NONLINEAR DYNAMO MODES AND TIMESCALES OF STELLAR ACTIVITY

G. Belvedere(*) and M. R. E. Proctor(**)

(*) *Istituto di Astronomia, Università di Catania, Italy.*

(**) *Dept. Applied Mathamtics and Theoretical Physics. University of Cambridge, UK.*

ABSTRACT. A simple mean-field model of a nonlinear stellar α-ω dynamo is considered, in which dynamo action is supposed to occur in a spherical shell, and where the main nonlinearity retained is the influence of the Lorentz force on the zonal flow field. The equations are simplified by truncating in the radial direction, while full latitudinal dependence is retained. The resulting nonlinear p.d.e.'s in latitude and time are solved numerically, and it is found that while regular dynamo wave type solutions are stable when the dynamo number D is sufficiently close to its critical value, there is a wide variety of stable solutions at larger values of D. Furthermore, two different types of dynamo can coexist at the same parameter values. Implications for fields in late-type stars are discussed.

1. INTRODUCTION

The observational evidence of magnetic activity with cyclical and non-cyclical behaviour in stars other than the Sun has suggested that the mechanism giving rise to stellar activity may operate in a variety of different ways. The physics of the interaction between rotation and convective modes is very complex and poorly understood, despite the increasingly detailed models that have been investigated in recent years.

So far, most theoretical work in stellar activity has been done in the framework of the α-ω linear theory and considering the simple theoretical basis, the results are encouraging, giving some useful general principles that may be expected to hold independently of the models.

However, there is no doubt that present and future

J. O. Stenflo (ed.), Solar Photosphere: Structure, Convection, and Magnetic Fields, 447–453.
© 1990 by the IAU.

https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press

theoretical research in stellar activity has to be carried out in the framework of the more rigorous and self consistent nonlinear approach, which in principle can describe a large variety of dynamo operation modes.

Here, a simple mean - field model of a nonlinear stellar dynamo is considered, in which dynamo action is supposed to occur in a spherical shell, and where the main nonlinearity retained is the influence of the Lorentz force on the zonal flow field.

Weiss, Cattaneo & Jones (1984) have constructed a low order system of ordinary differential equations describing stellar cycles, using a severely truncated representation of the spatial structure, and the simplest nonlinear couplings between the magnetic field and the mean rotation. They found, in various parameter ranges, regular cyclical behaviour, quasi - periodic oscillations and aperiodic cycles with features similar to those deduced from sunspot number observations and climatological evidence, including "grand minima". A drawback of their model is its local nature: the interaction between active regions in each hemisphere cannot be represented, and their equations cannot yield steady (non cyclical) activity although such solutions are known to be possible for linear α-effect dynamo models.

Our work develops the ideas of the above model. We retain the α-effect formalism, and the simplest nonlinear interactions, but use a representation of the spatial structure only in the radial direction thus allowing full latitudinal dependence of fields and flows.

While we do not claim to be simulating any particular stellar dynamo, we do feel that our model can capture many of the essential physical properties of the nonlinear interactions.

2. DERIVATION OF THE MODEL EQUATIONS

We begin with the axisymmetric mean field dynamo equations (see for example Parker 1955, Moffatt 1978) for the evolution of a magnetic field $\mathbf{B} = B(r,\theta)\hat{\boldsymbol{\phi}}+\nabla\times(A(r,\theta)\hat{\boldsymbol{\phi}})$ where r, θ are spherical polar coordinates and $\hat{\boldsymbol{\phi}}$ is the unit vector in the azimuthal direction. $B\hat{\boldsymbol{\phi}}$ is the toroidal and $\mathbf{B}_p=\nabla\times A\hat{\boldsymbol{\phi}}$ the poloidal part of $\mathbf{B}$. These equations take the form (in the presumed absence of any mean poloidal flow field)

$$\frac{\partial A}{\partial t} = \alpha F(r,\theta)B+\eta_t \left[\nabla^2 - \frac{1}{r^2\sin^2\theta} \right] A \qquad (1)$$

$$\frac{\partial B}{\partial t} = r\sin\theta \mathbf{B}_p\cdot\nabla \left[\frac{U(r,\theta)}{r\sin\theta} \hat{\boldsymbol{\phi}} \right] +\eta_t \left[\nabla^2 - \frac{1}{r^2\sin^2\theta} \right] B \qquad (2)$$

https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press

Here αF is the usual α-effect, with F representing its spatial structure and α its magnitude. Note also that, consistent with previous models of the solar dynamo, the differential rotation U is considered to be much more potent than the α-effect in producing toroidal field, and the latter term is thus omitted from (2). The quantity η_t is a turbulent diffusivity.

