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REGULARITY PROPERTIES IN VARIATIONAL ANALYSIS AND APPLICATIONS IN OPTIMISATION

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Regularity properties lie at the core of variational analysis because of their importance for stability analysis of optimisation and variational problems, constraint qualifications, qualification conditions in coderivative and subdifferential calculus and convergence analysis of numerical algorithms [3–5, 7, 21, 23]. The thesis is devoted to investigation of several research questions related to regularity properties in variational analysis and their applications in convergence analysis and optimisation.

Following the works by Kruger [13, 14], we examine several useful regularity properties of collections of sets in both linear and Hölder-type settings and establish their characterisations and relationships to regularity properties of set-valued mappings [16, 17].

Following the recent publications by Lewis *et al.* [19, 20], Drusvyatskiy *et al.* [6] and some others, we study application of the uniform regularity and related properties of collections of sets to alternating projections for solving nonconvex feasibility problems and compare existing results on this topic [15, 18, 22].

Motivated by Ioffe [8] and his subsequent publications [9–11], we use the classical iteration scheme going back to Banach, Schauder, Lyusternik and Graves to establish criteria for regularity properties of set-valued mappings and compare this approach with the one based on the Ekeland variational principle [12].

Finally, following the recent works by Anh and Khanh [1] on stability analysis for optimisation-related problems, we investigate calmness of set-valued solution mappings of variational problems [2].

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