reason for this is that the base of the hydrogen-rich envelope does not need to be compressed by the weight of overlying layers in order to supply enough energy to maintain the fluxes necessary to bloat these layers; the fluxes are maintained by helium burning and the base of the hydrogen-rich envelope is spatially much further from the electron-degenerate core than during the quiescent hydrogen-burning phase.

If wind mass loss during the interpulse phase occurs on a time scale long compared with the time between pulses (an "ordinary", "slow", or
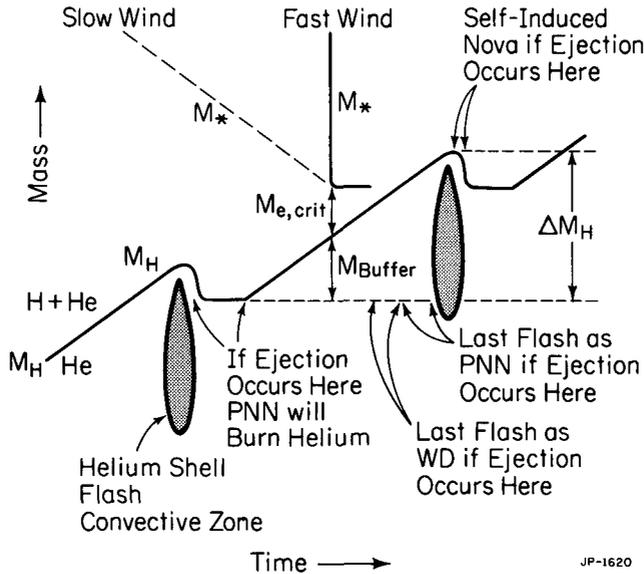
Figure 4. Schematic showing time variation of the location
in mass of: (1) the boundaries of the helium convective shell formed
during a thermal pulse [bordering shaded regions]; (2) the center of
the hydrogen-burning shell (during active hydrogen burning), or the
hydrogen-helium discontinuity (after dredge up and during the quiescent
helium-burning phase) [M_H]; (3) the surface of the star [M_*].
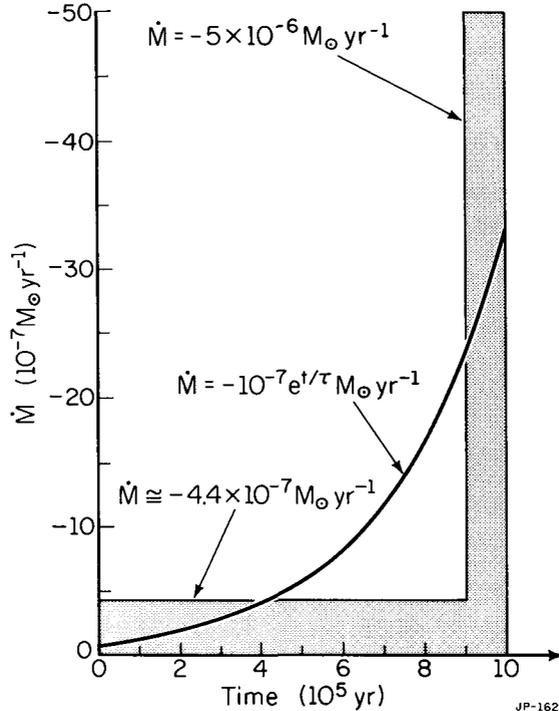
Figure 5. Hypothetical mass-loss rates as a function of
time. The area under the exponential (τ - 2.86 x 10^5 yr) is the same
as under the two-component curve consisting of a "slow" wind followed
by a "superwind".

"Reimer's" wind), one might guess that the sharp jolt that the star experiences during a thermal pulse (see the jump in surface luminosity in Fig. 3) might trigger mass loss at a much higher rate than average, and that departure from the AGB would be likely to occur most often at the time of a pulse. In this case, most central stars of planetary nebulae should be powered by helium burning.