The dynamical influence of the magnetic field enters the model through its effect on the differential rotation $U(r,\theta)$. We write $U=u_o+u$, where u_o is a prescribed velocity field and u is a perturbation driven directly by the mean Lorentz force, and subject to viscous damping. The simplest equation that encompasses these features of the evolution of u is

$$\rho \frac{\partial u}{\partial t} = \frac{1}{\mu_o} (\nabla\times B\times B)_\Phi + \rho\nu_t \left[\nabla^2 - \frac{1}{r^2\sin^2\theta} \right] u \tag{3}$$

where ν_t is a turbulent viscosity.

The aim, then, is to solve these equations in a spherical shell, representing the convection zone of the star in question. This implicitly assumes that the dynamo is operating throughout the stellar convection zone. However, suggestions have been made in recent years about the possibility that it is instead confined in the thin overshoot layer just beneath the bottom of the C.Z. (see e.g. Spiegel and Weiss 1980; Spruit and van Ballegooijen 1982). However, this is not a crucial point. What we want to show here is that in the non-linear regime a variety of dynamo operation modes arises in a spherical shell. We don't claim to describe real stars, but what *may* occur in real stars. Furthermore, in the context of the radial averaging that we later perform on the equations, the differences between the two scenarios will be manifested only in changes in the coefficients that appear in the truncated equations. We hope in future work to investigate the different consequences of confining the dynamo process to an overshoot layer, and of incorporating the results of the most recent helioseismological data (Brown and Morrow 1987; Brown et al. 1988).

Dimensionless equations are obtained by adopting the following scaling factors: $r_o=6.96\times10^8$m, $\eta_t=10^8$ m^2s^{-1}, $\tau_o=r_o^2/\eta_t\simeq 5\times10^9$s $\simeq$ 160y, $\Omega^*=2.57\times10^{-6}$rad s^{-1}, $B^*=\Omega^* r_o\sqrt{(\mu_o\rho^*)}$, $A^*=\eta_t\sqrt{(\mu_o\rho^*)}$, where ρ^* is an appropriate average of density across the c.z.. Thus three basic dimensionless parameters appear, namely r_b (radius of the bottom of the c.z),

https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press

$D = \alpha\,\Omega^* r_o^3/\eta_t^2$ (dynamo number), $P_m = \nu_t/\eta_t$ (magnetic Prandtl number).

We look for solutions of the form: $A=f(r)\chi(\theta,t)/\sin\theta$; $B=g(r)\psi(\theta,t)/\sin\theta$; $u=h(r)\,Q(\theta,t)/\sin\theta$, with the following boundary conditions: $A=0$ at $r=r_b$ (radiative zone a good conductor); A matches a potential field at $r=1$; $\partial(rB)/\partial r=0$ at $r=r_b$ (no tangential current); $B=0$ at $r=1$; $\partial(u/r)/\partial r=0$ at $r=r_b$, $r=1$ (no stress).

We adopt simple forms of f(r), g(r), h(r), which satisfy the boundary conditions. We also choose the α-effect variation $F(r,\theta)=a(r)\sin\theta\cos\theta$ and the basic zonal field u_o to match the observed latitudinal differential rotation at $r=1$, while reducing to solid body rotation (with the equatorial value of the surface angular velocity) at the base of the convection zone.

We seek a model problem in which the radial dependence is integrated out (radial truncation) so that A, B, u are functions of θ and t only.

Thus the radially truncated dimensionless equations are given by multiplying equations (1) to (3) by $r^2 f(r)$, $r^2 g(r)$, $r^2 h(r)$ respectively and integrating from r_b to 1. After some algebra, we obtain the equations (in dimensionless variables)

$$C_A \frac{\partial\chi}{\partial t} = C_A D \sin\theta\cos\theta\,\psi + \eta_1 \sin\theta \frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial\chi}{\partial\theta}\right] - \eta_2\chi \qquad (4)$$

$$C_B \frac{\partial\psi}{\partial t} = C_1 H \sin\theta \frac{\partial\chi}{\partial\theta} - C_2 \frac{\partial H}{\partial\theta}\chi\sin\theta + \frac{C_3 Q}{\sin\theta}\frac{\partial\chi}{\partial\theta} \qquad (5)$$

$$- C_4 \sin\theta \frac{\partial}{\partial\theta}(Q/\sin^2\theta)\,\chi + \eta_3 \sin\theta \frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right] - \eta_4\psi$$

