MOON'S PLANETARY PERTURBATIONS

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In ELP, the computation of planetary perturbations is about 20 years old. A better knowledge of lunar and planetary parameters, new planetary solutions under construction and progresses in numerical tools, are factors that should contribute to their improvements. The construction of planetary perturbations takes widely its inspiration from Brown's method. In a first step, we only consider the main problem (Earth, Moon, and Sun with a Keplerian motion). The solution of the main problem is actually of a high precision and is used as a reference (Chapront-Touzé, 1980). This solution is expressed in Fourier series of the 4 Delaunay arguments, with numerical coefficients, and partials with respect to integration constants.

1. Integration Method

The method based on the variation of arbitrary constants is described in (M.Chapront-Touzé, J.Chapront, 1980). Equations of Moon's motion are written in a rotating frame where the reference plane is the mean ecliptic. In this frame, the absolute acceleration is expressed by means of disturbing forces acting on the Moon, by the Sun, the Earth and a planet. It is the gradient of F which can be divided into several components : F_c related to the main problem, F_D and F_I giving rise to direct and indirect planetary perturbations. Equations of motion are initialy expressed in rectangular coordinates (x_i, u_i), $u_i = \dfrac{dx_i}{dt}$:

$$\frac{dx_i}{dt} = \frac{\partial \mathcal{H}_c}{\partial u_i}, \qquad \frac{du_i}{dt} = -\frac{\partial \mathcal{H}_c}{\partial x_i}$$

where $\mathcal{H}_c$ is the main problem's Hamiltonian. Then we perform a change of variables more appropriate to the lunar problem. Equations are thus expressed with the set of variables $\{z_1^0 = \frac{n'}{\nu}, z_2^0 = E, z_3^0 = \Gamma, w_1, w_2, w_3\}$, n' and ν being respectively Sun's and Moon's mean motions, Γ and E halves of the coefficients of the $\sin \overline{F}$ term in latitude and $\sin \overline{l}$ term in longitude. $w_i = b_i t + w_i^0$ are respectively the mean longitude, the longitude of the perigee and the longitude of the node of the Moon. The system becomes :

$$\left(\frac{dz_j^0}{dt}\right) = C^{-1}\left(\frac{\partial \mathcal{R}}{\partial w_i}\right)$$
$$\left(\frac{dw_j}{dt}\right) = (b_j) - [C^{-1}]^T\left(\frac{\partial \mathcal{R}}{\partial z_i^0}\right)$$

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$\mathcal{R}$ represents in either case, the direct or indirect disturbing force function. The C matrix is composed of $c_{i,j} = \dfrac{\partial c_i}{\partial z_j^0}$ where $c_i = \left\langle \sum_k u_k \dfrac{\partial x_k}{\partial w_j} \right\rangle$, the brackets meaning the constant term of the series. After integrating the system, the integration constants δz_j^0 are chosen to force the solution to be fitted on the main problem constants (ν, Γ, E).

2. Direct and Indirect Perturbations

For direct perturbations, function F_D is developped in Legendre polynomials of the variable $\Theta_P = \dfrac{1}{rD} \overrightarrow{EM} \cdot \overrightarrow{GP}$. Next a separation of the variables depending on the Moon from those depending on the planet is performed. This makes the integration easier in the case of small divisors. In F_D, the distance D between a planet and the Earth-Moon barycenter is developped in Fourier series of the planetary longitudes. For indirect perturbations, F_I is now concerned : Sun's elements are not any longer Keplerian but contain planetary perturbations. We use a planetary solution for the set : $\{\sigma_k, k = 1, \ldots, 6\} = \{a', \lambda', k' = e' \cos \varpi', h' = e' \sin \varpi', q' = \sin \frac{i'}{2} \cos \Omega', p' = \sin \frac{i'}{2} \sin \Omega'\}$. Note that : $\sigma_k = \sigma_k^{(0)} + \Delta\sigma_k$. As in the direct case, $\mathcal{R}_c = F_c - k\dfrac{m_E + m_M}{r}$ is developped in Legendre polynomials of the variable $\Theta' = \dfrac{1}{rr'} \overrightarrow{EM} \cdot \overrightarrow{GS}$.

3. Computation and Precision

Integrating the product of Moon and planet series, small divisors can appear and blow up the coefficients. Hence we sort coefficients and their associated frequencies in order to keep long period terms with small coefficients. However, terms with periods exceeding five thousand years are linearized. Once the series for the six variables z_j^0 and w_j are determined, we construct the coordinates substituing these results into longitude, latitude and radius vector expressions. Our computations let us hope a final precision for planetary perturbations of about 10^{-5}".

References

Chapront-Touzé M.: 1980, *Astron. Astrophys.*, **83**, 86
Chapront-Touzé M.and Chapront J.: 1980, *Astron. Astrophys.*, **91**, 233