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A NOTE ON 2-LOCAL MAPS

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Abstract The aim of this note is to characterize 2-local automorphisms and derivations on matrix rings over finite-dimensional division rings.

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1. Introduction

The study of local maps was initiated by Kadison [**18**] and Larson and Sourour [**20**]. In 1997, Šemrl [**29**] introduced the concepts of 2-local automorphisms and derivations on the algebra $B(H)$. Let A be an algebra. A (non-additive) map $\varphi : A \to A$ is called a 2-*local automorphism* if, for every $a, b \in A$, there exists an automorphism $\sigma_{a,b} : A \to A$ such that $\varphi(a) = \sigma_{a,b}(a)$ and $\varphi(b) = \sigma_{a,b}(b)$. Similarly, a (non-additive) map $\delta : A \to A$ is called a 2-*local derivation* if, for every $a, b \in A$, there exists a derivation $d_{a,b} : A \to A$ such that $\delta(a) = d_{a,b}(a)$ and $\delta(b) = d_{a,b}(b)$.

Local and 2-local maps have been studied on different operator algebras by many authors [**2**–**7**, **15**–**17**, **19**, **21**–**28**].

It is interesting to note that the study of local maps on finite-dimensional algebras is sometimes more difficult than in the infinite-dimensional case. In [**29**], Šemrl described 2-local automorphisms on the algebra $B(H)$, all bounded linear operators on the infinite-dimensional separable Hilbert space H. However, for the case when H is finite dimensional, Šemrl's original proof was long and involved tedious computations. A similar description for the finite-dimensional case appeared later, in [**19**, **24**]. Our first goal is to describe 2-local automorphisms on matrix algebras over finite-dimensional division rings.

Theorem 1.1. *Let K be a finite-dimensional division algebra over its centre Z with characteristic not 2, and let $M_n(K)$, $n \geqslant 1$, be the ring of $n \times n$ matrices over K. Then every 2-local automorphism of $M_n(K)$ is an automorphism or an anti-automorphism* *of $M_n(K)$. Moreover, if $n \geqslant 2$, then every 2-local automorphism of $M_n(K)$ is an automorphism of $M_n(K)$.*

This result is a generalization of theorems due to Molnar [**24**] and Kim and Kim [**19**] obtained for $M_n(\mathbb{C})$. It also generalizes a theorem by Chebotar *et al.* [**5**, Theorem 5.3], where 2-local automorphisms of finite-dimensional division rings K with characteristic 0 were described. It is interesting to note that the case of anti-automorphism (if $n = 1$) is really possible (see [**5**, Example 5.4]).

Our second theorem gives a description of 2-local derivations on matrix algebras over finite-dimensional division rings.

Theorem 1.2. *Let K be a finite-dimensional division algebra over its centre Z with characteristic not 2, and let $M_n(K)$, $n \geqslant 1$, be the ring of $n \times n$ matrices over K. Then every 2-local derivation of $M_n(K)$ is a derivation.*

This result is a generalization of Kim and Kim's theorem [**19**] obtained for $M_n(\mathbb{C})$.

Finally, motivated by [**5**, Theorem 2.1], we prove the following result.

Theorem 1.3. *Let K be a division ring with centre Z and let $M_n(K)$, $n \geqslant 2$, be the ring of $n \times n$ matrices over K. Suppose that $\alpha : M_n(K) \to M_n(K)$ is a bijective additive map such that*

$$\alpha(a^{-1})\alpha(a) = \alpha(b^{-1})\alpha(b) \neq 0 \quad \text{for all invertible } a, b \in M_n(K).$$

Then $\alpha = \lambda\varphi$, where $\varphi : M_n(K) \to M_n(K)$ is an automorphism or an anti-automorphism and $\lambda = \alpha(1) \in Z$.

This result is connected with the well-known Hua theorem [**14**] and it generalizes some results of [**5**, **10**].

2. 2-local automorphisms and derivations on matrix algebras over division rings

Let K be a finite-dimensional division algebra over its centre Z, and let $M_n(K)$ be the ring of $n \times n$ matrices over K. We denote by e_{ij} the matrix unit, that is, the matrix which has a one in the (i, j)-position and zeros elsewhere.

Let $\mathrm{tr} : K \to Z$ be a reduced trace of K and $\mathrm{Tr} : M_n(K) \to Z$ be the trace map of $M_n(K)$ defined by $\mathrm{Tr}(A) = \mathrm{tr}(a_{11}) + \mathrm{tr}(a_{22}) + \cdots + \mathrm{tr}(a_{nn})$ if $A = \sum_{i,j} a_{ij}e_{ij} \in M_n(K)$.

