MOLLIN, R. A. *Quadratics* (CRC Press, Boca Raton–New York–London–Tokyo, 1996), xx + 387 pp., 0 8493 39839, $74.95.

This is an unusual and remarkable book about quadratic number fields and will evidently become the definitive modern reference work on the subject. It incorporates many results and techniques from the last decade and development is ongoing. The author along with his associates and students has been a major contributor to these advances, although we also single out the name of H. C. Williams in this context. The book is so stuffed with results that the reviewer has opted to use this space to try to convey the flavour of its contents rather than on comparisons, opinions and (minor) criticisms.

A quadratic number field has the form $\mathbb{Q}(\sqrt{D})$, where D is a square-free integer (positive or negative) and is usually labelled by its discriminant $\Delta(= D$ or $4D)$. It is *real* if $\Delta > 0$ and *complex* (a term preferred to the customary "imaginary") if $\Delta < 0$. Its ring of integers is $\mathbb{Z}[\omega_\Delta]$, where $\omega_\Delta = (1+\sqrt{D})/2$ or $\sqrt{D}$ and the minimal polynomial of $-\omega_\Delta$ (the "Euler–Rabinowitsch" polynomial) is denoted by F_Δ. The (ideal) class group is denoted by C_Δ with order the class number h_Δ. In particular the integers have unique factorisation if and only if $h_\Delta = 1$ and the "class number 1 problem" for a set of discriminants Δ relates to identifying those Δ in the set for which $h_\Delta = 1$, usually showing that the set is finite.

It is well-known that the quadratic polynomial $(F_{-163}(x) =)x^2 + x + 41$ returns prime values for each $x = 0, 1, \ldots, 39(= [|\Delta|/4 - 1])$. Many are indeed aware that this is connected with the facts that $h_{-163} = 1$ and that there are no larger (negative) Δ with such a property because the class number 1 problem for complex fields has been completely solved by Baker–Heegner–Stark in the 1960s. Prior to that it was known that there could be at most one more complex field with $h_\Delta = 1$ and this would yield a counterexample to the Generalised Riemann Hypothesis (GRH). The book analyses in great depth the structures of quadratic fields which give rise to such phenomena. However, instead of operating in terms of quadratic forms and genera (as has been standard practice), the author works with explicit descriptions of the ideals of the field and for real fields (which tend to be more difficult because of the presence of infinitely many units) uses the continued fraction expansions of associated elements. The reason is that in the end these more basic concepts lead to deeper results and they are more suited to algorithmic development and explicit computation.

In brief, an ideal I typically has the form $I = \langle a, b + c\omega_\Delta \rangle$, where c divides (a, b) and $N(I)$ (the norm of I) is ac. An ideal I is said to be *primitive* if $c = \pm 1$ and *reduced* if it contains no (non-zero) element α such that both α and its conjugate are less than $N(I)$ in absolute value. We then have, for example, that every class C_Δ contains a primitive ideal of norm less than the "Minkowski bound" M_Δ. Moreover, if $\Delta < 0$, every ideal class contains a unique reduced ideal or a conjugate pair. If $\Delta > 0$, there are correspondences between quadratic irrationals and primitive ideals and between such irrationals with purely periodic expansions and reduced ideals. Moreover, given a primitive ideal, there is a cycle of reduced ideals in the same class (corresponding to irrationals of the form $\gamma_i = (P_i + \sqrt{D})/Q_i$) which constitutes the all-important *infrastructure* of the class (a term coined by D. Shanks). What we are endeavouring to draw from the above (inadequate) summary is that from such concrete and comprehensible concepts the author derives a plethora of highly detailed results.

