$$\frac{\triangle ABC}{\triangle PQR}=\frac{\triangle ACD}{\triangle PRS}=\frac{\triangle ADE}{\triangle PST}=\frac{\triangle ABCDE}{\triangle PQRST}=\frac{AB^2}{PQ^2}.$$

Or, *similar polygons may be divided into the same number of similar triangles, which bear to one another the same ratio as the polygons, and this ratio is the square of the ratio of corresponding sides.*

(4) Let $\triangle ABC$ have A a right angle; draw AD the perpendicular from A to BC. Then $\triangle s$ ABC, ABD, ACD, being equiangular, may be regarded as three maps of the same three places in a country, on different scales. And $\triangle ABC = \triangle ABD + \triangle ACD$. Therefore any area of the map represented by ABC = sum of corresponding areas of the maps represented by ABD, ACD.

$$\therefore \quad BC^2 = AB^2 + AC^2.$$

The above are meant as illustrations of the corresponding propositions in geometry, or as the "proofs" necessary for a working knowledge of the propositions, in a preliminary course of geometry which includes similar figures.

P. PINKERTON.

Internal and External Bisectors, and an Example of Continuity.—I. To draw quickly a good figure of A, B, C, I, I_1, I_2, I_3, etc. Draw a circle, and a chord BC. Mark L the middle point of the arc below BC by the "engineer's method," viz., with B as centre and a radius as near BL as can be judged by the eye, make a mark on the arc, with C as centre and the same radius make another mark on the arc, judge by the eye the middle point of the arc between these marks; this is L. With centre L, radius LB or LC, describe a circle: I and I_1 lie on this circle. Mark M the middle point of the arc above BC. With centre M, radius MB or MC, describe a circle: I_2 and I_3 lie on this circle. Now I and I_1 lie on AL; I_2 and I_3 on AM. Mark A on the first circle, so that MA lies conveniently on the paper. The various collinearities and perpendicularities justify the figure to the eye; the properties of the mid-points of II_1, etc., I_2I_3, etc., and the loci of I, I_1, I_2, I_3 as A varies, are emphasised.

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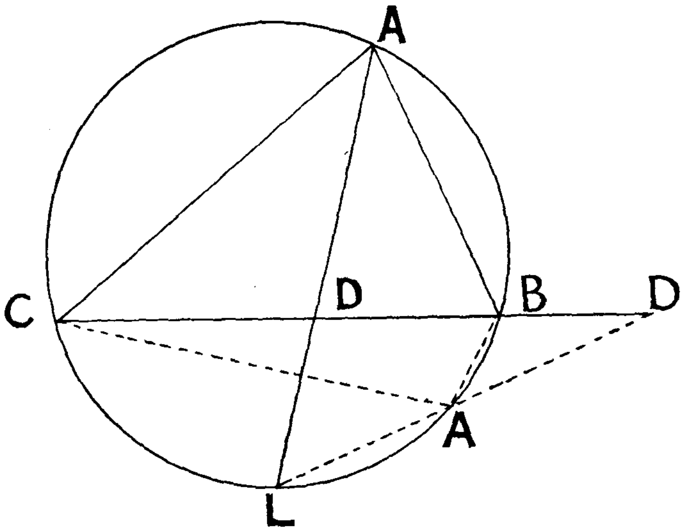
II. An example of Continuity. Let ABC be a triangle inscribed in a circle; let L be the mid-point of the arc below the base BC; let AL cut BC in D.



Then we have

(1) $BD/DC = BA/AC$

(2) $CD = \frac{b}{b+c}$ of $a = \frac{ba}{b+c}$

(3) $BD = \frac{c}{b+c}$ of $a = \frac{ca}{b+c}$

(4) $AD^2 = BA \, . \, AC - BD \, . \, DC$

(5) $AD = \sqrt{bc(a+b+c)(b+c-a)}/(b+c)$.

Imagine A to move round the circle towards B and to continue its motion *past* B and take up the new position A. L is now the mid-point of the arc *above* the base BC of $\triangle$ABC; LA is the *external* bisector of $\widehat{A}$ of $\triangle$ABC; D lies *without* BC. The side AB or c has gone *through zero* to the new AB, so that AB or c in the formulae (1)–(5) must have the *negative* sign prefixed to give the corresponding property for AD an *external* bisector of $\widehat{A}$ of

$\triangle$ABC. Also DB has gone *through zero* to the new DB, so that the *negative* sign must likewise be prefixed to DB in formulae (1)–(5), to obtain formulae regarding $\triangle$ABC and AD the external bisector of $\widehat{A}$. No other term in the formulae has gone through zero, except AD, which practically occurs only in the second power. Hence if the *external* bisector of $\widehat{A}$ of $\triangle$ABC meet BC produced in D,

(1) gives $-\mathrm{BD}/\mathrm{DC} = -\mathrm{BA}/\mathrm{AC}$ or $\mathrm{BD}/\mathrm{DC} = \mathrm{BA}/\mathrm{AC}$;

(2) gives $\mathrm{CD} = \dfrac{ba}{b-c}$;

(3) gives $-\mathrm{BD} = \dfrac{-ca}{b-c}$ or $\mathrm{BD} = \dfrac{ca}{b-c}$;

(4) gives $\mathrm{AD}^2 = -\mathrm{BA}\,.\,\mathrm{AC} + \mathrm{BD}\,.\,\mathrm{DC}$;
or $\mathrm{AD}^2 = \mathrm{BD}\,.\,\mathrm{DC} - \mathrm{BA}\,.\,\mathrm{AC}$;

(5) gives $|\mathrm{AD}| = \sqrt{bc(a+b-c)(c+a-b)}/(b-c)$.

Further remarks of a similar nature will occur to the reader.

P. PINKERTON.

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