Very strong feelings are sometimes aroused when those of us in the model-making business use the terms "superwind" and "slow wind" or their variants. This useage is, of course, an oversimplification, but unless and until the observers can provide us with the rate at which an AGB star of a given total mass and core mass loses mass, or can tell us where in a thermal-pulse cycle final departure from the AGB occurs, I cannot appreciate the objections raised to an admitted oversimplification of a complicated physical process that neither observers nor theo-·reticians can yet quantitatively describe. To extend this thought, consider an AGB model star of initial mass $1.5 M_\odot$ and initial core mass $0.5 M_\odot$ which loses mass at the hypothetical rate of $10^{-7} M_\odot \, yr^{-1}$ x $\exp(t/2.86 \times 10^5 yr)$. At the end of $10^6 yr$ it will have lost $0.9 M_\odot$ and its core mass will have grown to $0.6 M_\odot$, so that it must of necessity leave the AGB. The hypothetical mass-loss rate is shown by the solid curve in Figure 5. Shown also in Figure 5 is another hypothetical mass-loss rate consisting of a "slow" wind of magnitude $4.44 \times 10^{-7} M_\odot \, yr^{-1}$ which operates for $0.9 \times 10^6 yr$ and a "superwind" of magnitude $0.5 \times 10^{-5} M_\odot \, yr^{-1}$ which operates for $10^5 yr$. The area under the two-mode function is identical with that under the exponential curve, and the outcome, inso-far as we are interested in when departure from the AGB occurs, is the same in the two cases. I am not convinced that the two-mode function is less likely to eventually fit the facts than the exponential; as far as I understand it, there are, in fact, insufficient facts to be able to draw observationally based curves in the plane of Figure 5.

3   POST-AGB HELIUM SHELL FLASHES AND THE BORN AGAIN AGB PHASE

There are six main types of evolution for model post-AGB stars, four for those which depart from the AGB while burning hydrogen, and two for those which depart while burning helium. In describing the hydrogen burners, it is convenient to define an angle $\phi = M_{buf}/\Delta M_H$. Although all illustrations here will be made for models of ~ $0.6 M_\odot$ and metallicity Z = 0.001, the behavior of models of larger or smaller mass is qualitatively the same, and the classification in terms of ϕ is expected to be essentially mass invariant. It is, however, composition dependent.

Model stars which depart from the AGB with $\phi < 0.75$ evolve for ~ $10^4 yr$ as luminous, hot central stars of planetary nebulae until the mass M_e in their hydrogen-rich envelope drops below about $10^{-4} M_\odot$ (see Figure 6 for evolution in the HR diagram). At this point, hydrogen-burning by CN cycle reactions is extinguished, and the model dims by about a factor of ten in luminosity in a matter of only $10^3 yr$.   Although mass loss at rates observed for PN central stars (~ 10^{-9}-$10^{-7} M_\odot \, yr^{-1}$) abstracts mass from the surface and thus accelerates the rate of
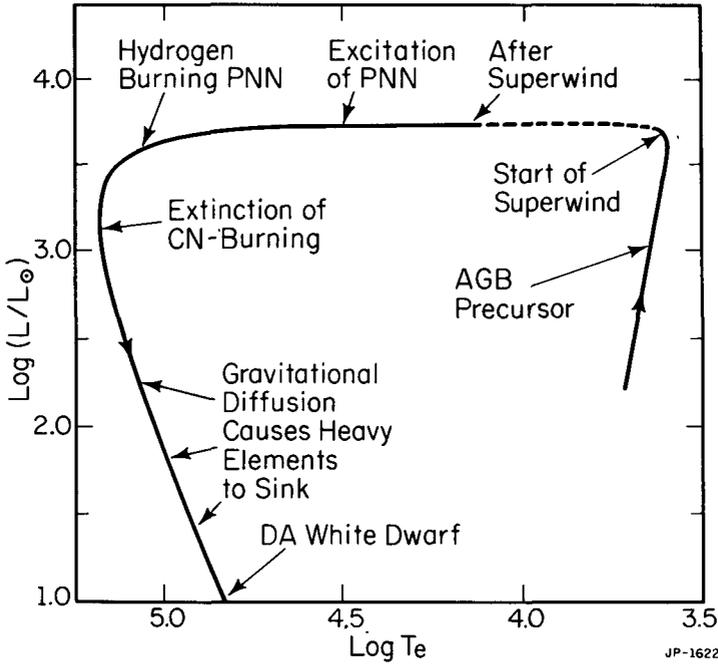
Figure 6.   Evolution in the HR diagram for models with
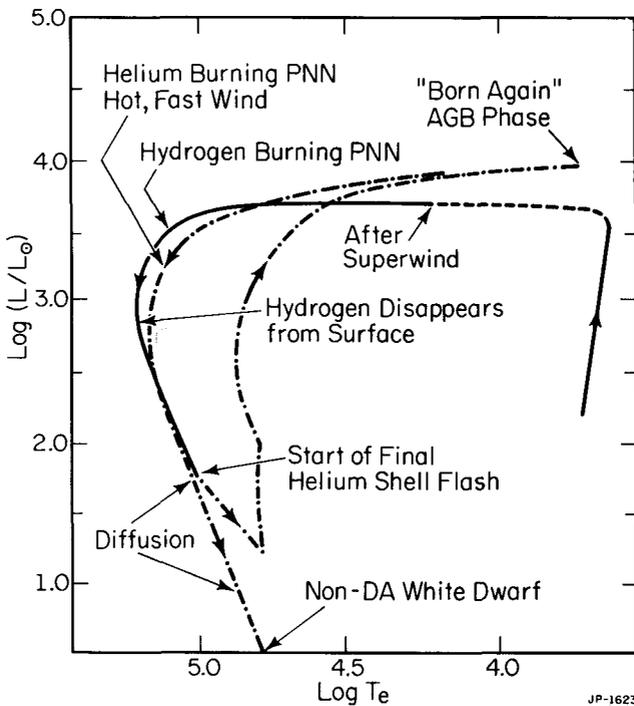buffer masses between ~ 0.15 and 0.75 times the maximum possible.