$$C_\Omega \frac{\partial Q}{\partial t} = -(C_3+C_4)\frac{\psi}{\sin\theta}\frac{\partial\chi}{\partial\theta} - \frac{C_4}{\sin\theta}\chi\frac{\partial\psi}{\partial\theta} \qquad (6)$$

$$+ P_m\left[\nu_1 \sin\theta \frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial Q}{\partial\theta}\right] - \nu_2 Q\right],$$

where C_A, C_B, C_Ω, the C_i, η_i and ν_i are constants that depend only on the functions f,g,h and $H(\theta) = 1-.189\,P_2(\cos\theta) -.0394\,P_4(\cos\theta)$ gives the latitudinal part of the differential rotation (Durney 1974). Note that it is not

https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press

necessary to define the form a(r); this reinforces our earlier remarks about it not being too necessary to decide the radial dependence of α.

Equations (4) to (6) are to be solved with the boundary conditions,appropriate for axisymmetric fields: $\chi = \psi = Q = 0$, $\theta = 0, \pi$.

In practice, the computations are actually carried out only in the hemisphere $0 \leq \theta \leq \pi/2$, with the symmetry conditions $\psi = \frac{\partial \chi}{\partial \theta} = \frac{\partial Q}{\partial \theta} = 0$ at $\theta = \pi/2$ to simulate a poloidal field of dipole type.

The equations were solved using an explicit time - stepping method of DuFort - Frankel type, for various values of D, P_m, r_b and various initial conditions. The equations were represented on a spatial mesh, with between 20 and 60 mesh intervals. Convergence with respect to temporal and spatial resolution was checked, and found to be quite satisfactory. No problems were experienced at $\theta = 0, \pi$, in spite of the coordinate singularity there. This appears to be due, firstly to a careful treatment of the second order differences so as to give an accurate representation near the poles, and secondly because χ, ψ, Q all vanish quadratically as $\theta \longrightarrow 0, \pi$.

3 . RESULTS AND DISCUSSION

It is well known that highly truncated nonlinear dynamo models can exhibit irregular oscillations as shown by Weiss et al.. However these models suffer from the deficiency of having no latitudinal resolution. Not only does this prevent the construction of butterly diagrams, but it also means that steady solutions of the dynamo equations cannot be found. Our model, on the other hand, is able to describe propagation of dynamo waves and the spherical geometry makes possible a wide range of nonlinear phenomena. In particular, the symmetry between successive cycles may be broken, leading to a preponderance of one polarity over the other.

Furthermore, solutions have been found which spend a long time to an (unstable) steady state and then show a pulsed behaviour. Though these solutions do not closely resemble the modulation of the solar cyle (e.g. the Maunder minimum), they do show that significant variations in activity are possible. Weiss et al.'s calculations showed Maunder minimum - like behaviour but only for the most severely truncated model. The pulsed behaviour occurs for values of the dynamo number ranging (in the particular case P_m=0.1) from D $\simeq$ 625 to beyond D = 2000. Solutions are asym-

https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press

metric until D ≃ 1800, but symmetric above this value.

In our model dynamo action occurs for D large than about 70. From D ≃ 70 to D ≃ 340 we get periodic symmetric stable solutions. From D ≃ 325 to D ≃ 750 there are stable solutions which are still periodic but asymmetric. However quasi - periodic stable solutions are also allowed in the range D ≃ 350 - 640. The latter resemble quasi - periodic solutions found by Weiss et al. Thus, D ≃ 350 seems to correspond to a point of subcritical bifurcation.

Therefore our results show that two or three different forms of stable solution can coexist in suitable ranges of the dynamo number, depending on the initial conditions. This suggests that there might be large differences in the activity signatures of very similar stars.

Figure 1. shows the time variation of the toroidal field (Psi), the poloidal field (Chi), the differential rotation perturbation (Q) and the related butterfly diagram for four different values of α = 310,750,550,1600 corresponding (top to bottom) to periodic symmetric, periodic asymmetric, quasi periodic and pulsed solutions.

Our calculations were carried out principally for P_m=0.1, but no significant differences appear for other values, provided P_m is not too large. We did find, though, that the bifurcation structure is sensitively affected by the assumed form of the radial dependence of the zonal velocity perturbation h(r) and maybe, by the radial dependences of the poloidal and toroidal fields, but we have not investigated the latter point. Indeed, for some forms of h(r), we did not find asymmetric or pulsed behaviour. Therefore, a further investigation should include a better description of the radial dependencies (ideally by incorporating full radial resolution).