We first recall the following result about the reduced trace (see, for example, [**9**, p. 148, Lemma 4]).

Lemma 2.1. *There exists an $a \in K$ such that $\mathrm{tr}(a) \neq 0$.*

Lemma 2.2. *If A is non-zero in $M_n(K)$, then there exists a $B \in M_n(K)$ such that $\mathrm{Tr}(AB) \neq 0$.*

Proof. We denote A by $\sum_{i,j} a_{ij}e_{ij}$. Since $A \neq 0$, say $a_{st} \neq 0$ in K for some $1 \leqslant s, t \leqslant n$. By Lemma 2.1, we can pick an $a \in K$ such that $\mathrm{tr}(a) \neq 0$. Let $B = a_{st}^{-1}ae_{ts}$. We have $AB = \sum_i a_{it}a_{st}^{-1}ae_{is}$ and so $\mathrm{Tr}(AB) = \mathrm{tr}(a) \neq 0$ as desired. □

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Now we can describe 2-local automorphisms of matrix algebras over finite-dimensional division rings using some ideas from [**5**, **24**].

Proof of Theorem 1.1. Let $\varphi : M_n(K) \to M_n(K)$ be a 2-local automorphism. For every $x, y \in M_n(K)$, there exists an automorphism $\sigma_{x,y}$ on $M_n(K)$ such that $\varphi(x) = \sigma_{x,y}(x)$ and $\varphi(y) = \sigma_{x,y}(y)$. By [**13**, Theorem 4.3.1], there exists an invertible $c \in M_n(K)$ such that $\sigma_{x,y}(x) = cxc^{-1}$ and $\sigma_{x,y}(y) = cyc^{-1}$. Therefore,

$$\varphi(x)\varphi(y) = \sigma_{x,y}(x)\sigma_{x,y}(y) = cxyc^{-1} \tag{2.1}$$

and so

$$\mathrm{Tr}(\varphi(x)\varphi(y)) = \mathrm{Tr}(xy) \quad \text{for all } x, y \in M_n(K). \tag{2.2}$$

Let $\{k_1, k_2, \dots, k_m\}$ be a basis of K over Z. We claim that $\varphi(k_i e_{jl})$, $1 \leqslant i \leqslant m$, $1 \leqslant j, l \leqslant n$, are linearly independent over Z. Assume on the contrary that there exist λ_{ijl} in Z not all zero, say $\lambda_{i_0 j_0 l_0} \neq 0$, such that

$$\sum_{i,j,l} \lambda_{ijl}\varphi(k_i e_{jl}) = 0.$$

By Lemma 2.1, there exists an $a \in K$ such that $\mathrm{tr}(a) \neq 0$. Since $\sum_i \lambda_{ij_0l_0} k_i \neq 0$, let

$$b = \biggl(\sum_i \lambda_{ij_0l_0} k_i\biggr)^{-1} a.$$

It follows from (2.2) and the linearity of the trace map that

$$\begin{aligned}
0 &= \mathrm{Tr}\biggl(\biggl[\sum_{i,j,l} \lambda_{ijl}\varphi(k_i e_{jl})\biggr]\varphi(be_{l_0j_0})\biggr) \\
&= \sum_{i,j,l} \lambda_{ijl}\,\mathrm{Tr}(\varphi(k_i e_{jl})\varphi(be_{l_0j_0})) \\
&= \sum_{i,j,l} \lambda_{ijl}\,\mathrm{Tr}(k_i b e_{jl} e_{l_0j_0}) \\
&= \sum_i \lambda_{ij_0l_0}\,\mathrm{tr}(k_i b) \\
&= \mathrm{tr}\biggl(\biggl(\sum_i \lambda_{ij_0l_0} k_i\biggr) b\biggr) \\
&= \mathrm{tr}(a),
\end{aligned}$$

which is a contradiction. Therefore, the $\varphi(k_i e_{jl})$, $1 \leqslant i \leqslant m$, $1 \leqslant j, l \leqslant n$, are linearly independent over Z and hence span $M_n(K)$ over Z.