Figure 7.   Evolution in the HR diagram for models with
buffer masses between 0.75 and 0.85 times the maximum possible.

evolution in the HR diagram, the model star will in any case arrive at the turning point of highest surface temperature with $M_e \sim$ few x 10^{-4} $M_\odot$, a value which has come to be known as a "thick" hydrogen envelope. After the phase of rapid dimming, the model then proceeds to cool as a white dwarf, with its luminosity being supplied primarily by the thermal energy of ions in the interior. Gravitational settling forces helium and the heavy elements to sink below the surface. However, diffusion also insures that there will be a tail of hydrogen that extends deeply into the star. As long as ϕ is larger than a critical value (estimated to be ~ 0.15), no further episode of hydrogen burning will occur, and, even if mass loss from the surface continues, the inner tail of hydrogen will be shielded from loss. To reduce the total amount of hydrogen in the star to less than $\sim 10^{-13} M_\odot$, as is argued in some quarters (see section IV), would require a most unusual wind indeed. If such a wind does not occur, then the real analogues of the models with $0.15 < \phi < 0.75$ should become DA white dwarfs.

Up to the point that hydrogen burning by CN-cycle reactions is extinguished, the evolution of models with $0.75 < \phi < 0.85$ (shown in Figure 7) is essentially identical with that of models with $0.15 < \phi < 0.75$. Following the extinction of nuclear burning in both cases, as the flux of energy through it is reduced, the hydrogen-rich layers contract rapidly, increasing the weight on the helium-rich buffer zone below, thus compressing and heating matter in this zone. When $\phi < 0.75$, the compression and heating is not sufficient to ignite helium-burning reactions, and the story of nuclear burning is by and large over, except for a very mild burning of hydrogen via the pp chains (provided mass loss does not abstract too much more of the hydrogen surface layer). For $\phi > 0.75$, howevever, the compression and heating is sufficient; the star experiences a final helium shell flash, as first predicted by Masayuki Fujimoto over a decade ago (Fujimoto 1977). Since hydrogen is not burning, there is no entropy barrier to overcome, and the convective shell which is formed in the helium buffer zone extends into hydrogen-rich layers, further decreasing M_e.

What happens after this is not known precisely. The star may return to the AGB (see Figure 7) as an R CrB-like star burning helium quiescently in a shell of mass $\sim 0.01 M_\odot$ and hydrogen at the base of an envelope of mass $\sim 0.001 M_\odot$ in which the abundance by mass of hydrogen is very small (say, $< 10^{-4} M_\odot$, giving a total mass of hydrogen in the star $< 10^{-7} M_\odot$). Or, helium burning could continue to be the only source of surface luminosity, and the same wind which caused departure of the progenitor from the AGB could abstract more hydrogen-rich matter from the star.