Further developments of the present model will include runs with different values of r_b, to look at dynamos in stars with different depths of the c.z. and, of course, with different rates of rotation (this can be modelled by varying the dynamo number).

REFERENCES

Brown, T.M., and Morrow, C.A.: 1987, Astrophys. J. 314, L21.

Brown, T.M., Christensen - Dalsgaard, J.,Dziembowski, W.A., Goode, P., Gough, D.O. and Morrow, C.A.: 1989, Astrophys, J. in press

Durney, B.R.: 1974, Astrophys. J. 190, 211

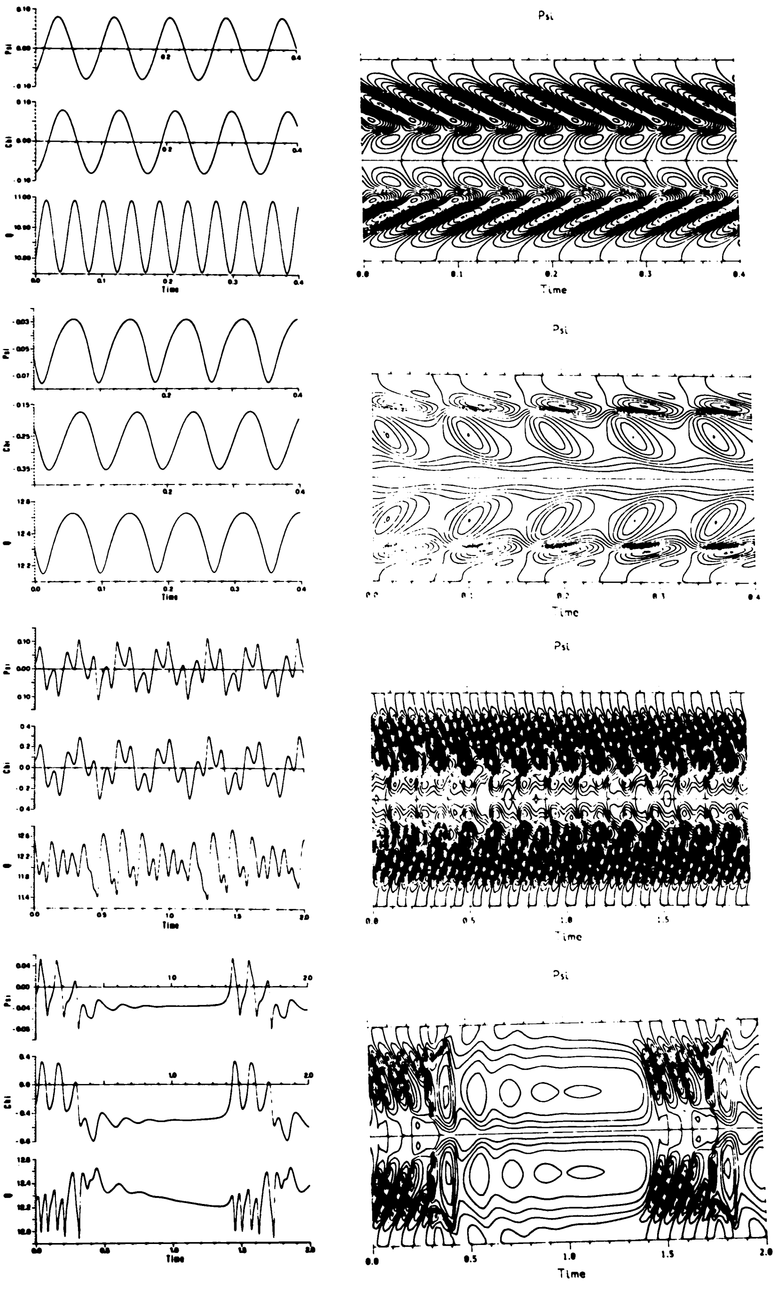
Moffatt, H.K.: 1978, *Magnetic Field Generation in Electrically Conducting Fluids*, Cambridge Univ. Press

Parker, E.N.: 1955, Astrophys. J. 122, 293

https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press

Spiegel, E.A. and Weiss, N.O.: 1980, Nature 287, 616
Spruit, H.C. and Van Ballegooijen, A.A.: 1982, Astron. Astrophys. 106, 58
Weiss, N.O., Cattaneo, F. and Jones, C.A.: 1984, Geophys. Astrophys. Fluid Dyn. 30, 305

FIGURE 1



https://doi.org/10.1017/S0074180900044454 Published online by Cambridge University Press