We can now prove the linearity of φ over Z. For each $u, v \in M_n(K)$ and for every i, j, l, we find from (2.2) that

$$\begin{aligned}\operatorname{Tr}(\varphi(u+v)\varphi(k_i e_{jl})) &= \operatorname{Tr}((u+v)k_i e_{jl})\\ &= \operatorname{Tr}(uk_i e_{jl}) + \operatorname{Tr}(vk_i e_{jl})\\ &= \operatorname{Tr}(\varphi(u)\varphi(k_i e_{jl})) + \operatorname{Tr}(\varphi(v)\varphi(k_i e_{jl}))\\ &= \operatorname{Tr}((\varphi(u)+\varphi(v))\varphi(k_i e_{jl})).\end{aligned}$$

Since the $\varphi(k_i e_{jl})$ span $M_n(K)$ over Z, we have

$$\operatorname{Tr}((\varphi(u+v) - \varphi(u) - \varphi(v))x) = 0 \quad \text{for all } x, u, v \in M_n(K).$$

By Lemma 2.2, we have $\varphi(u+v) - \varphi(u) - \varphi(v) = 0$. That is, $\varphi(u+v) = \varphi(u) + \varphi(v)$ for all $u, v \in M_n(K)$.

For each $\alpha \in Z$ and $u \in M_n(K)$, there exists an automorphism $\sigma_{u,\alpha u}$ such that $\varphi(u) = \sigma_{u,\alpha u}(u)$ and $\varphi(\alpha u) = \sigma_{u,\alpha u}(\alpha u)$. Then

$$\varphi(\alpha u) = \sigma_{u,\alpha u}(\alpha u) = \alpha\sigma_{u,\alpha u}(u) = \alpha\varphi(u).$$

That is, φ is a linear map on $M_n(K)$ over Z. Being a 2-local automorphism, φ is injective and hence is surjective, since $M_n(K)$ is finite dimensional over Z.

Note that, for each $u \in M_n(K)$, there exists an automorphism σ_{u,u^2} such that $\varphi(u) = \sigma_{u,u^2}(u)$ and $\varphi(u^2) = \sigma_{u,u^2}(u^2)$. Then $\varphi(u^2) = \sigma_{u,u^2}(u^2) = \sigma_{u,u^2}(u)^2 = \varphi(u)^2$ for all $u \in M_n(K)$. Therefore, φ is a Jordan automorphism. Since the characteristic of K is not 2, it follows from the Herstein theorem [**11**] that φ is an automorphism or an anti-automorphism.

Finally, let $n > 1$. Suppose that φ is an anti-automorphism. Substituting $x = e_{11}$ and $y = e_{12}$ in (2.1), we obtain $0 = \varphi(yx) = \varphi(x)\varphi(y) = cxyc^{-1}$, which is a contradiction. □

We shall now describe 2-local derivations of matrix algebras over finite-dimensional division rings.

Proof of Theorem 1.2. Let $\delta : M_n(K) \to M_n(K)$ be a 2-local derivation. For each $x, y \in M_n(K)$, there exists a derivation $d_{x,y}$ on $M_n(K)$ such that $\delta(x) = d_{x,y}(x)$ and $\delta(y) = d_{x,y}(y)$. By the proposition in [**13**, p. 100], there exists an invertible $c \in M_n(K)$ such that $[c, xy] = d_{x,y}(xy) = d_{x,y}(x)y + xd_{x,y}(y) = \delta(x)y + x\delta(y)$. Thus, we have

$$0 = \operatorname{Tr}([c, xy]) = \operatorname{Tr}(\delta(x)y + x\delta(y)) \quad \text{and so} \quad \operatorname{Tr}(\delta(x)y) = -\operatorname{Tr}(x\delta(y)).$$

Therefore,

$$\begin{aligned}\operatorname{Tr}(\delta(u+v)z) &= -\operatorname{Tr}((u+v)\delta(z))\\ &= -\operatorname{Tr}(u\delta(z)) - \operatorname{Tr}(v\delta(z))\\ &= \operatorname{Tr}(\delta(u)z) + \operatorname{Tr}(\delta(v)z)\\ &= \operatorname{Tr}((\delta(u)+\delta(v))z)\end{aligned}$$

and so

$$\mathrm{Tr}((\delta(u+v)-\delta(u)-\delta(v))z)=0 \quad \text{for all } u,v,z\in M_n(K).$$

By Lemma 2.2, we have $\delta(u+v)-\delta(u)-\delta(v)=0$. That is, $\delta(u+v)=\delta(u)+\delta(v)$ for all $u,v\in M_n(K)$.