In any case, the "born again" AGB phase must be of very short duration, and the model, now with a total mass of hydrogen much less than $10^{-4} M_\odot$, quickly moves to high temperatures, nearly retracing the path it followed earlier as a hydrogen-burning PN central star. Since helium burning keeps the model at high luminosities and surface temperatures for $\sim 3 \times 10^4$ yr, wind mass loss at rates typical of real PN central stars may be expected to abstract all of the remaining hydrogen from the real analogue of the model before diffusion inward has had a chance to hide it. The real analogue becomes a luminous DO star and then,
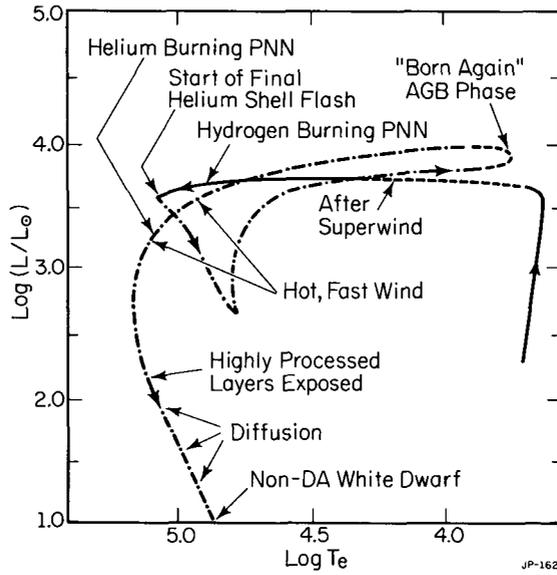
Figure 8.   Evolution in the HR diagram for models with
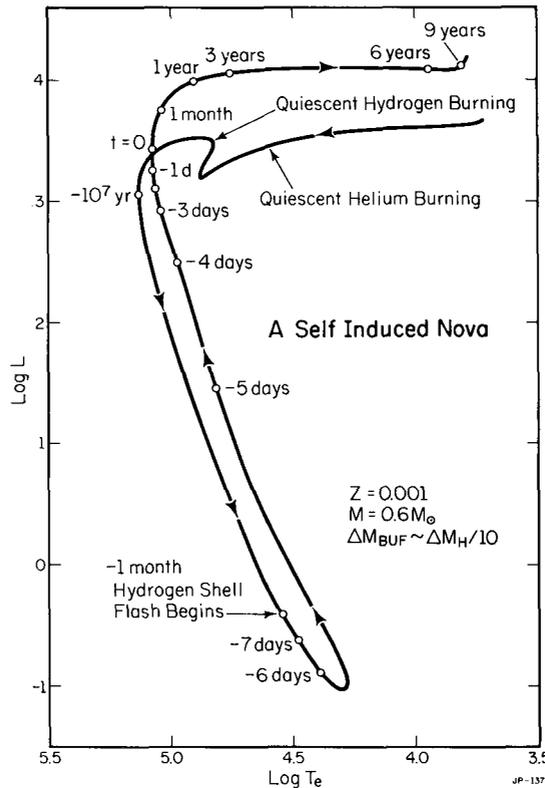buffer masses between 0.85 and 1.00 times the maximum possible.

Figure 9.   Evolution in the HR diagram for models which
depart the AGB during a helium flash or during the quiescent helium-
burning phase with a thick layer of hydrogen at their surfaces. Models
which depart from the AGB while burning hydrogen, but with $\phi < \phi_{min} \sim$
0.15 (?) will also experience the "self induced" nova phenomenon.

after helium burning is extinguished, cools to become a non-DA white
dwarf.

Models with $0.85 < \phi < 1.0$ have a large enough buffer mass as they
depart from the AGB that, during their tenure as hydrogen-burning PN
central stars, the buffer mass grows to be as large as the critical
value (ΔM_H) which leads to a shell helium flash on the AGB. But, since
hydrogen is still burning, an entropy barrier prevents contact between
the convective shell formed in the buffer layer and hydrogen-rich
regions. Therefore, model evolution can be followed in a straight-
forward fashion, with the result shown in Figure 8. The model returns
to the AGB, formally remaining there for a time proportional to M_e.
Since the surface characteristics of the model are similar to those of
its AGB precursor, a superwind should operate once again, and since
hydrogen-rich layers now occupy a region in the density-temperature
plane which was occupied by the hydrogen-rich matter lost in the first
superwind phase, this superwind should reduce M_e considerably. But,
reduction of M_e again forces the model to return to the blue, where it
continues to burn helium on a long time scale and lose mass from its
surface by a hot, fast wind. Eventually, a real analogue of this model
star should evolve into a non-DA white dwarf.