We illustrate with specimens drawn from various chapters. By means of the infrastructure Diophantine equations such as $x^2 - Dy^2 = -3$ $(D > 0)$ can be completely solved (§3.5). Chapter 4 contains a full account of prime producing polynomials, both real and complex. Further, if $\Delta < 0$, then $h_\Delta = 2$ if and only if $F(\Delta) = 2$ (§4.1), where $F(\Delta)$ is the maximum number of prime factors of $F_\Delta(x)$ for $0 \le x \le [|\Delta|/4 - 1]$. Though results with $\Delta > 0$ are generally more difficult, progress on discriminants of "ERD-type" (for which $D = s^2 + r, r|4s$) has been greater. For example, relating to Chowla's conjecture (1976), the class number 1 problem for such discriminants has been solved with a set of 43 values of Δ with $h_\Delta = 1$ plus one further possible exception which would be a counterexample to GRH. An elementary proof (using quadratic residue covers) is given of the conjecture of Shanks (1969) on the class number 1 question for

$\Delta(>0) \equiv 1 \pmod 8$ for which the Q_i are powers of 2 (§7.1). An important key exchange system of Buchmann and Williams in cryptology uses the infrastructure of real quadratic fields and large values of h_Δ (§8.2).

We briefly survey further features of this volume. There are eighty pages of tables on fundamental units, class numbers and the structure of the class group for $D < 2000$. There are also sections on the significance of the GRH and the philosophy of computer-assisted proofs as well as appendices on analytic material and the conventional theory of quadratic forms. Throughout one becomes aware of the advantages that the power of modern computing provides in terms of insights that can lead to proofs of results which might otherwise have been unsuspected or in some cases yield counterexamples to what had seemed reasonable conjectures. On the other hand many plausible conjectures and questions are stated which remain to be settled and might serve to motivate interested readers.

With its many footnotes giving items of mathematical, historical and personal interest (some anecdotal), its full sets of references and examples and its wealth of curiosities and numerical facts, this volume is bound to be stimulating for expert and student alike. Certainly it is a browser's delight. Nevertheless, from an early stage (for example, Theorem 1.3.2 on page 16) it touches on the frontiers of knowledge and in many places it is a tour along such frontiers, so that a serious reader should not expect to proceed quickly. It will certainly become an invaluable handbook on "quadratics".

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RAMM, A. G. and KATSEVICH, A. I. *The Radon transform and local tomography* (CRC Press, Boca Raton–New York–London–Tokyo, 1996), xviii + 485 pp., 0 8493 9492 9, $79.95.

The Radon transform of a function $f : \mathbb{R}^2 \to \mathbb{R}$ gives the line integrals of f along straight lines in the plane. Thus the Radon transform $\hat{f}$: {lines in $\mathbb{R}^2$} $\to \mathbb{R}$ is defined as

$$\hat{f}(\ell) = \int_\ell f(x) d\ell$$

and there are obvious extensions to integrals of functions on $\mathbb{R}^n$ over translates of subspaces of a given dimension. A natural problem to ask is: how can one recover f knowing $\hat{f}$ or, in other words, what is the inverse of the transform? An inversion formula was published by Johann Radon in 1917, but this attracted little interest for many years.

It was in the 1970s with the development of X-ray and computer technology that the Radon transform and its inversion became enormously important. Given a section of a head or body, the local density of X-ray absorption at the point x, given by $f(x)$ say, varies considerably between different types of bone and tissue. The attenuation of X-rays along a line represents the integral of f along the line and this can be measured using X-ray photography. Thus a large number of X-ray photographs taken in different directions enable the Radon transform of f to be estimated. Inversion of the transform gives f, providing a "map" of the head or body section that enables tumours, blood clots, etc. to be detected. The development of a practical scanner to implement this idea led to the Nobel prize in medicine being awarded to Cormack and Hounsfield in 1979 and to the birth of the science of tomography.

Since then an enormous amount of theoretical and practical mathematics of tomography has been developed, with functional analysis and Fourier transforms playing an important role. Many problems arise: for example practical data is discrete rather than continuous, it is more convenient to work with point source rather than parallel beam X-rays, the inversion problem is ill-posed given a finite set of X-ray photographs, and the data is often incomplete – for example the data may be known for only a limited cone of directions.

In the first few chapters this book presents standard tomography theory, including results on the range and uniqueness of the Radon transform, inversion formulae and reconstruction