Finally, for each $u\in M_n(K)$, there exists a derivation d_{u,u^2} such that $\delta(u)=d_{u,u^2}(u)$ and $\delta(u^2)=d_{u,u^2}(u^2)$. Then

$$\delta(u^2)=d_{u,u^2}(u^2)=d_{u,u^2}(u)u+ud_{u,u^2}(u)=\delta(u)u+u\delta(u) \quad \text{for all } u\in M_n(K).$$

Therefore, δ is a Jordan derivation. Since the characteristic of K is not 2, we see that δ is a derivation by the Herstein theorem [**12**]. □

3. A generalization of Hua's theorem

In 1949, Hua [**14**] proved that every bijective additive map α on a division ring K satisfying $\alpha(aba)=\alpha(a)\alpha(b)\alpha(a)$ and $\alpha(1)=1$ is an automorphism or an anti-automorphism. This result was reformulated by Artin as: any bijective additive map α on a division ring K satisfying $\alpha(a^{-1})=\alpha(a)^{-1}$ and $\alpha(1)=1$ is an automorphism or an anti-automorphism [**1**, Theorem 1.15]. The same result was established for the $n\times n$ matrix rings over a division ring K in case when $K\neq \mathrm{GF}(2)$, the Galois field of two elements [**10**]. In [**5**], the authors removed the condition of $\alpha(1)=1$ in Hua's result and prove the following.

Theorem 3.1 (Chebotar *et al.* [5, Theorem 2.1]). *Let K be a division ring with centre Z and $\alpha : K\to K$ be a bijective additive map such that*

$$\alpha(a^{-1})\alpha(a)=\alpha(b^{-1})\alpha(b) \quad \textit{for all non-zero } a,b\in K.$$

Then $\alpha=\lambda\varphi$, where $\varphi : K\to K$ is an automorphism or an anti-automorphism and $\lambda=\alpha(1)\in Z$.

We shall generalize this result to matrix algebras over division rings. We begin with some technical results.

Lemma 3.2. *Let K be a division ring with centre Z such that $K\neq \mathrm{GF}(2)$ and let $M_n(K)$, $n\geqslant 2$, be the ring of $n\times n$ matrices over K. Suppose that $\alpha : M_n(K)\to M_n(K)$ is a surjective additive map. If $\mu\in M_n(K)$ satisfies $[\mu,\alpha(y)]=0$ for all invertible $y\in M_n(K)$ with $y-1$ invertible, then $\mu\in Z$.*

Proof. We claim first that $[\mu,\alpha(ke_{ij})]=0$ for all $k\in K$ and $1\leqslant i,j\leqslant n$. If $k=0$, then the above equality holds automatically. Let $0\neq k\in K$ and $1\leqslant i,j\leqslant n$. In the case when $i\neq j$, we pick $h\in K$ such that $h\neq 0,1$. Let $y_1=h+ke_{ij}$ and $y_2=h$; we find that y_l and y_l-1 are invertible and so $[\mu,\alpha(y_l)]=0$ for $l=1,2$. Therefore, $[\mu,\alpha(ke_{ij})]=[\mu,\alpha(y_1)-\alpha(y_2)]=0$. In the case when $i=j$, we consider $y=ke_{ii}+e_{12}+e_{23}+\cdots e_{n-1n}+e_{n1}$. Since y and $y-1$ are invertible, we have $[\mu,\alpha(y)]=0$. It follows from the above case that

$$0=[\mu,\alpha(e_{12})]=[\mu,\alpha(e_{23})]=\cdots=[\mu,\alpha(e_{n-1n})]=[\mu,\alpha(e_{n1})],$$

and so $[\mu, \alpha(ke_{ii})] = 0$. Since α is a surjective additive map, by the claim, we have $\mu \in Z$ as desired. □

Our next goal is the case when $K = \mathrm{GF}(2)$.

Lemma 3.3. *Suppose $K = \mathrm{GF}(2)$ and $n \geqslant 2$. Let α be a surjective additive map of $M_n(K)$ and $\mu \in M_n(K)$.*

(i) *If μ satisfies $[\mu, y] = 0$ for all invertible $y \in M_n(K)$, then $\mu \in K$.*

(ii) *If μ satisfies $[\mu, \alpha(y)] = 0$ for all invertible $y \in M_n(K)$, then $\mu \in K$.*

Proof. (i) Let $i \neq j$. By assumption, we have $[\mu, 1] = 0$ and $[\mu, 1 + e_{ij}] = 0$, and therefore $[\mu, e_{ij}] = 0$. Further, since

$$\Big[\mu, e_{ii} + e_{ij} + e_{ji} + \sum_{k\neq i,j} e_{kk}\Big] = 0 \quad \text{and} \quad \Big[\mu, e_{ij} + e_{ji} + \sum_{k\neq i,j} e_{kk}\Big] = 0,$$

it follows that $[\mu, e_{ii}] = 0$. Hence, $\mu \in K$ as desired.

