7
Units and conventions

To define the units and conventions used in this book, and to set the stage for the subsequent analysis, we conclude this introduction by writing Maxwell's equations for the electromagnetic field in vacuum with sources. With the use of Heaviside–Lorentz (rationalized c.g.s.) units these equations are[1]

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \rho \\ \nabla \cdot \mathbf{H} &= 0 \\ \nabla \times \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} &= \mathbf{j} \\ \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} &= 0 \end{aligned} \tag{7.1}$$

Here ρ and $\mathbf{j}$ are the local charge and current density; the former is measured in e.s.u. and the latter in e.m.u. where 1 e.m.u = 1 e.s.u./c. The Lorentz force equation and fine structure constant are given respectively by

$$\begin{aligned} \mathbf{F} &= e\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{H}\right) \\ \frac{e^2}{4\pi\hbar c} &= \alpha = \frac{1}{137.04} \end{aligned} \tag{7.2}$$

Introduce the antisymmetric electromagnetic field tensor

$$F_{\mu\nu} = \begin{pmatrix} 0 & H_3 & -H_2 & -iE_1 \\ -H_3 & 0 & H_1 & -iE_2 \\ H_2 & -H_1 & 0 & -iE_3 \\ iE_1 & iE_2 & iE_3 & 0 \end{pmatrix} \tag{7.3}$$

[1] In this case the magnetic field is $\mathbf{H} \equiv \mathbf{B}$.

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Straightforward algebra then shows that Maxwell's equations can be written in covariant form as

$$
\begin{aligned}
\frac{\partial}{\partial x_\nu} F_{\mu\nu} &= j_\mu \\
\varepsilon_{\mu\nu\rho\sigma} \frac{\partial}{\partial x_\sigma} F_{\nu\rho} &= 0
\end{aligned} \tag{7.4}
$$

Here $\varepsilon_{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor and repeated Greek indices are summed from one to four. Also

$$
\begin{aligned}
x_\mu &= (\mathbf{x}, ict) \\
j_\mu &= (\mathbf{j}, i\rho)
\end{aligned} \tag{7.5}
$$

The second set of Maxwell's Equations can be satisfied identically with the introduction of a vector potential

$$
\begin{aligned}
F_{\mu\nu} &= \frac{\partial}{\partial x_\mu} A_\nu - \frac{\partial}{\partial x_\nu} A_\mu \\
A_\mu &= (\mathbf{A}, i\Phi)
\end{aligned} \tag{7.6}
$$

Comparison with Eq. (7.3) then allows the identification

$$
\begin{aligned}
\mathbf{H} &= \nabla \times \mathbf{A} \\
\mathbf{E} &= -\nabla\Phi - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}
\end{aligned} \tag{7.7}
$$

While k and q are used interchangeably in the following for the four-momentum transfer in inclusive electron scattering $(\mathrm{e}, \mathrm{e}')$, with a direction defined in context,[2] when the coincidence process $(\mathrm{e}, \mathrm{e}'\,\mathrm{X})$ is discussed, k is reserved for the four-momentum transfer of the electron to the target and q for the four-momentum of the produced particle X.

[2] Unfortunately, this is common usage. The four-momentum transfer will be denoted $(\boldsymbol{\kappa}, i\omega/c)$, with a direction again defined in context.

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