Up to the point that hydrogen-burning by CN-cycle reactions is first
extinguished (at the bluest point along an evolutionary track) and
diffusion becomes important, the evolution of models with $\phi \sim 0.15$ or
less is the same as that of models with $0.15 < \phi < 0.75$. Thereafter,
howevever, carbon diffusing through the small buffer layer from below
and hydrogen diffusing inward through the buffer layer from above (see
Figure 2) meet within the buffer zone at such high temperatures and at
such high abundances that CN-cycle burning is activated again and the
model experiences a hydrogen shell flash which carries it back to the
giant branch in the fashion of a slow nova, as shown by the dash-dot
portion of the evolutionary track in Figure 9 (from Iben and MacDonald
1986).

The calculation whose results are described in Figure 9 has actually
been done for a model which departs from the AGB while it is burning
helium quiescently. The mass in the envelope at departure is $M_e \sim$
10^{-3} $M_\odot$, and this illustrates the statement made earlier that the value
of M_e required for departure from the AGB during helium burning depends
on how far into the quiescent helium-burning phase the model has
progressed. As a consequence of helium burning, the mass of the buffer
zone is steadily reduced, but the hydrogen-rich envelope of the model
continues to contract and the temperature at the base of this envelope
continues to rise until hydrogen-burning is again ignited, and this
adds mass to the buffer zone. Once both hydrogen- and helium-burning
are extinguished, the mass of the buffer zone is $\sim 10^{-3} M_\odot$, giving $\phi \sim$
0.1; the mass of the hydrogen-rich layer remaining is the canonical few
x $10^{-4} M_\odot$. The configuration of the model at this point could also have
been achieved by waiting until the end of the quiescent helium-burning
phase before removing all but $\sim 10^{-3} M_\odot$ of hydrogen-rich matter from the
surface.

In any case, the "self induced" nova phenomenon occurs: (1) when the
model star departs from the AGB shortly after the end of the quiescent
helium-burning phase with $(M_{buf} + M_e - \text{few x } 10^{-4} M_\odot) < M_{buf,crit} \sim$
0.15; and (2) when departure from the AGB occurs either during the
helium-flashing stage or during the quiescent helium-burning stage, <u>if</u>
enough hydrogen, say $M_e > M_{e,dif}$, is retained at the surface. If M_e is
too small, then there will not be enough fuel to produce a thermal run-
away along the white dwarf cooling sequence. Obviously, calculations
need to be carried out to determine both $M_{e,dif}$ and $M_{buf,crit}$.

A major consequence of the final hydrogen shell flash is, of course,
the reduction in the total amount of hydrogen in the star. Nuclear
burning will destroy the hydrogen tail that has been built up by
diffusion prior to the flash. As a giant, more hydrogen-rich material
will be abstracted from the surface by a stellar wind. The net result
is that, when the flash has run its course and the star again becomes a
white dwarf, the mass of the hydrogen envelope will be smaller (perhaps
much smaller) than when the star first became a white dwarf.

The final type of theoretical behavior is obtained by abstracting mass
from a helium shell flashing or quiescent helium-burning model until so
little hydrogen is left in surface layers that no final hydrogen-
burning phase can occur. Obviously, if all of the hydrogen were
abstracted, the existence of DA stars would be difficult to explain,
except perhaps by some convoluted scenario involving accretion from the
interstellar medium. However, something close to the almost complete
abstraction of hydrogen is receiving increasingly serious attention, as
will be described in the next section.

4 THE INCREDIBLY THIN HYDROGEN ENVELOPE SCENARIO
There are several lines of evidence which may be used to
argue against the scenario I have just described. These arguments have
been summarized recently by Fontaine and Wesemael (1987), Koester
(1987), and Shipman (1988). Most of the arguments call into question
the possibility that DA white dwarfs have "thick" hydrogen envelopes,
as follows from evolutionary theory when it is assumed that most
central stars of planetary nebulae are burning hydrogen and that
stellar winds are not very effective after the extinction of hydrogen-
burning by CN-cycle reactions.

For example, it is often stated that a DA white dwarf model cannot
pulsate as a ZZ Ceti star if the mass of hydrogen in its envelope
exceeds $\sim 10^{-7} M_\odot$, a value which is three orders of magnitude smaller
than suggested by the evolutionary calculations, given the assumptions
just made. A more accurate statement is that model white dwarfs of the
appropriate luminosity do not pulsate at surface temperatures
consistent with the observations (even though they may indeed pulsate)
unless, when a "standard" treatment of convection is assumed, the
hydrogen envelope is chosen to be "thin" (namely less massive than
$\sim 10^{-7} M_\odot$). It is not out of the question, however, that the standard
theory of convection is the culprit, and not the mass of the hydrogen
envelope. For that matter, some other aspect of the input physics,

such as the opacity or the ionization equation of state, could be at
fault.