(ii) Since α is additive, we can see from the above proof that $[\mu, \alpha(e_{ij})] = 0$ for all $1 \leqslant i, j \leqslant n$. From the fact that α is surjective and additive, it follows that $\mu \in K$. □

Proof of Theorem 1.3. Let $z = \alpha(1^{-1})\alpha(1) \neq 0$; then $z = \alpha(a^{-1})\alpha(a) = \alpha(a)\alpha(a^{-1})$ and so

$$\alpha(a)z = \alpha(a)(\alpha(a^{-1})\alpha(a)) = (\alpha(a)\alpha(a^{-1}))\alpha(a) = z\alpha(a)$$

for all invertible $a \in M_n(K)$. By Lemmas 3.2 and 3.3(ii), we have $z \in Z$.

Suppose first that $K \neq \mathrm{GF}(2)$. Let $\lambda = \alpha(1)$ and let $\varphi : M_n(K) \to M_n(K)$ be defined by $\varphi(a) = \lambda^{-1}\alpha(a)$ for all $a \in M_n(K)$. Then φ is a bijective additive map on $M_n(K)$ with $\varphi(1) = 1$. If we can claim $\lambda \in Z$, then we will have $\varphi(a^{-1})\varphi(a) = z^{-1}\alpha(a^{-1})\alpha(a) = z^{-1}z = 1$ for all invertible $a \in M_n(K)$. Hence, φ is an automorphism or an anti-automorphism in light of [**10**] and so the proof will be complete.

Let $x, y \in M_n(K)$ be invertible elements such that $x - y^{-1}$ is invertible. Thus, we have the following beautiful identity due to Hua:

$$(x^{-1} - (x - y^{-1})^{-1})^{-1} = x - xyx. \tag{3.1}$$

Set $x = 1$ and let y be an invertible element such that $y - 1$ is invertible (and hence $1 - y^{-1} = y^{-1}(y - 1)$ is invertible). Applying α to (3.1) and using $\alpha(a^{-1}) = z\alpha(a)^{-1}$, we obtain

$$\begin{aligned}\alpha(y) &= \lambda - \alpha((1-(1-y^{-1})^{-1})^{-1}) \\ &= \lambda - z\alpha(1-(1-y^{-1})^{-1})^{-1} \\ &= \lambda - z(\lambda - \alpha((1-y^{-1})^{-1}))^{-1} \\ &= \lambda - z(\lambda - z(\lambda - \alpha(y^{-1}))^{-1})^{-1} \\ &= \lambda - (\lambda^{-1} - (\lambda - z\alpha(y)^{-1})^{-1})^{-1} \\ &= \lambda - (\lambda^{-1} - (\lambda - (z^{-1}\alpha(y))^{-1})^{-1})^{-1} \\ &= \lambda - (\lambda - \lambda^{-1}\alpha(y)\lambda) \\ &= \lambda^{-1}\alpha(y)\lambda.\end{aligned}$$

Hence, $[\lambda, \alpha(y)] = 0$ for all invertible $y \in M_n(K)$ with $y - 1$ invertible. By Lemma 3.2, we have $\lambda \in Z$ as desired.

Suppose next that $K = Z = \mathrm{GF}(2)$. Let a be an invertible element of $M_n(K)$. It follows from $0 \neq z = \alpha(a)\alpha(a^{-1}) \in K$ that $\alpha(a)$ is invertible. Therefore, α is an invertibility-preserving map. Since α is a bijective map on the finite set $M_n(\mathrm{GF}(2))$, it maps singular matrices to singular matrices. It follows from Dieudonné's [**8**] result that α must have the form of $\alpha(X) = UXV$ or $\alpha(X) = UX^tV$, where $U, V \in M_n(K)$ are invertible and t is the transpose map.

Say $\alpha(X) = UXV$. Let a be an invertible element in $M_n(K)$. It follows from

$$\alpha(a^{-1})\alpha(a) = \alpha(1)^2$$

that $Ua^{-1}VUaV = UVUV$, i.e. $[VU, a] = 0$ for all invertible a. Therefore, $VU \in K$ by Lemma 3.3(i) and so $UV = VU$. Hence, we have $\alpha(1) = UV = VU \in K$ and $\alpha(X) = UXV = UV(V^{-1}XV) = \alpha(1)(V^{-1}XV)$ as desired. Similar arguments can be applied for the case $\alpha(X) = UX^tV$. The proof is completed. □

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