In [3], where $\widetilde{\mathrm{Inv}}(\mathfrak{U})$ was introduced to prove an Ax–Kochen–Eršov-type result, it was claimed that this semigroup is always well-defined and commutative. We disprove both statements, provide $\sim_{\mathrm{D}}$-invariants, and show independence of $S^{\mathrm{inv}}(\mathfrak{U})/\sim_{\mathrm{D}}$ from the choice of $\mathfrak{U}$ to contradict the Independence Property.

Theorem A. *There is a supersimple theory of SU-rank 2 in which $\sim_D$ is not a congruence with respect to $\otimes$, and where $\geq_D$ differs from nonforking-domination. Moreover, in the Random Graph $\widetilde{\mathrm{Inv}}(\mathfrak{U})$ is not commutative.*

Theorem B. *If $p_0 \geq_D p_1$ and p_0 is definable, finitely satisfiable in some small model, generically stable, or weakly orthogonal to q, then so is p_1.*

Theorem C. *If there are only boundedly many $\sim_D$-classes, then T is* NIP.

Beyond the above results from [4], we reduce the study of $\widetilde{\mathrm{Inv}}(\mathfrak{U})$ in o-minimal context to proving that every invariant type is equivalent to a product of 1-types, and show this to hold in Real Closed Fields. This yields a complete characterisation both in this theory and, using results from [1], in that of Real Closed Valued Fields. We also survey the stable case, compute $\widetilde{\mathrm{Inv}}(\mathfrak{U})$ in several other theories, including that of dense meet-trees, and show its well-definedness in certain expansions of the latter studied in [2].

Theorem D. *In Real Closed Fields, $(\widetilde{\mathrm{Inv}}(\mathfrak{U}),\otimes)$ is well-defined and isomorphic to the semilattice of finite subsets $(\mathscr{P}_{fin}(X),\cup)$, where X is the set of convex subrings of $\mathfrak{U}$ which, for some small A, are fixed by the stabiliser* $\mathrm{Aut}(\mathfrak{U}/A)$.

Theorem E. *In dense meet-trees, $\widetilde{\mathrm{Inv}}(\mathfrak{U})$ has the form $\mathscr{P}_{fin}(X)\oplus\bigoplus_{|\mathfrak{U}|}\mathbb{N}$.*

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Abstract prepared by Rosario Mennuni.
E-mail: R.Mennuni@tutanota.com
URL: http://etheses.whiterose.ac.uk/26807/

José Miguel Blanco, ***An Implicative Expansion of Belnap's Four-Valued Matrix: A Modal Four-Valued Logic Without Strong Modal Lukasiewicz-Type Paradoxes*** (in Spanish), University of Salamanca, Spain, 2018. Supervised by Gemma Robles and José M. Méndez. MSC: 03B47, 03B45, 03B50, 03B53. Keywords: substructural logics, relevance logics, modal logics, many-valued logics.

Abstract

Towards the end of his life, the great Polish logician J. Łukasiewicz developed the four-valued modal system known as Ł. This system validated theses as $(MA \wedge MB) \to M(A \wedge B)$

or $L(A \vee B) \to (LA \vee LB)$, which are part of what it is known as strong modal Łukasiewicz-type paradoxes. Because of this, this system was strongly criticized. On the other hand, in [Brady, 1982], R. T. Brady presents his relevant logic BN4, a four-valued version of the relevant implication system R. Taking this background into account, the main goal of this research is to build a system that works as a companion of BN4 (just like E does with respect to R) and lacks the paradoxes that can be found in Łukasiewicz's system. Firstly, we define the matrix M4, which is the base for all the systems that we develop later. We then introduce two different semantics, i.e., the four-valued semantics related to the matrix and a bivalent Belnap-Dunn type semantics, and we show that both semantics are equivalent. Next, the system that we have labeled FDF4, which is based on FDE, is defined. We prove that this system is both sound and complete in the strong sense and that it is indeed an axiomatization of the M4 matrix. Afterwards, we define a system based on E that we name EF4, for which we also prove strong soundness and completeness and how it originates from the M4 matrix, all of this based on the fact that EF4 is a system equivalent to FDF4. With respect to EF4, two different modalities are presented: the first one, which in this case is equivalent to the inherent modality of E, is developed from the interdefinitional extensions used by Łukasiewicz, and the second one, from the proposal of J. Y. Beziau related to the approach of J. M. Font and M. Rius that in its turn is linked to the Portuguese algebraic tradition led by A. Monteiro. In this way, we get two different modal systems, EF4-M and EF4-Ł. For the former, we give just one axiomatization, while for the latter, we supply up to four different ones. For both systems, EF4-M and EF4-Ł, we prove soundness and completeness. Furthermore, EF4 is provided with a reduced ternary relational semantics, as well as with a 2-set-up ternary relational semantics, and it is proven that it is sound and complete with respect to both semantics. Finally, it is shown that the system FDF4 is also sound and complete with respect to both aforementioned relational semantics and that the 2-set-up semantics is a particular case of the reduced semantics.

Abstract prepared by José Miguel Blanco.
E-mail: jmblanco@usal.es
URL: http://hdl.handle.net/10366/139440

Patrick Walsh, ***A Categorical Characterization of Accessible Domain***, Carnegie Mellon University, USA, 2019. Supervised by Wilfried Sieg. MSC: 00A30, 03G30, 03D70. Keywords: category theory, Hilbert's finitist program, inductive definitions, algebraic set theory.

Abstract

Inductively defined structures are ubiquitous in mathematics; their specification is unambiguous and their properties are powerful. All fields of mathematical logic feature these structures prominently: the formulas of a language, the set of theorems, the natural numbers, the primitive recursive functions, the constructive number classes and segments of the cumulative hierarchy of sets.

This dissertation gives a mathematical characterization of a species of inductively defined structures, called *accessible domains*, which include all of the above examples except the set of theorems. The concept of an accessible domain comes from Wilfried Sieg's analysis of proof-theoretic practices, starting with his dissertation (contributed to [1]). In particular, he noticed the special epistemological character of elements of an accessible domain: they can always be uniquely identified with their build-up. Generally, the unique build-up of elements justifies the principles of induction and recursion.

I use category theory to give an abstract characterization of accessible domains. I claim that accessible domains are all instances of *initial algebras for endofunctors*. Grounded in the historical roots of Sieg's discussions, this dissertation shows how the properties of initial algebras for endofunctors and accessible domains coincide in a satisfying and natural way.

Filling out this characterization, I show how important examples of accessible domains fit into this broad characterization. I first characterize some accessible domains by relatively