On the other hand, there may be no reason why a radiatively driven wind
might not continue to operate after hydrogen-burning by CN-cycle
reactions ceases and reduce the mass of hydrogen remaining in surface
layers to the amount suggested by current calculations with "standard"
physics. Indeed, by adopting a hypothetical wind which does not
violate energy and momentum conservation laws (e.g., Mv = L/c, where M
is the mass loss rate, L is the stellar luminosity, c is the velocity
of light, and v is some factor times the escape velocity from the
star), it is possible to abstract most of the mass of hydrogen
remaining after the cessation of hydrogen burning by the time the ZZ
Ceti region is reached (Iben and Tutukov 1986).

Thus, this particular argument for a thin hydrogen envelope does not
destroy the overall scenario described in section III, particularly
when it is remembered that winds already play a major role in this
scenario. In fact, by invoking a wind which continues during the
cooling phase, one acquires an additional advantage: the potential for
explaining why the frequency of non-DA dwarfs increases with decreasing
luminosity in spite of the fact that all white dwarfs in the solar
vicinity should experience episodes of accretion as they pass through
interstellar clouds.

A much more formidable argument against the thick shell scenario (even
as modified by invoking winds during the cooling phase) is based on the
fact that there are no known DA white dwarfs with surface temperatures
larger than 75000K, although DO white dwarfs with surface temperatures
well in excess of 100000K are numerous, and on the fact that there is a
distinct absence of non-DA white dwarfs in the surface temperature
range 30000-45000K. The argument assumes that all white dwarfs have a
common origin, and therefore must begin their cooling phase as hot DO
white dwarfs, turn into DA white dwarfs when their surface temperature
drops below some critical value between 75000K and 45000K (presumably
depending on differences in initial mass and composition, and so
forth), then reemerge as non-DA white dwarfs as they cool to surface
temperatures below 30000K (Liebert, Fontaine, and Wesemael 1987;
Fontaine and Wesemael 1987).

The detailed argument requires that the total mass of hydrogen in
surface layers of a DO white dwarf be less than ~ $10^{-13}M_\odot$. Presumably,
the white dwarf is born with this incredibly small mass of hydrogen,
most of which is apparently hidden in a long tail below the surface.
As the star cools, gravitational diffusion brings hydrogen to the
surface, and drives heavier elements into the interior, so that the
star becomes a DA white dwarf (but, why is the transition not sharp in
the DA distribution at 45000K as well?). Then, as it cools still
further to a surface temperature of about 30000K, a convective zone
that appears in the region of helium ionization below the thin hydrogen
layer is clever enough to reach up to grab hydrogen and devour it,
making some stars (not all!) turn once again into non-DA white dwarfs.

The specifics of this scenario clearly have some loose ends to tidy up.
However, the "extremely thin" (as opposed to thin) hydrogen envelopes
demanded by the scenario have some support from current interpretations
of the X-ray spectra of DO white dwarfs (Holberg 1988). If the hydro-
gen envelopes really are this thin, then it is very difficult to escape
the conclusion that AGB evolution is invariably terminated either
during a helium shell flash or during the ensuing quiescent helium
burning phase, and that all central stars of bright planetary nebulae
must be sustaining themselves by burning helium rather than hydrogen.
A linear pulsation analysis by Kawaler (1988) suggests that hydrogen-
burning central stars must pulsate at some point (although a linear
analysis cannot predict amplitudes) and a search for pulsations in real
central stars (Robinson 1988) reveals that none are pulsating (above
some noise limit); the interpretation favored by the authors is that
hydrogen-burning central stars are ruled out, an interpretation which
to me seems a bit premature when based on their results alone. How-
ever, when coupled with the current interpretation of X-ray spectra,
with the absence of DA white dwarfs with surface temperatures greater
than 75000K, and with the gap in the non-DA distributions at surface
temperatures in the range 30000-45000K, the evidence for "extremely
thin" hydrogen envelopes becomes very impressive. The evidence is not
yet compelling, because there are other arguments, based on other
observational data, which can be used to defend the thick envelope
scenario. One of these has been constructed by Schonberner (1986), who
presents evidence in the number-luminosity distribution for DA white
dwarfs for the rapid envelope contraction phase that, in models with
thick envelopes, follows immediately upon the extinction of hydrogen
burning by CN-cycle reactions. This rapid phase of envelope
contraction does not occur in helium-burning models.

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